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V.N.Pervushin, Nguyen Suan Han, R.A.Azimov

TO THE PROOF OF MANIFEST RELATIVISTIC INVARIANCE OF TRANSVERSE VARIABLES IN QED

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1. Introduction

The physical transverse variables play an important role in QED[1-6]. They are associated with the Coulomb gauge, a unique gauge, in which one has been a success to calculate the O (α^6) corrections to the Lamb shift (as in terms of transverse variables it is easy to separate the binding effect from the radiative corrections). However, just for transverse fields a consistent renormalization is not yet accomplished.

In this paper we show, that the proof of relativistic invariance of the transverse variables in gauge theories can be accomplished on the level of the Feynman diagram and present a manifest relativistic-invariant expression for the electron Green function.

2. Formulation of the problem

It is well known [7,8] that the difficulty of electromagnetic field quantization is the singular Lagrangian. To removo this difficulty, account is to be taken of the term $\Delta\mathcal{L} = -\frac{1}{2d}(\partial_\mu A_\mu)^2$, which makes it possible to quantize all field components on equal footing.

The introduction of the superfluous longitudinal variables changes the singularity of the electron Green function

$$G(p) \sim (p^2 - m^2)^{-1 + \frac{\alpha}{2\pi}(3-d)} . \qquad (1)$$

In particular, for the Landau (d=0) and the Feynman (d=1) gauges instead of the usual pole the branch point appears, and the residue of the Green function

$$\lim_{\hat{p} \to m} (\hat{p} - m) G(p) \qquad (2)$$

is equal to zero. To reconstract physically right analytical properties, it is necessary to choose nonsingular longitudinal propagators, or to take into account the asymptotical interaction with longitudinal components. In both the cases we contradict the supposition about quantization of all field components on equal footing. In the context of such a relativistic approach the dependence of the Green function on the choice of gauge is to be considered as an inevitable defect of quantization. There is a general opinion that fermion Green functions to a certain extent are nonphysical quantities, as their analytical properties do not reflect the gauge-invariant content of a gauge theory. As to the calculation of the same quantities in the Hamiltonian scheme of quantization of transverse variables, this

scheme now is discredited by the manifest relativistic-noninvariant expression for the Green function.[1,2,3]

On the other hand there is a general proof of relativistic invariance of QED on the level of the transformation properties of field operators[4,5,6]. The question arises why the relativistic covariant quantization approach leads to manifest relativistic noninvariance on the level of Feynman diagrams.

To answer this question, let us consider the very scheme of the proof of relativistic invariance[5] on a simple example of a free electromagnetic field

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}^2 = \frac{1}{2}(\partial_0 A_i - \partial_i A_0)^2 - \frac{1}{2}(\varepsilon_{ijk}\partial_j A_k)^2 . \qquad (3)$$

In the Hamiltonian approach we first have to choose the time axis $x_\mu \ell_\mu^0 = t$; $\ell_\mu^0 = (1,0,0,0)$ and to consider the equation for $A_0 = (\ell_\mu^0 A_\mu)$ (the Gauss equation)

$$\partial_i^2 A_0 = \partial_i \partial_0 A_i \Rightarrow A_0 = -\frac{1}{4\pi}\int d^3x \frac{\partial_i \partial_0 A_i}{|\vec{x} - \vec{y}|} = \frac{1}{\partial^2}\partial_i \partial_0 A_i \qquad (4)$$

as a constraint equation.

Lagrangian (3) on the constraint equation depends only on two variables

$$\mathcal{L}[A_0(A_i), A_i] = \mathcal{L}^T = \frac{1}{2}(\partial_0 A_i^T)^2 - \frac{1}{2}(\varepsilon_{ijk}\partial_j A_k^T)^2 , \qquad (5)$$

$$A_i^T(A_i) = \delta_{ik}^T A_k \quad ; \quad \delta_{ik}^T = (\delta_{ik} - \partial_i \frac{1}{\partial^2}\partial_k) , \qquad (6)$$

which are in a nonlocal manner connected with the initial field A_k. The nonlocal variables (6) are gauge invariant under the initial field transformation

$$A_i^T(A_i + \partial_i \lambda) = A_i^T(A_i) . \qquad (7)$$

Formulae (3) - (6) are not consistent with the general scheme of choice of gauge conditions that is applied to relativistic gauges[7]; one gauge-invariant condition (that does not fix gauge) allows one to remove just two nonphysical variables. We have only the arbitrariness in the choice of the time axis $\ell_\mu^0 = (1,0,0,0)$. We can choose as ℓ_μ^0 any vector, connected with ℓ_μ^0 by the Lorentz transformation.

