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ELECTROWEAK ONE-LOOP CORRECTIONS TO THE DECAY OF THE CHARGED VECTOR BOSON

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1. Introduction

A unique approach is emerging of precision calculations of electroweak radiative corrections to basic parameters and processes within the Glashow-Weinberg-Salam (GWS) theory /1,2/. It seems worthwile to treat the W-decay width within this approach as has recently been done for the Z-boson decay /3/. Whereas electroweak corrections to W-decays into two quarks have not been considered so far in the literature, we feel also some usefulness of a re-examination of decays into leptons /4,5/ [1]:

$$W \to \bar{l}\nu(\gamma), \quad l = e, \mu, \tau, \tag{1}$$

$$W \to q_u \bar{q}_d(\gamma), \quad q_u = u, c, \quad q_d = d, s, b. \tag{2}$$

The calculational scheme is defined as follows:

1. On mass shell renormalization combined with the relations

$$\sin^2\theta_W \equiv S_\theta^2 = 1 - \frac{M_W^2}{M_Z^2}, \quad g = e/S_\theta . \tag{3}$$

2. Numerical determination of the gauge boson sector through α, G_μ, M_Z. Especially, it follows that

$$M_W = M_Z\left[\frac{1}{2} + \frac{1}{2}\left(1 - \frac{4A^2}{M_Z^2}\right)^{1/2}\right]^{1/2}, \tag{4}$$

$$A = A_0\,(1-\delta r)^{-1/2}, \quad A_0 = \left(\frac{\pi\alpha}{\sqrt{2}G_\mu}\right)^{1/2}, \quad \delta r = \frac{\alpha}{4\pi}X . \tag{5}$$

The δr is shown in Fig. 1. It contains the electroweak one-loop corrections to muon decay /1,7/ and is also responsible for the radiative mass shift of the gauge boson masses $M_{W,Z}$ compared to $M_{W,Z}^{(0)}$ as being calculated in Born approximation from other observables.

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[1] Although we do not consider the production of heavy fermions, the inclusion of the mass-corrected phase-space factor /6/ would lead to a quite good description of such process based on the results presented here.

Alternatively, instead of M_Z one may choose the W-boson mass as an experimental imput and derive M_Z iteratively:

$$M_Z = M_W \left(1 - \frac{A^2}{M_W^2}\right)^{-1/2} \tag{4a}$$

3. The use of electroweak form factors allows a clear and simple interpretation of observables in the GWS theory without need of process-dependent changes of basic parameters of the theory.
4. Instead of the fine structure constant α, the μ-decay Fermi constant G_μ is used for the normalization of weak processes:

$$\frac{g^2}{8M_W^2} \equiv \frac{\pi\alpha}{2s_\Theta^2 M_W^2} = \frac{G_\mu}{\sqrt{2}}\left[1 - \delta r + O(\alpha^2)\right]. \tag{6}$$

The last point deserves some comment. The only independent coupling constant of the theory is chosen to be α, and it should be used to calculate cross sections and to define Born approximations. The δr perturbatively connects α and G_μ. The $\alpha \equiv \alpha(0)$ is a truly low energy coupling constant, whereas G_μ even if measured from a low energy process (e.g. muon decay) is connected with an energy scale of the order of $M_{Z,W}$, the masses of the exchanged particles which determine the dynamics of weak processes. Inserting into (6) instead of α (0) the $\alpha(M_Z) \approx 1/128$ drastically reduces the correction δr to some smaller $\delta r'$. So, G_μ is the preferred coupling for all weak processes. Along the lines described we calculate the electroweak form factors ρ^W of W-boson decays (1,2) as functions of m_t, the t-quark mass, and of M_H, the higgs boson mass. To be physically well-defined, hadronic partial widths should remain finite in the limit of vanishing masses of the produced light quarks (u, d, s). We take into account the undetected emission of one real photon without any cut corresponding to the proper definition of a width. Together with the loop corrections, this ensures an infrared finite result and, in accordance with the Kinoshita-Lee-Nauenberg theorem /8/, the absence of mass singularities. Masses of quarks of "intermediate weight" m_c and m_b, also may be neglected both due to the same reason and since their proper mass effects are very small, $m_b^2 / M_W^2 \approx 3\text{x}10^{-3}$. Therefore, we only take into account non-vanishing mass corrections from the top-quark which may be treated

within perturbative QCD /9/. For quark production we add the QCD-correction due to one-gluon exchange. In Section 2 the derived formulae are presented, and Section 3 contains numerical results. In the Appendix some details on bremsstrahlung and on the loop functions used may be found.

