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FINITE N=1 SUPERSYMMETRIC GRAND UNIFIED THEORIES

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1. INTRODUCTION

Modern elementary-particle physics is spread by the idea of symmetry. According to the common opinion the symmetry increases with increasing energy or decreasing distance. Grand-unification hypothesis is based on the assumption that strong, weak and electromagnetic interactions are unified into a single interaction with a simple gauge group. However, the standard GUT's leave many unresolved questions and undefined parameters. In the absence of direct experimental tests the main criterion for the theory to be correct becomes its aesthetic attractiveness and self-consistency. At the same time total unification should clearly include gravity. Here, however, we meet with serious problems, first of all the problem of divergences. Available versions of quantum gravity are nonrenormalizable.

The most promising scheme for consistent quantum theory including gravity is based on superstrings [1]. In this approach the fundamental object is a supersymmetric string, quantization of which requires a ten-dimensional space-time. The spectrum of its normal modes defines the spectrum of elementary particles of an effective local theory. The symmetry group of superstring theory is practically fixed: it is SO(32) or $E_8 \times E_8$. However, as a result of reduction from the 10-dimensional to 4-dimensional space-time symmetry is reduced and defined by the topological properties of a 6-dimensional compact manifold.

The superstring theory is probably free from ultraviolet divergences, which in turn may lead to a finite local theory on the Planck scale and further on to a finite GUT. However, numerous variants of compactification complicate the choice and give no unique way how to reduce the symmetry group of superstring theory to that of GUT's.

This problem can be approached from another side, from low energies, trying to construct a field theory without divergences. Such examples do already exist. They are based on extended supersymmetry [2] and historically have arised through the compactification of type I superstring on a six-torus [3]. It has been realized that an effective low-energy theory is N = 4 supersymmetric Yang-Mills theory finite in all orders of perturbation theory [4]. Later on a class of finite N = 2 supersymmetric models has been found [5]. Unfortunately, they are unsatisfactory from a phenomenological point of view due to the presence of mirror partners of ordinary particles nonobserved experimentally.

The search for finite theories was continued. It has been shown [6] that N = 1 supersymmetric Yang-Mills theories finite at a one-loop level can be constructed. A complete classification of chiral N = 1 theories with a simple gauge group which satisfy a one-loop-finiteness condition has been given [7]. All these theories are described in terms of N = 1 superfields by a real superfield V together with a set of matter chiral superfields Φ^a. The requirement of finiteness restricts the number of these fields and their gauge group representation.

It is remarkable that the one-loop finiteness automatically leads to the absence of divergences at a two-loop level [6,8]. However, contrary to extended supersymmetry, absence of any known symmetry does not allow us to prove a nonrenormalization theorem [9] and generalize this result to all orders of perturbation theory. Analysis of the three-loop approximation has shown [10] that the gauge superfield propagator is also finite while the chiral one is in general divergent. Imposing further constraints on higher group Casimirs one can achieve finiteness at the three-loop level, however, nothing can be said about higher orders.

In the present paper we propose an algorithm to construct N = 1 supersymmetric Yang-Mills theory finite to all orders of perturbation theory. Necessary and sufficient conditions for finiteness are defined already at the one-loop level. The resulting Yukawa couplings represent some functions of the gauge one defined order by order of perturbation theory. As an example we consider the so-called N = 4d model. The realistic finite Grand unification theory is proposed based on N = 1 supersymmetric SU(5) model. The three-loop calculation of Yukawa couplings is performed.

2. ONE-LOOP FINITENESS

Consider a general renormalizable N = 1 supersymmetric gauge theory (we use the superfield notation of ref. [6]):

$$S = \int d^4x \Big[\int d^4\theta \bar{\Phi}_a (e^{gV})^a_b \Phi^b - \frac{\mathrm{tr}}{g^2 C_G} \int d^2\theta W^\alpha W_\alpha + \int d^2\theta \frac{1}{3!} d^{abc} \Phi^a \Phi^b \Phi^c + \text{h.c.} + \text{gauge-fixing} + \text{ghost} \Big]. \qquad (1)$$