The nonlocality of variables (6) does not demand a radical change of the quantization procedure. Variables (6) lead only to the nonlocal transverse commutation relation

$$i[E_i^T(\vec{x},t), A_j^T(\vec{y},t)] = \delta_{ij}^T \delta^3(\vec{x} - \vec{y}) \qquad (8)$$

that means the disappearance of the longitudinal electric field

$\partial_i E_i^T = 0$ in the operator sense (but not on the state vectors $\partial_i E_i |\Phi\rangle = 0$). Just the nonlocal character of operators (6) and of the commutation relation (8) is a decisive point for the proof of relativistic invariance of the transverse variables [5]. Another decisive step is the choice of the gauge-invariant energy-momentum and angular momentum tensors [5]

$$T^S_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - g_{\mu\nu}\mathcal{L} \ , \quad M^S_{0k} = \int d^3x\,(x_i T^S_{0k} - x_k T^S_{00}) \tag{9}$$

known as the Belifante tensors [10]. The canonical tensor $T^c_{\mu\nu} = F_{\mu\lambda}\partial_\lambda A_\nu - g_{\mu\nu}\mathcal{L}$ differs from (9) by the total derivative $\partial_\lambda X_{\lambda\mu\nu}$ that gives an essential contribution to integral M^S_{0k} (9).

Due to gauge invariance on the constraint equation (4) tensor (9) is also expressed in terms of the nonlocal gauge-invariant variables. (Note that the field A0 in eq.(9) does not play the role of the Langrange factor).

The Belifante tensor (9) in terms of the quantum variables (6), (7) produces a closed algebra of the Poincare group and the following transformation properties of the operator $A^T_\mu = (0, A^T_i)$

$$\delta A^T_\mu = i\varepsilon_k [M^S_{0k}, A^T_\mu] = \delta^0_L A^T_\mu + \partial_\mu \Lambda \ , \tag{10}$$

where δ^0_L is the usual Lorentz transformation, and

$$\Lambda = \varepsilon_k \frac{1}{\partial^2}\partial_0 A^T_k$$

is an auxiliary gauge transformation which de facto means that the field operator in the new reference frame satisfies a new transversality condition connected with the new time axis $\ell_\mu = \ell^0_\mu + \delta^0_L \ell^0_\mu$ since after every Lorentz transformation (L) a new gauge appears ($L\ell^0_\mu$), the Hamiltonian approach is not consistent with the term "the choice of gauge". Just this fact represents a basic obstacle for understanding of the manifest relativistic invariance of the Hamiltonian approach and for calculating the manifest relativistic-invariant Green function. For this reason we shall avoid the term "the Coulomb gauge".

3. The one-loop calculation of the Green functions in QED

Let us consider the QED

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \bar\Psi[\gamma_\mu(i\partial_\mu - eA_\mu) - m]\Psi \ ; \quad S = \int d^4x\,\mathcal{L}(x). \tag{11}$$

We can construct the gauge-invariant nonlocal variable (see [10,11]) like (6)

$$\hat A^T_i(A_i) = v(\hat A_i + \partial_i)v^{-1} \ , \quad \Psi^T = v\Psi \ , \tag{12}$$

where

$$\hat A_\mu = ieA_\mu \ , \quad v = \exp\{\int^t \tfrac{1}{\partial^2}\partial_i\partial_0 \hat A_i dt'\} = \exp\{\tfrac{1}{\partial^2}\partial_i \hat A_i\} \ . \tag{13}$$

The variables are constructed by using the explicit solution of the Gauss equation $\delta S/\delta A_0 = 0$

$$A_0(A_i) = \frac{1}{\partial^2}(\partial_i\partial_0 A_i - j_0) \tag{14}$$

and by construction they are invariant under the gauge transformation of the initial field

$$\hat A^g_i = g(\hat A_i + \partial_i)g^{-1} \ ; \quad \Psi^g = g\Psi \ ; \ (g = \exp\{ie\lambda(x,t)\}). \tag{15}$$