2. Formulae

The following matrix element corresponds to the diagrams of Fig. 2:

$$M_{ij} = g K_{ij} \mathcal{F}_{ij}^W \epsilon_\mu^W(q) \bar{u} \gamma_\mu (1+\gamma_5) u, \tag{7}$$

where K_{ij} is the Kobayashi-Maskawa mixing matrix ($K_{ij} = \delta_{ij}$ for leptons), $\epsilon_\mu^W(q)$ is W-boson polarization vector and the form factor $\mathcal{F}^W = 1 + \delta\mathcal{F}^W$ may be taken from /7/. Compared to Z-boson decays /3/, there are two differences. While Z-boson decay is described by two form factors $\mathcal{F}_{1,2}^Z$ in the approximation adopted here, for the decays (1,2) the full matrix element is projected by the W-boson couplings onto one structure $\gamma_\mu(1+\gamma_5)$. On the other hand, the gauge-invariant separation of photonic corrections from pure weak ones is possible in Z-boson decays but not in W-boson decays. Intrinsically, the electrodynamics of a charged, massive vector boson is well-defined only in conjunction with some other, broken gauge interaction. As a consequence, a gauge-invariant QED correction to W-decay cannot be separated. To get a physically meaningful result, one has to add to $|M|^2$ of (7) the bremsstrahlung contribution δf^b of Fig.3. Finally, the normalization g^2 of the widths will be replaced by G_μ (see(6)):

$$\Gamma_{ij}^W = |K_{ij}|^2 C_i \frac{G_\mu}{\sqrt{2}} \frac{M_W^3}{6\pi} \rho_{ij}^W \left[1 + \frac{\alpha_s(M_W)}{\pi} D_i\right], \tag{8}$$

$$\rho_{ij}^W = 1 - \delta r + \delta f_{ij}, \quad \delta f_{ij} = 2\delta\mathcal{F}_{ij}^W + \delta f_{ij}^b. \tag{9}$$

The electroweak radiative corrections in (8) are contained in one form factor ρ^W which measures the channel-dependent deviation of the coupling strength from G_μ. The explicit expression for ρ^W is derived in the Appendix. Into (8) we included the one-gluon-exchange correction whose strength is determined by the strong interaction constant $\alpha_s(M_W)$. For quark production is

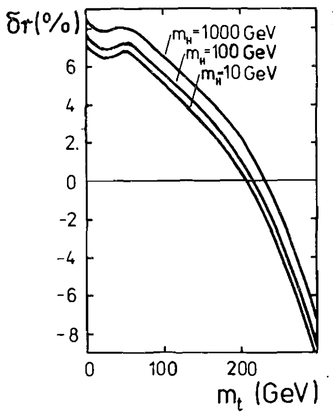
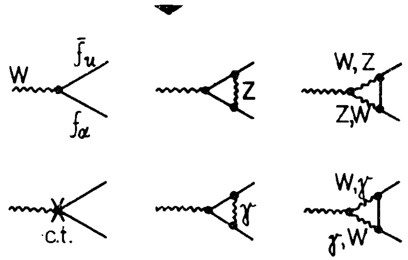


Fig. 1. The electroweak correction factor δr as function of m_t .

Fig. 2. Vertex corrections to W-decay into two light fermions in the unitary gauge.



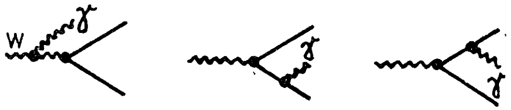


Fig. 3. Bremsstrahlung contributions to W-decay.

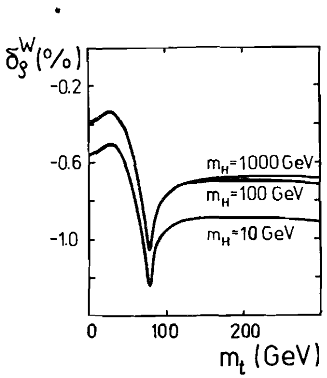


Fig. 4. The electroweak form factor $\delta\rho^W = \rho^W - 1$ for W-decay.

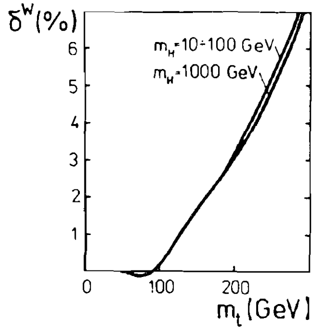


Fig. 5. The total dependence of Γ^W on m_t as defined in (15).