A chiral superfield Φ^a is in a reducible representation R of the gauge group G. The index a is a multi-index, it runs over an irreducible representations A and members of a given irreducible representation s, i.e., $a = \{A, s\}$. Here $V^a_b = V^i (R_i)^a_b$ and $(R_i)^a_b = (R^A_i)^s_t$. Matrices of an irreducible representation satisfy the following conditions

$$[R_i, R_j] = i f_{ijk} R_k, \qquad R^a_{ib} R^b_{ic} = C_A \delta^a_c, \qquad (2)$$
$$R^a_{ib} R^b_{ja} = \delta_{ij} \sum_A T_A, \qquad f_{ijk} f_{\ell jk} = C_G \delta_{i\ell}.$$

Action (1) is invariant under G if

$$d_{abc} (R_i)^c_d + d_{dac} (R_i)^c_b + d_{bdc} (R_i)^c_a = 0,$$

where d_{abc} is totally symmetric in a, b and c.

The theory is finite in the one-loop approximation if the following constraints are fulfilled [6]:

$$\sum_A T_A = 3 C_G \quad \text{and} \quad S^E_A = \delta^E_A C_A, \qquad (3)$$

where S^E_A is defined by

$$d_{abc} \bar{d}^{ebc} = 2 S^e_a g^2 = 2\delta^r_s S^E_A g^2. \qquad (4)$$

We note that these results were obtained by using N = 1 superfields and the background-gauge formalism. In this case the problem is reduced to the calculation of propagators of the fields present in action (1). Finiteness of propagators then means the absence of infinite charge renormalization and finiteness of all amplitudes.

Equation (4) in fact leads to a single coupling constant theory where all Yukawa couplings are equal to the gauge one. Indeed, picking out a purely tensorial structure corresponding to a concrete realization of the chiral superfield interaction, we get from eq. (4) that Yukawa couplings d_i are proportional to g. However, this equality may fail in higher orders due to the renormalization if it is not fixed by some symmetry. In general, the gauge coupling g and Yukawa ones d_i are renormalized in a different manner. In the three-loop approximation the gauge β-function vanishes when eqs. (3) and (4) are fulfilled, while that of the Yukawa coupling differs from zero [10,11]. This means that in general we have a set of couplings to be taken into account in the analysis of divergences. The requirement of finiteness in all orders of perturbation theory reduces the arbitrariness in the choice of Yukawa couplings expressing them as functions of g.

3. AN ALGORITHM TO CONSTRUCT A FINITE THEORY

As has been mentioned, the theory is finite if this is true for field propagators, i.e., all the anomalous dimensions of the fields vanish. Consider the anomalous dimensions of the gauge superfield V and independently renormalized chiral superfields Φ_i in a general multicoupling situation. Introducing the notation $h_i \equiv d_i^2/16\pi^2$, $a \equiv g^2/16\pi^2$ we have

$$\gamma_a(a, \{h_i\}) = A_{10}a + A_{20}a^2 + \sum_i A_{2i}h_i a + A_{30}a^3 + \sum_i A_{3i}h_i a^2 + \sum_{i,j} A_{3ij}h_i h_j a + \dots , \tag{5}$$

$$\begin{aligned}\gamma_i(a, \{h_i\}) = {} & B^i_{10}a + \sum_j B^i_{1j}h_j + \\ & + B^i_{20}a^2 + \sum_j B^i_{2j}h_j a + \sum_{j,k} B^i_{2jk}h_j h_k + \\ & + B^i_{30}a^3 + \sum_j B^i_{3j}h_j a^2 + \sum_{j,k} B^i_{3jk}h_j h_k a + \sum_{j,k,\ell} B^i_{3jk\ell}h_j h_k h_\ell + \dots .\end{aligned} \tag{6}$$

We are interested in vanishing the r.h.s. of eqs.(5) and (6). As far as the coefficients A and B are nonzero, this is possible only if h_i are some functions of a.

The problem is: to choose the Yukawa couplings h_i in the form

$$h_i = \alpha_i a + \beta_i a^2 + \gamma_i a^3 + \dots \tag{7}$$

in order to provide all the anomalous dimensions to vanish in all orders of pertirbation theory. We will show below that this problem is solvable [12].