On a quantum level variables (12) coincide with the ones employed in ref. [5]. By analogy with ref. [5] is easy to show that the Belifante tensor

$$T^S_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} + \bar\Psi\gamma_\nu(i\partial_\mu - eA_\mu)\Psi - g_{\mu\nu}\mathcal{L} + \frac{i}{2}\partial_\lambda(\bar\Psi\Gamma_{\lambda\mu\nu}\Psi)$$

$$\Gamma_{\lambda\mu\nu} = \frac{1}{2}[\gamma_\lambda, \gamma_\mu]\gamma_\nu - g_{\nu\mu}\gamma_\lambda - g_{\nu\lambda}\gamma_\mu$$

expressed in terms of (12) provides the following transformation properties of the operator A^T_μ, Ψ^T

$$\begin{aligned}
&\delta A^T_\mu = i\varepsilon_k [M^S_{0k}, A^T_\mu] = \delta^0_L A^T_\mu + \partial_\mu\Lambda \ , \\
&\delta\Psi^T = i\varepsilon_k [M^S_{0k}, \Psi^T] = \delta^0_L \Psi^T - ie\Lambda\Psi^T , \\
&A^T_\mu = (-\tfrac{1}{\partial^2} j^T_0, A^T_i), \\
&\Lambda(A^T, \Psi^T) = \frac{\varepsilon_k}{\partial^2}(\partial_0 A^T_k + \partial_k A^T_0).
\end{aligned} \tag{16}$$

A general structure of the gauge operator Λ can be easily found on a classical level by the usual Lorentz transformation of quantity v (13)

$$\delta^0_L\left(\frac{1}{\partial^2}\partial_i A_i\right)\Big|_{\partial_i A_i = 0} = \frac{\varepsilon_k}{\partial^2}(\partial_0 A_k + \partial_k A_0).$$

Following the Schwinger method [5] we reproduced the relativistic transformation of the gauge-invariant variables (12).

Let us calculate the electron Green function by the formula

$$(2\pi)^4\delta^4(p-q)\, i\, G(p) = \int d^4x\, d^4y\, e^{ipx-iqy}\langle 0|T\,\psi^T(x)\,\bar{\psi}^T(y)|0\rangle \tag{17}$$

where ψ^T, $\bar{\psi}^T$ are operators in the Heisenberg representation.

In the one-loop approximation $G(p)$ has the form

$$G(p) = G_0(p) + G_0(p)\Sigma(p)G_0(p) + O(\alpha^2), \tag{18}$$

where $\Sigma(p)$ is the electron self-energy of an order e^2 which contains the contributions from transverse fields and the Coulomb interaction

$$\Sigma(p) = \int\frac{(dq)}{q_\mu^2}\left[\left(\delta_{ij} - \frac{q_i q_j}{\vec{q}^{\,2}}\right)\gamma_i G_0'\gamma_j + \gamma_0 G_0'\gamma_0\frac{q_\mu^2}{\vec{q}^{\,2}}\right], \tag{19}$$

where

$$(dq) = \frac{e^2}{(2\pi)^4}\, i\, d^4q\,;\quad q_\mu^2 = q_0^2 - \vec{q}^{\,2} = q^2\,;\quad G_0' = G_0(p-q).$$

Let us prove the invariance of the Green functions (17) under the Lorentz transformation of the operators ψ^T, $\bar{\psi}^T$. By "invariance" we shall understand the equality[6]

$$G_0(p') = S_{p'p}\, G_0(p)\, S^{-1}_{p'p} \tag{20}$$

that is, we shall take into account the Lorentz transformation of the γ-matrices. In this case $\delta_L^c G_0(p) = 0$. It is known[6] that (19) can be represented by a sum of the invariant $\Sigma_F(p)$ and noninvariant $\Delta\Sigma(p)$ terms:

$$\Sigma_F(p) = -\int(dq)\frac{1}{q^2}\gamma_\mu G_0'\gamma_\mu\,, \qquad \delta_L^c\Sigma_F(p) = 0\,,$$

$$\Delta\Sigma(p) = \int\frac{(dq)}{q_\mu^2\,\vec{q}^{\,2}}\left[\hat{q}G_0'\hat{q} + \not{q}\,G_0'\hat{q} + \hat{q}G_0'\not{q}\right]. \tag{21}$$

$$\hat{q} = \gamma_\mu q_\mu\,,\quad \not{q} = \vec{\gamma}\vec{q}.$$

The response of $\Delta\Sigma(p)$ to the Lorentz transformation (20) can be got by changing the integration variables in eq (21)

$$\delta_L^c q_0 = \varepsilon_k q_k\,,\qquad \delta_L^c q_k = \varepsilon_k q_0\,.$$

$$\delta_L^c\Delta\Sigma(p) = \varepsilon_k\int\frac{(dq)}{q_\mu^2\,\vec{q}^{\,2}}\left[B_k G_0'\hat{q} + \hat{q}G_0' B_k\right],$$

where

$$B_k = q_k\gamma_0 + \gamma_k q_0 - \frac{2q_0 q_k}{\vec{q}^{\,2}}q_i\gamma_i - \frac{q_0 q_k}{\vec{q}^{\,2}}\hat{q}\,. \tag{22}$$