$C_i/3 = D_i = 1$ and for lepton production $C_i = 1$, $D_i = 0$. Since gluons are flavour-blind, these corrections are the same both in W- and Z-boson decays, and we may take them from the latter /6/.

3. Results

If not stated otherwise, the discussion will be based on the following set of parameters: M_Z = 93 GeV, m_t = 40 GeV, M_H = 100 GeV. From (4,5) one iteratively derives δr = 0.070 ± 0.002, A = 38.65 GeV and M_W = 82.03 GeV. A systematic tabulation of M_W for a wide range of parameters may be found in /10/. The error quoted for δr is due to the hadronic vacuum polarization /10/. The error correlation between M_Z , M_W , and A may be derived most simply from the relation $s_\theta M_W = A$, an expression equivalent to (4,4a):

$$(M_W^2 - A^2)\, dM_Z^2 - (2M_W^2 - M_Z^2)\, dM_W^2 = M_Z^2\, dA^2,$$

$$dA = \frac{1}{2} A_0\, d(\delta r). \tag{10}$$

An immediate consequence of (10) is that for an experimental error of δM_Z = 100 MeV as is expected from e^+e^--annihilation near the Z-boson pole at SLC/LEP, the uncertainty in M_W is mainly due to δM_Z [1]:

$$\delta M_W = \frac{M_W^2 - A^2}{2M_W^2 - M_Z^2}\,\frac{M_Z}{M_W}\,\delta M_Z = 1.2\,\delta M_Z . \tag{11}$$

Equation (11) estimates the influence of an experimental uncertainty in M_Z on the theoretical prediction of Γ^W , namely for δM_Z = 100 MeV: $\delta\Gamma^W/\Gamma^W = 3\delta M^W/M^W = 0.45\%$.

In Born approximation, both the leptonic and hadronic decay channels of the W-boson have equal probability (up to the factor $|K_{ij}|^2 C_i$):

$$\Gamma^W_{0ij} = |K_{ij}|^2 C_i\, \frac{G_\mu}{\sqrt{2}}\, \frac{M_W^3}{6\pi} = |K_{ij}|^2 C_i \left(\frac{M_W}{82.0}\right)^3 0.242\,\mathrm{GeV}. \tag{12}$$

[1] For δM_Z = 50 MeV the error of A has to be taken into account too.

In case of three generations, the total width becomes

$$\Gamma^{W}_{0,tot} = \Gamma^{W}_{0,t} + \left(\frac{M_W}{82.0}\right)^3 2.174\,\mathrm{GeV}, \tag{13}$$

where we have separated out the decay rate into final states containing a t-quark, since m_t is yet unknown.

The discussion of radiative corrections in the scheme chosen has to comprise the influence of M_W and ρ^W, both of them being dependent on m_t and M_H (of course, m_t is representative for any quark doublet with a large mass splitting). For given M_W, the one-loop corrections are contained in ρ^W shown in Fig. 4. As may be explicitly seen from (9, A.4-6), the influence of the charges of the final particles is extremely small so that we may neglect the channel-dependence of electroweak corrections henceforth:

$$\rho^{W}_{\nu\ell} - \rho^{W}_{q_u \bar{q}_d} = 0.046\,\% \,. \tag{14}$$

The magnitude of $\delta\rho = \rho - 1$ is typically of an order of 0.7% . As has been expected in the Introduction, it is really small compared to $\delta r \simeq$ 7% and $\delta\rho^W \simeq$ 6% and is due to the use of G_μ instead of α (0).

An interesting feature of observables is their dependence on yet undetermined parameters of the theory, the masses m_t and M_H. A smooth dependence of the form factor ρ^W on M_H is negligible in the interval 100-1000 GeV, and is weak between 10 and 100 GeV (smaller than 0.1%). The influence of m_t is more interesting as may be seen from Fig. 4. Whereas $\delta\rho^W$ approaches a nearly constant value of -0.8% for large m_t, one observes a sharp peak at about 82 GeV; the t-quark mass has its largest influence for comparably small values. This may easily be understood. As a result of the renormalization procedure, certain self-energy functions and their derivatives contribute to M_W, ρ^W, Γ^W. Those of them which represent loops with t-quark exchange show some threshold behaviour near $m_t = M_Z/2$ and $m_t = M_W$ (for $m_b = 0$) leading to the peak of ρ^W (see also the Appendix). The Γ^W-dependence on m_t is as follows:

$$\frac{d\Gamma^W}{\Gamma^W} = 3\,\frac{dM_W}{M_W} + d\rho^W = -\frac{3}{2}\,\frac{M_Z^2 A_0 A}{M_W^2 (2M_W^2 - M_Z^2)}\, d(\delta r) + d\rho^W$$
$$= -0.58\, d(\delta r) + d\rho^W. \tag{15}$$