Starting from the two-loop approximation the coefficients of eqs.(5) and (6) are scheme-dependent. This is also true when due to eqs.(3), (4) the one- and two-loop anomalous dimensions vanish. In this sense eqs.(7) can be considered as a transition to another renormalization scheme such as $h_i = h_i(\tilde{a}, \tilde{h}_i)$, $a = a(\tilde{a}, \tilde{h}_i)$, and the equality $\tilde{h}_i = \alpha_i \tilde{a}$ leads to the cancellation of divergences in all orders of perturbation theory. Hence, the coefficients β_i, γ_i, etc., in eqs.(7) depend on the calculation procedure and in some schemes (regularizations) may vanish. This happens when the regularization does not break the symmetry responsible for the divergence cancellation. And vice versa, nonzero coefficients may lead to the restoration of symmetry if the intermediate regularization breaks it. Such an example will be considered below.

Substituting now h_i given by eqs.(7) into eqs.(5), (6), we get

$$\begin{aligned}\gamma_a(a) = {} & A_{10}a + (A_{20} + \sum_i A_{2i}\alpha_i)a^2 + \\ & + (A_{30} + \sum_i A_{3i}\alpha_i + \sum_{i,j} A_{3ij}\alpha_i\alpha_j + \sum_i A_{2i}\beta_i)a^3 + \dots ,\end{aligned} \tag{8}$$

$$\begin{aligned}\gamma_i(a) = {} & (B^i_{10} + \sum_j B^i_{1j}\alpha_j)a + \\ & + (B^i_{20} + \sum_j B^i_{2j}\alpha_j + \sum_{j,k} B^i_{2jk}\alpha_j\alpha_k + \sum_j B^i_{1j}\beta_j)a^2 + \\ & + (B^i_{30} + \sum_j B^i_{3j}\alpha_j + \sum_{j,k} B^i_{3jk}\alpha_j\alpha_k + \sum_{j,k,\ell} B^i_{3jk\ell}\alpha_j\alpha_k\alpha_\ell + \\ & + \sum_j B^i_{2j}\beta_j + \sum_{j,k} B^i_{2jk}(\alpha_j\beta_k + \alpha_k\beta_j) + \sum_j B^i_{1j}\gamma_j)a^3 + \dots .\end{aligned} \tag{9}$$

The requirement of vanishing of γ_a and all γ_i then leads to the equations:

1 loop:

$$A_{10} = 0. \tag{10a}$$

$$B^i_{10} + \sum_j B^i_{1j}\alpha_j = 0. \tag{10b}$$

These eqs. are nothing else than eqs. (3) and (4). Equation (10a) is satisfied by an appropriate choice of matter fields and their representations (3), and eqs. (10b) determine the coefficients α_i. This system of linear equations has a solution when the number of eqs., i.e., the number of independently renormalized fields, does not exceed that of Yukawa couplings. In the case of equality the matrix B_1 becomes a square one, and the solution is unique if it is nonsingular, i.e., $\det B_1 \neq 0$.

2 loops:

$$A_{20} + \sum_i A_{2i}\alpha_i = 0, \tag{11a}$$

$$B^i_{20} + \sum_j B^i_{2j}\alpha_j + \sum_{j,k} B^i_{2jk}\alpha_j\alpha_k + \sum_j B^i_{1j}\beta_j = 0. \tag{11b}$$

Equation (11a) looks like a new constraint on α_i, but it is not so. It is satisfied automatically for α_i determined from eqs. (10b). This fact has been checked by direct computation [6], and it is a consequence of a general theorem [13] which states:

If an N = 1 supersymmetric gauge theory is finite at L-loops (i.e., all the Green functions are finite), then the gauge propagator is finite in (L + 1) loop, i.e., the (L + 1)-loop anomalous dimension of the gauge field vanishes.

Equations (11b) define the coefficients β_i. The existence of a solution here is guaranteed by that of eqs. (10b), because the homogeneous part contains the same matrix B_1. No new constraint arises. Since according to [6,8] the one-loop finiteness automatically leads to the cancellation of two-loop divergences, all the coefficients $\beta_i = 0$.