The total Lorentz transformation for the Green function contains also the gauge transformation (16)

$$\delta_\Lambda\left[(2\pi)^4\delta^4(p-q)\, i\, G(p)\right] = ie\int d^4x\, d^4y\, e^{ipx-iqy}\cdot$$

$$\cdot\left[\langle 0|T(\psi^T(x)\bar{\psi}^T(y)\Lambda(y))|0\rangle - \langle 0|T(\Lambda(x)\psi^T(x)\bar{\psi}^T(y))|0\rangle\right]. \tag{23}$$

Using the explicit form for Λ (16), we get the following expression

$$\delta_\Lambda\Sigma = -\varepsilon_k\int\frac{(dq)}{q_\mu^2\,\vec{q}^{\,2}}\left[B_k G_0'(\hat{p}-m) + (\hat{p}-m)G_0' B_k\right], \tag{24}$$

where B_k is given by formula (23). As

$$G_0(p-q)(\hat{p}-m) = 1 + G_0(p-q)\hat{q}$$

$$\int(dq)\frac{B_k}{q_\mu^2\,\vec{q}^{\,2}} = 0 \tag{25}$$

we get that the total response of (12) to the Lorentz transformation (16) is equal to zero.

$$\delta_{L\ t.t.}\Sigma(p) = \left(\delta_L^0 + \delta_\Lambda\right)\Sigma(p) = 0\,. \tag{26}$$

Therefore, it is sufficient to calculate expression (17) in the rest frame of the electron $p_\mu = (p_0, \vec{0})$, for the choice $f_\mu^0 = (1,0,0,0)$

$$\Sigma(p_\mu) = \int\frac{(dq)}{q_\mu^2}\frac{2}{\hat{p}-\hat{q}+m} - \int\frac{(dq)}{\vec{q}^{\,2}}\gamma_0\frac{1}{\hat{q}+m}\gamma_0\,. \tag{27}$$

In another reference frame $p_\mu' = (p_0', \vec{p}\,')$ due to transformation (16), we have to take into account auxiliary diagrams (23), i.e., to change the gauge

$$q_i A_i^T \;\Rightarrow\; \left(q_\mu - p_\mu\frac{qp}{p^2}\right)A_\mu(q) = 0\,.$$

Finally, we get for $\Sigma(p)$ the same expression (27) up to the change $\vec{p} \to \vec{p}\,'$. In the dimensional regularization the integral (27) is equal to

$$\Sigma(p_\mu) = \frac{\alpha}{4\pi}\left[m(3\mathcal{D} + 4) - \mathcal{D}(\hat{p}-m)\right] + \Sigma_R(p_\mu)\,,$$

where

$$\mathcal{D} = \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi$$

$$\Sigma_R(p_\mu) = \frac{\alpha}{2\pi}\left\{-\frac{\hat{p}}{4} + \int_0^1 dx\,[x\hat{p} - m]\ln\left(1 - \frac{p^2}{m^2}x\right)\right\} =$$

$$= \frac{\alpha}{2\pi}(\hat{p} - m)^2\left\{\frac{\hat{p}+m}{p^2}\left[\ln\left(\frac{m^2 - p^2}{m^2}\right)\right]\left[1 + \frac{\hat{p}(\hat{p}-m)}{2p^2}\right] - \frac{\hat{p}}{2p^2}\right\}$$

(28)

The self-energy (28) has no infrared divergences (the residue of the electron Green function is equal to unity)and allows the renormalization

$$\Sigma(\hat{p} = m) = \delta m = \frac{\alpha m}{4\pi}(3\mathcal{D} + 4),$$

$$\Sigma'(\hat{p} = m) = Z - 1; \quad Z = 1 - \frac{\alpha}{4\pi}\mathcal{D}.$$

These results represent a solution to the renormalization problem on the mass shall for transverse variables.

Misunderstanding refs. /1,2,3/ consists not only in ignoring the right transformation properties (16) in the construction of $\Sigma(p)$, but also in a nonphysical choice of the initial vector ℓ_μ^0 (the time axis). For example, in expression (19) where $p_\mu = (p_0, \vec{p} \neq 0)$ the vector $\ell_\mu^0 = (1,0,0,0)$ is chosen so that the electron has a velocity different from that of its statical Coulumb field. This choice leads to the manifest Lorentz noninvariance nonrenormalization, and infrared divergence. On the other hand transition (19) to the rest frame of the electron $p_\mu = (p_0, \vec{p} = 0)$ does not remove these difficulties as we simultaneously turn the initial gauge ℓ_μ^0 (the disagreement of velocities of the electron and its field remains).