In Fig. 5 we plot

$$\delta^W = \frac{\Gamma^W(m_t, M_H) - \Gamma^W(40\,\mathrm{GeV}, M_H)}{\Gamma^W(40\,\mathrm{GeV}, M_H)} \,. \tag{16}$$

This quantity exhibits the combined dependence of Γ^W through M_W and ρ^W on m_t, M_H at fixed M_Z. In contrast to the form factor ρ^W, δ^W is nearly independent of m_t up to m_t = 100 GeV and then strongly raises due to δr. Evidently, the electroweak radiative corrections are not necessarily small in the scheme chosen here. The best prospects to measure Γ^W are in the reaction $e^+e^- \longrightarrow W^+W^-$ at the threshold where the shape of the cross section is determined mainly by $(E - E_{thr})/\Gamma^W$ /11/. This reaction also allows a precise determination of M_W, may be, with even better accuracy than that of M_Z /12/. Then, one should use M_W as an experimental input and calculate M_Z from Eq. 4a. Within such an approach all the radiative corrections in W-decay are contained in ρ^W, which simplifies the above analysis considerably. As a consequence, now they are small (not exceeding 1.5%) even for quite large masses of the t-quark and the higgs boson. They are comparable to the uncertainty from the gauge boson masses (at δM^W = 100 MeV, $\delta\Gamma^W/\Gamma^W \simeq$ 0.5%) and, of relevance in the quark channel, to the error in QCD-corrections ($\delta\alpha_s/\alpha_s \simeq$ 20% for $\alpha_s/\pi \simeq$ 5%, yielding to a $\delta\Gamma^W/\Gamma^W \simeq$ 0.8%). A plot of Γ^W against M_W, with parameter m_t, has been shown for the leptonic decay channel in /5/. Strictly speaking, this may be compared to our results only when we adopt M_W as an independent input quantity since $\rho^W[M_Z, M_W(M_Z, m_t, M_H); m_t, M_H]$ is different from $\rho^W[M_Z(M_W, m_t, M_H), M_W, m_t, M_H]$. But since this difference is $O(\alpha^2)$, one may neglect it. Really, we agree with /5/ within the accuracy of their figures.

To summarize, in a slightly different and more transparent representation we reproduce the results of /5/ for the leptonic and show additionally for the quarkonic decay channels of the W-boson that electroweak radiative corrections are below one percent for a wide range of m_t, M_H if Γ^W is written in terms of M_W, G_μ.

This is smaller than the expected experimental accuracy for Γ^W so that the Born approximation (12,13) remains applicable with high precision.

Appendix

Here we present the expressions contributing to the electroweak form factor ρ^W:

$$\rho^W = 1 - \delta r + 2\delta\mathcal{F}^W + \delta f^b. \tag{A.1}$$

The bremsstrahlung of Fig. 2 for the process $W \to f_u \bar{f}_d \gamma$ has to be integrated over the complete photon phase space. The matrix element is

$$M_b = \bar{u}(K_u)\Big\{ |Q_d|\, O_\mu \frac{2K_{d\alpha} + \hat{p}\gamma_\alpha}{2K_d p} + |Q_u| \frac{2K_{u\alpha} + \gamma_\alpha \hat{p}}{2K_u p} O_\mu -$$

$$- \frac{1}{q\cdot p}\Big[q_\alpha \delta_{\nu\mu} + p_\mu \delta_{\alpha\nu} - \big(p_\nu + a(q_\nu - p_\nu)/2\big)\delta_{\alpha\mu}\Big]\Big\} u(K_d)\, \epsilon_\alpha^\gamma(p)\, \epsilon_\mu^W(q), \tag{A.2}$$

where $a = 0$ (1) corresponds to the unitary ('t Hooft - Feynman) gauge and $O_\mu = \gamma_\mu(1+\gamma_5)$. The calculation has been done using SCHOONSCHIP /13/. It is a nontrivial but a straight-forward application of the methods developed in /14/. The result is the same in both gauges:

$$\delta f^b = \frac{\alpha}{\pi}\Big\{ P_{IR}\Big[-(1 + Q_u^2 + Q_d^2) + Q_u^2 \ln\frac{M_W^2}{m_u^2} + Q_d^2 \ln\frac{M_W^2}{m_d^2}\Big] +$$

$$+ \frac{77}{24} + \frac{11}{4} Q_u Q_d - \frac{\pi^2}{3}(Q_u^2 + Q_d^2) - \frac{1}{4}\Big(Q_u^2 \ln^2\frac{M_W^2}{m_u^2} + Q_d^2 \ln^2\frac{M_W^2}{m_d^2}\Big)\Big\}. \tag{A.3}$$