3 loops:

$$A_{30} + \sum_i A_{3i}\alpha_i + \sum_{i,j} A_{3ij}\alpha_i\alpha_j + \sum_i A_{2i}\beta_i = 0, \tag{12a}$$

$$B^i_{30} + \sum_j B^i_{3j}\alpha_j + \sum_{j,k} B^i_{3jk}\alpha_j\alpha_k + \sum_{j,k,\ell} B^i_{3jk\ell}\alpha_j\alpha_k\alpha_\ell + \sum_j B^i_{2j}\beta_j + \sum_{j,k} B^i_{2jk}(\alpha_j\beta_k + \alpha_k\beta_j) + \sum_j B^i_{1j}\gamma_j = 0. \tag{12b}$$

The situation here is entirely like that of two-loops. The first eq. is satisfied automatically due to the mentioned theorem. Direct calculations have also been performed[10]: Equations (12b) give us the coefficients γ_i, and again we have the same matrix B_1.

Obviously, the same mechanism will take place in all orders. Choosing h_i as functions of a (like eqs. (7)), one can always make anomalous dimensions to vanish; and this, in turn, means the finiteness of the theory in all orders of perturbation theory, i.e., all the physical amplitudes calculated in an arbitrary beforehand known order will be finite. In any given regularization, even if it breaks supersymmetry, the coefficients $\alpha_i, \beta_i, \gamma_i$, etc., once calculated will lead to the cancellation of all divergences. Regularization dependence will cancel in the expressions for physical observables.

Thus, the necessary and sufficient condition for finiteness is the existence of a solution of eqs. (10): a) eqs. (3) and (4) should be fulfilled; b) the number of independently renormalized fields should not exceed the number of Yukawa couplings; i.e., the matrix B_1 should be nonsingular.

4. THE RESTORATION OF SYMMETRY BY THE YUKAWA-COUPLING FINE-TUNING

The fact that to provide finiteness, we need the fine tuning of Yukawa couplings in every order of pertirbation theory may provoke the feeling of dissatisfaction, however, this situation is not new. The same tuning helps to restore the symmetry of a given classical Lagrangian at the quantum level if it is broken by regularization. Consider an example of the restoration of extended supersymmetry in the component field formulation of N = 4 supersymmetric Yang-Mills theory. Here the use of ordinary dimensional regularization obviously breaks supersymmetry because transition to $4-2\epsilon$ dimensions breaks the balance between bosonic and fermionic degrees of freedom. This leads to nonvanishing of the Yukawa β-function in the two-loop approximation[14]. At the same time the gauge β-function that should be equal to the Yukawa one due to the N = 4 invariance does vanish and gets a nonzero contribution only at the three-loop level. If in this situation one restores a broken supersymmetry by fine tuning of the Yukawa coupling, i.e.; puts $h = a + \frac{2}{3}a^2$, then the two-loop Yukawa β-function vanishes, and hence according to the theorem[13] there vanishes the three-loop gauge β-function. This fact has been checked in refs.[15]. Thus, a fine tuning of the couplings provides the restoration of supersymmetry and leads to a finite supersymmetric theory.

5. N = 4d MODEL

As an example illustrating the algorithm proposed to construct finite theories we consider the so-called N = 4d model[10]. This model contains the same set of fields as N = 4 supersymmetric Yang-Mills theory, namely, one gauge superfield V and three chiral superfields Φ^A in the ajoint representation of a gauge group. The difference from N = 4 theory is that instead of a f-type Yukawa interaction here is a d-type coupling. The superpotential is

$$W_Y = \frac{\lambda}{3!}\int d^4x\, d^2\theta \sum_{A=1}^{3} d_{ijk}\,\Phi^i_A\Phi^j_A\Phi^k_A . \tag{13}$$

Here d_{ijk} is a totally symmetric group structure constant. For definiteness we shall consider the group SU(n), $n \geq 3$.

Realization of the proposed algorithm for the N = 4d model looks as follows:

(i) Use of the properties of d_{ijk} in SU(n) group to calculate the anomalous dimensions of the fields V and Φ_A.