Thus, the choice of ℓ_μ^0 must be defined by a physical formulation of the problem.

It is easy to be convinced of that the point transformations A_i^T

$$A_i^T = A_i^L + \partial_i \lambda(A^L)$$

that effectively lead to another gauge, with the simultaneous transformation of the electron operator

$$\psi^T = \exp(ie\lambda^L)\,\psi^L \tag{29}$$

$$\langle 0|T(\psi^T\bar{\psi}^T)|0\rangle_T \equiv \langle 0|T(e^{ie\lambda^L}\psi^L\,\bar{\psi}^L e^{-ie\lambda^L})|0\rangle_L$$

do not change result (28)

If the right hand side is calculated in the gauge $\partial_\mu A_\mu^L = 0$, then $\lambda^L(A) = \frac{1}{\partial^2}\partial_0 A_0^L$; $\ell_\mu^0 = (1,0,0,0)$. For the gauge L^d, fixed by the auxiliary term $-(\partial_\mu A_\mu)^2/2d$, the function $\lambda^d(A) = \frac{1}{\partial^2}\partial_i A_i^d$ and does not depend an d.

The operator transformations of electrons (29) lead to auxiliary diagrams that reconstruct result (27) for the above gauges (L, L^d).

4. The discussion of results

The proof by Schwinger /5/ of relativistic invariance of the Hamiltonian approach of the quantization of the transverse variable is essentially based on nonlocality of this variable and on the gauge-invariant Belifante energy-momentum tensor. The essence of the proof consists in that the gauge of physical transverse nonlocal variables follows the relativistic transformation of the time axis. These covariant properties of the transverse variables are lost if we pass from the operator approach to the functional integral

$$Z(\bar{\eta}^T, \eta^T) = \int d\psi^T d\bar{\psi}^T d^4A_\mu^T\,\delta(\partial_i A_i^T)\,e^{iS + i\int d^4x(\bar{\eta}^T\psi^T + \bar{\psi}^T\eta^T)} \equiv$$

$$\equiv \int d\psi^d d\bar{\psi}^d d^4A_\mu^d \exp\left\{iS + i\int d^4x\left[\frac{1}{2d}(\partial_\mu A_\mu)^2 + \bar{\eta}^d\psi^d + \bar{\psi}^d\eta^d\right]\right\},$$

$$\left(\bar{\eta}^d = \bar{\eta}^T e^{i\lambda^d}, \quad \lambda^d = \frac{1}{\partial^2}\partial_k A_k^d\right). \tag{30}$$

On this level the nonlocal transverse variables which follow their time axis turn into one of the ordinary gauges (into the Coulomb gauge). Just in such a context there appear the statements of the manifest relativistic noninvariance of the Coulomb gauge and of the dependence of Green functions on the choice of gauge. As we have seen above, these statements become meaningless on the level of the Hamiltonian operator approach that serves as a foundation of the functional integral (30). Thus, the proof of manifest Lorentz-invariance of the theory, based on the quantization only of physical transverse variables, can be made on the level of the Feynman diagram. The results of the paper solve the problem of renormalization of physical quantities on the mass shell for the transverse variables /1,2,3/.

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Первушин В.Н., Нгуен Суан Хан, Азимов Р.А. E2-86-128

К доказательству явной релятивистской инвариантности поперечных переменных в КЭД

Проводится квантование электродинамики в терминах поперечных физических переменных. На всех этапах построения теории: 1/ при выборе поперечных переменных, 2/ выборе тензоров энергии импульса и момента количества движения, 3/ квантовании и 4/ описании диаграмм Фейнмана, сохраняется калибровочная и релятивистская инвариантность. Впервые для поперечных переменных вычислены релятивистски-инвариантные собственная энергия электрона и вершинная часть. Полученные результаты полностью решают для физических переменных проблему перенормировок физических величин на массовой поверхности.

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Pervushin V.N., Nguyen Suan Han, Azimov R.A. E2-86-128

To the Proof of Manifest Relativistic Invariance of Transverse Variables in QED

The quantization of electrodynamics in terms of transverse physical variables is accomplished. At all the steps of the theory construction: I) the choice of transverse variables, II) the choice of energy-momentum tensor, III) quantization, IV) the Feynman diagram description we preserve the manifest gauge and relativistic invariance. For the transverse variables the relativistic-invariant self-energy of the electron is calculated. The results completely solve the problem of renormalization of physical quantities on the mass shell for the physical variables.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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