The sum of δf^b and of diagrams of Fig. 1 with photon exchange (which are taken from /7/) is infrared-finite:

$$\delta f_{QED} = \frac{\alpha}{\pi}\Big[\frac{85}{18} - \frac{\pi^2}{3} + \frac{3}{4} Q_u Q_d \Big]. \tag{A.4}$$

It differs from the result quoted in /15/ for $Q_u = 0$ (leptonic decay mode) in the 't Hooft Feynman gauge by a nonlogarithmic gauge-

dependent term from the vertex correction. The rest of diagrams of Fig. 1 together with $-\delta r = -(\alpha/4\pi)X$ will be used to define $\delta\mathcal{F}^W$:

$$\delta f^W = \frac{\alpha}{4\pi s_\theta^2}\Big\{ W(-1) - W(0) + W_F(-1) + \frac{5}{8}R(1+R) - \frac{11}{2} - \frac{9R}{4(1-R)}\ln R +$$

$$+ \Big[-1 + \frac{1}{2R} + \frac{2(1-R)^2}{R} Q_u Q_d\Big]\Big[V_1(W,Z) + \frac{3}{2}\Big] + 2R\Big[V_2(W,W,Z) + \frac{3}{2}\Big]\Big\}. \tag{A.5}$$

Here, $R = 1 - s_\theta^2$, the W-functions and $V_{1,2}$ again are from /7/, their t-quark mass dependence from /3/. A closer inspection shows that the peaking of ρ^W mainly is due to $W_F(-1)$, the finite part of the W-boson wave function renormalization constant which has an extremum at $r = m_t^2/M_W^2 = 0.862$ for $m_t = 76.2$ GeV, $M_W = 82.0$ GeV, if $M_Z = 93.0$ GeV. The m_t-dependence of $W_F(-1)$ is

$$W_F(-1, r) = W(-1, 0) + \ln r - \frac{r}{2} - r^2 + (1 - r^3)\ln\Big|1 - \frac{1}{r}\Big|.$$

Finally, we get the gauge-independent co-factor of G_μ in W-decay

$$\rho^W = 1 + \delta f^W + \delta f_{QED}. \tag{A.6}$$

The channel-dependence is contained in two terms:

$$\rho_{\nu\ell} - \rho_{qq'} = \frac{\alpha}{\pi}\Big\{\frac{3}{4} + \frac{1-R}{R}\Big[V_1(W,Z) + \frac{3}{2}\Big]\Big\}(Q_\nu Q_\ell - Q_q Q_{q'}) = 0.046\%, \tag{A.7}$$

for $M_Z = 93$ GeV, $M_H = 100$ GeV, $m_t = 40$ GeV. The dependence on m_t, M_H is only due to their influence on M_W (i.e. of order α^2).

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Бардин Д.Ю., Риманн С., Риманн Т. E2-86-169
Однопетлевые электрослабые поправки к распаду заряженного векторного бозона

В рамках стандартной теории вычислены электрослабые радиационные поправки к парциальным ширинам распадов W-бозона, $\Gamma(W \to l\bar{\nu}, \bar{u}d, \bar{c}s)$. Результаты представлены в терминах электрослабого формфактора ρ^W. Изучается его зависимость от массы t-кварка m_t и массы бозона Хиггса M_H. Показано, что типичная величина $\rho^W - 1$ – порядка 1%, разница $\rho^W_{l\nu} - \rho^W_{qq}$, очень мала (0,045 %). Детально обсуждается используемая схема вычислений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Bardin D.Yu., Riemann S., Riemann T. E2-86-169
Electroweak One-Loop Corrections to the Decay of the Charged Vector Boson

The electroweak radiative corrections to the decay widths of the W-boson, $\Gamma(W \to l\bar{\nu}, \bar{u}d, \bar{c}s)$, have been calculated in the standard theory. The results are presented in terms of an electroweak form factor ρ^W, and their dependence on m_t and M_H (masses of t-quark and higgs boson) is studied. Typically, $\rho^W - 1$ is of an order of one per cent. The difference $\rho^W_{l\nu} - \rho^W_{qq}$, is negligible, 0.045%. The calculational scheme used is described in detail.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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