(ii) Substitution of $h \equiv \lambda^2/16\pi^2$ in the form of eq. (7)

$$h = \alpha a + \beta a^2 + \gamma a^3 + \ldots$$

into the expressions for anomalous dimensions (8,9) to determine the coefficients α, β, γ, etc.

Based on the three-loop calculations of ref. [10] we realize the proposed program up to the three-loop level. The result is

$$h = \frac{2n^2}{n^2-4} a \left(1 - \frac{48n^2(n^2-10)}{(n^2-4)^2} \zeta a^2 + \ldots\right), \tag{14}$$

where n refers to the group SU(n), and $\zeta \equiv \zeta(3)$ is the Riemanian ζ-function. We assert that all the amplitudes calculated within the N = 4d model by using the dimensional regularization with Yukawa coupling (14) will be finite up to the three-loop level. The question about any symmetry of the N = 4d model responsible for its finiteness is still open.

6. FINITE REALISTIC SUPERSYMMETRIC SU(5) MODEL

We apply the proposed method for constructing a finite Grand unified theory based on a supersymmetric SU(5) model. As has been mentioned, a complete classification of the models satisfying the one-loop finiteness condition (3,4) has been given in ref. [7]. It presents a model based on SU(5) gauge group with five types of chiral super-multiplets: 5, $\bar{5}$, 10, $\overline{10}$ and 24. If one additionally requires:

a) at least, three families of quarks and leptons without mirror partners,

b) necessary elementary Higgs fields that can be arranged to break SU(5) gauge group up to $SU(3)\times SU(2)\times U(1)$ and subsequently to $SU(3)\times U(1)$,

c) the absence of axial anomalies,

there arises a set of fields characterized by the weights of each multiplet, respectively: (4, 7, 3, 0, 1). This example was considered in detail in ref. [16,17]. The available chiral fields can be assigned as follows: 24 are Higgs fields which will break SU(5) down to $SU(3)\times SU(2)\times U(1)$; $4(5+\bar{5})$ are also Higgs fields the part of which will be used for breaking the electroweak gauge group; the remaining fields $3(\bar{5}+10)$ are identified with matter fields.

The fields are labelled as follows [16]

a) matter fields

$$\bar{5}:\ \Psi_{\alpha i}, \quad i = 1, 2, 3$$

$$10:\ \Lambda_i^{\alpha\beta}, \quad i = 1, 2, 3$$

b) Higgs fields

$$5:\ \Phi_a^{\alpha}, \quad a = 1, 2, 3, 4$$

$$\bar{5}:\ \tilde{\Phi}_{\alpha a}, \quad a = 1, 2, 3, 4$$

$$24:\ \Sigma_\beta^\alpha .$$

Here $\alpha, \beta = 1, \ldots, 5$ are the SU(5) indices. The matrix Σ is traceless, and Λ is antisymmetric.

In the general case the Yukawa type interaction can be described by the following superpotential:

$$\begin{aligned} W_Y = {} & A_{ij}^a \tilde{\Phi}_{a\alpha} \Psi_{i\beta} \Lambda_j^{\alpha\beta} + \frac{1}{8} B_{ij}^a \Phi_a^\alpha \Lambda_i^{\beta\gamma} \Lambda_i^{\delta\sigma} \epsilon^{\alpha\beta\gamma\delta\sigma} + \\ & + C_{ab} \tilde{\Phi}_{a\alpha} \Phi_b^\beta \Sigma_\beta^\alpha + \frac{1}{3} D \Sigma_\beta^\alpha \Sigma_\gamma^\beta \Sigma_\alpha^\gamma + \frac{1}{2} E_{ab}^1 \tilde{\Phi}_{a\alpha} \tilde{\Phi}_{b\beta} \Lambda_i^{\alpha\beta} + \\ & + F_{ia} \Psi_{i\alpha} \Phi_a^\beta \Sigma_\beta^\alpha + \frac{1}{2} G_{ij}^k \Psi_{i\alpha} \Psi_{j\beta} \Lambda_k^{\alpha\beta}, \end{aligned} \tag{15}$$

where $\Sigma_\beta^\alpha = \frac{1}{\sqrt{2}} (\lambda^k)_\beta^\alpha \Sigma^k$, λ^k is an SU(5) matrix, and summation over repeated indices is understood. The last three terms would lead to a large violation of the baryon and lepton number at the tree-level and to a B-L nonconservation and usually are ignored.

The one-loop finiteness conditions are*

$$\begin{aligned} \tilde{\Phi}: \quad & 4 \sum_{i,j} A_{ij}^a \overset{*}{A}{}_{ij}^b + \frac{24}{5} \sum_e C_{ae} \overset{*}{C}_{be} + 4 \sum_{i,e} E_{ae}^i \overset{*}{E}{}_{be}^i = \frac{12}{5} g^2 \delta_{ab}, \\ \Phi: \quad & 3 \sum_{i,j} B_{ij}^a \overset{*}{B}{}_{ij}^b + \frac{24}{5} \sum_e C_{ea} \overset{*}{C}_{eb} + \frac{24}{5} \sum_i F_{ia} \overset{*}{F}_{ib} = \frac{12}{5} g^2 \delta_{ab}, \end{aligned} \tag{16}$$

*According to ref. [6] we put $g = g_{SUSY} = \sqrt{2} g_{NON\text{-}SUSY}$.

$$\Psi:\quad 4\sum_{a,k} A^a_{ik}\overset{*}{A}{}^a_{jk} + \frac{24}{5}\sum_a F_{ia}\overset{*}{F}_{ja} + 4\sum_{\ell,k} G^k_{i\ell}\overset{*}{G}{}^k_{j\ell} = \frac{12}{5}g^2\delta_{ij},$$

$$\Lambda:\quad 2\sum_{a,k} A^a_{ki}\overset{*}{A}{}^a_{kj} + 3\sum_{a,k} B^a_{ik}\overset{*}{B}{}^a_{jk} + 2\sum_{a,b} E^i_{ab}\overset{*}{E}{}^j_{ab} + 2\sum_{k,\ell} G^i_{k\ell}\overset{*}{G}{}^j_{k\ell} = \frac{18}{5}g^2\delta_{ij}, \tag{16}$$

$$\Sigma:\quad \sum_{a,b} C_{ab}\overset{*}{C}_{ab} + \frac{21}{5}D\overset{*}{D} + \sum_{i,a} F_{ia}\overset{*}{F}_{ia} = 5g^2.$$

They leave rather large arbitrariness in the matrices A, B, C, D, E, F, G. In ref. [16,17] some specific examples of solutions to these equations have been considered. We discuss below some generalization of one of these examples which provides us with a theory finite to all orders.

Consider a superpotential

$$W_Y = d_1\sum_{i=1}^{3}\bar{\Phi}^\alpha_i\Psi^\beta_i\Lambda^{\alpha\beta}_i + \frac{d_2}{8}\sum_{i=1}^{3}\epsilon^{\alpha\beta\gamma\delta\sigma}\Phi^\alpha_i\Lambda^{\beta\gamma}_i\Lambda^{\delta\sigma}_i + d_3\bar{\Phi}^\alpha_4\Phi^\beta_4\Sigma^\alpha_\beta + \frac{d_4}{3}\Sigma^\alpha_\beta\Sigma^\beta_\gamma\Sigma^\gamma_\alpha + d_5\sum_{i=1}^{3}(\bar{\Phi}^\alpha_i + \Psi^\alpha_i)\Phi^\beta_i\Sigma^\alpha_\beta. \tag{17}$$

Superpotential (17), contrary to that of ref. [16], contains individual couplings for each interaction. To get a nonsingular matrix B_1 (10b), we have also to introduce the fifth interaction $\sim d_5$. The second term in this interaction leads to B-L nonconservation and should be suppressed. In our case the constant d_5 is not chosen arbitrary, rather it is defined from eq. (10b).

Superpotential (17) is chosen so that there are five arbitrary constants d_i and five independently renormalized fields, namely, $\Psi_i, \Phi_i, \Phi_4, \Lambda_i$ and Σ. The fields $\bar{\Phi}_i$ and $\bar{\Phi}_4$ are renormalized like Ψ_i and Φ_4, respectively, because they enter into the potential (17) in the same way and belong to the same representation of SU(5). Substituting eq. (17) into eq. (16) we get the following linear system of eqs. for a_i:

$$\begin{aligned} \Psi_i:&\quad 4a_1 + \frac{24}{5}a_5 = \frac{12}{5},\\ \Phi_i:&\quad 3a_2 + \frac{48}{5}a_5 = \frac{12}{5},\\ \Phi_4:&\quad \frac{24}{5}a_3 = \frac{12}{5},\\ \Lambda_i:&\quad 2a_1 + 3a_2 = \frac{18}{5},\\ \Sigma:&\quad a_3 + \frac{21}{5}a_4 + 2a_5 = 5. \end{aligned} \tag{18}$$

Thus, the matrix B_1 is nonsingular

$$B_1 = \begin{pmatrix} 4 & 0 & 0 & 0 & \frac{24}{5}\\ 0 & 3 & 0 & 0 & \frac{48}{5}\\ 0 & 0 & \frac{24}{5} & 0 & 0\\ 2 & 3 & 0 & 0 & 0\\ 0 & 0 & 1 & \frac{21}{5} & 2 \end{pmatrix}; \quad \det B_1 = \frac{72576}{25} \neq 0.$$

Solving eqs. (18) we get

$$a_1 = \frac{3}{5},\quad a_2 = \frac{4}{5},\quad a_3 = \frac{1}{2},\quad a_4 = \frac{15}{14},\quad a_5 = 0. \tag{19}$$

Hence, to provide one-loop finiteness, one does not need a B-L violating interaction in agreement with ref. [16,17].

Since the one-loop finiteness automatically leads to the two-loop one, all the $\beta_i = 0$.

In the three-loop approximation, using the expression for propagators [10,11], with the help of "SCHOONSCHIP" computer program [18], we get the following system of eqs. for γ_i:

$$\begin{aligned} &4\gamma_1 + \frac{24}{5}\gamma_5 = \frac{96}{25}K,\\ &3\gamma_2 + \frac{48}{5}\gamma_5 = -\frac{222}{125}K,\\ &\frac{24}{5}\gamma_3 = \frac{432}{25}K,\\ &2\gamma_1 + 3\gamma_2 = \frac{18}{125}K,\\ &\gamma_3 + \frac{21}{5}\gamma_4 + 2\gamma_5 = \left(\frac{621}{20} - \frac{1215}{49}\right)K,\quad K = 6\zeta(3). \end{aligned}$$

This gives

$$\gamma_1 = \frac{24}{25}K, \quad \gamma_3 = \frac{18}{5}K, \quad \gamma_5 = 0. \quad \gamma_2 = -\frac{74}{125}K, \quad \gamma_4 = \frac{867}{1372}K, \tag{20}$$

Thus, to provide finiteness of all amplitudes up to the three-loop level, one should connect the couplings h_i with a in the following manner:

$$h_1 = \frac{3}{5}a(1 + \frac{48}{5}\zeta(3)a^2), \quad h_2 = \frac{4}{5}a(1 - \frac{111}{25}\zeta(3)a^2), \tag{21}$$

$$h_3 = \frac{1}{2}a(1 + \frac{216}{5}\zeta(3)a^2), \quad h_4 = \frac{15}{14}a(1 + \frac{867}{245}\zeta(3)a^2), \quad h_5 = 0.$$

Note that at the three-loop level four couplings are sufficient for finiteness. Like in the one- and two-loop approximation $h_5 = 0$. This is due to the fact that anomalous dimensions of the fields Ψ, Φ and Λ up to three-loops are connected by the relation

$$\frac{1}{2}\gamma_\Psi + \gamma_\Phi = \gamma_\Lambda. \tag{22}$$

Combinatorical arguments based on that the first two terms in eq. (17) are symmetric in Ψ, Φ and Λ, probably, allow us to assert that eq. (22) is valid in all orders like for $\gamma_\Psi = \gamma_{\tilde{\Phi}}$. But even if it is not so, the B-L violating interaction is strongly suppressed.

Superpotential (17) can be generalized by introducing a mixing matrix of the Kobayashi-Maskawa type. For W_Y this would result in nondiagonal interaction like [16] $\sum_{i,j} K_{ij}\tilde{\Phi}_i\Psi_i\Lambda_j$ or $\sum_{i,j} K_{ij}\tilde{\Phi}_j\Psi_i\Lambda_j$, where K is some unitary matrix. In the first case together with the second term in (17) this leads to nondiagonal propagators in the three-loop order that does not allow us to achieve finiteness. The second case is free from this difficulty: the theory remains finite with eqs. (21) being unchanged.

So far we have considered the massless case. One can add mass terms. The most general supersymmetric mass terms are given by

$$W_M = d_3 M_{ab}\tilde{\Phi}_a\Phi_b + \frac{1}{2}d_4 M_0 \Sigma^2.$$

Here coefficients M_0 and M_{ab} have a dimension of mass and are not restricted by the finiteness requirement. The potential $W_Y + W_M$ is extremized by

$$\Sigma = M_0 \mathrm{diag}(2, 2, 2, -3, -3)$$

that breaks SU(5) to SU(3)×SU(2)×U(1) in the usual fashion.

The introduction of explicit soft supersymmetry breaking terms [17] is also possible. By providing the one-loop finiteness by an appropriate choice of supersymmetry breaking parameters, higher-loop finiteness can be achieved by fine-tuning of them as calculable functions of the gauge coupling a like eq. (7).

7. CONCLUSION

In conclusion we summarize some results:

i) We propose a method to construct finite field theories which provides us with a whole class of quantum field theory models without ultraviolet divergences. This class is not restricted by the known examples of N = 2 or N = 4 supersymmetries.

ii) The proposed finite unified supersymmetric theories, which can be called FUST's, are distinguished among all GUT's. They contain a fixed set of fields (a fixed number of families) and lead to strict connections between the amplitudes of different processes due to a single coupling constant.

iii)As has been mentioned in the introduction, reducing the superstring theory from a 10 to 4-dimensional space-time one can get a finite GUT with the gauge group depending on the way of compactification. It would be very interesting to find such a compactification that leads to a theory obtained by our method. Then, may be, it would become clear what kind of symmetry (if any) is responsible for the cancellation of divergences.

iv) The existence of a whole class of finite quantum field theory models with interaction brings us back to the foundations of quantum field theory. Absence of divergences allows us to make formal expressions meaningful in the interaction representation, such as the state vector, Hamiltonian, etc. This means that a nonformal meaning there acquire both the interaction representation and the Shrödinger representation. Thus, the well-known problem of quantum field theory, which is usually ignored in modern pragmatic approaches, finds its solution in finite theories.

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НЕТ ЛИ ПРОБЕЛОВ В ВАШЕЙ БИБЛИОТЕКЕ?

Вы можете получить по почте перечисленные ниже книги, если они не были заказаны ранее.

Ермушев А.В., Казаков Д.И., Тарасов О.В.

E2-86-17

Конечные теории великого обьединения с N = 1 суперсимметрией

Предложен алгоритм построения N = 1 суперсимметричных теорий Янга-Миллса, конечных во всех порядках теории возмущений. Необходимые и достаточные условия конечности определяются уже в однопетлевом приближении. Класс таких теорий достаточно широк и не исчерпывается N = 4 или N = 2 суперсимметричными моделями. Рассмотрен ряд примеров конечных теорий. Предлагается реалистическая конечная N = 1 суперсимметричная SU(5) модель Великого обьединения с тремя поколениями фермионов и необходимыми хиггсовскими мультиплетами.

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Finite N = 1 Supersymmetric Grand Unified Theories

We propose an algorithm to construct N = 1 supersymmetric Yang-Mills theories finite in all orders of perturbation theory. Necessary and sufficient conditions for finiteness are determined already in the one-loop approximation. The class of such theories is wide enough and not exhausted by N = 4 or N = 2 supersymmetric models. Several examples of finite theories are considered. Finite realistic N = 1 supersymmetric Grand Unifield theory is proposed which is based on SU(5) gauge group and contains three fermion families with necessary Higgs multiplets.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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