

# объединенны" институт ядериых исследований <br> дусиа 

E2-86-125
B.Z.Kopeliovich, N.N.Nikolaev*,
I.K.Potashnikova

## QCD SUGGESTED HIGH-ENERGY

ASYMPTOTICS
OF THE DIFFRACTION PROTON-PROTON SCATTERING
AND THE COSMIC RAY DATA

Submitted to AlI-Union School
on Inelastic Hadron Interactions
beyond Accelerator Energies (Nor-Amberd, 1985)

[^0]
## I. INTRODUCTION. THE SUPERCRITICAL POMERON

The high-energy behaviour of the hadron-hadron scattering amplitudes is governed by their $j$ - plane singularities. The conventional attitude is that a rise of $\sigma_{\text {tot }}(\mathrm{pp})$ implies that the pomeron $\mathbb{P}$ is the single supercritical pole with the $j$ - plane trajectory

$$
\begin{equation*}
j=\alpha_{\mathbb{P}}\left(\vec{q}^{2}\right)=1+\Delta-\alpha_{\mathbb{P}^{\prime}}^{\prime} \cdot \vec{q}^{2} \tag{1}
\end{equation*}
$$

The one-pomeron exchange gives the bare amplitude ( normalization is such that $\vec{\sigma}_{\text {tot }}=\operatorname{Im} f(0)$ )

$$
\begin{equation*}
f_{\mathbb{P}}(\vec{q})=i h_{\mathbb{P}}(\vec{q}) \cdot \exp \left[\Delta \cdot \xi-\alpha_{\mathbb{P}}^{\prime} \cdot \xi \cdot \vec{q}^{2}\right] \tag{2}
\end{equation*}
$$

Here $\vec{q}$ is the momentum transfer; $\xi=\ln \left(S / S_{0}\right)-i \pi / 2 ; S=2 m_{p} E_{3}$ $S_{0}=1(\mathrm{GeV} / \mathrm{c})^{2}$; the residue $h_{\mathbb{P}}(\vec{q})$ is usually parametrized as

$$
\begin{equation*}
h_{\mathbb{P}}(\vec{q})=h_{\mathbb{P}}(0) \cdot \exp \left[-\frac{1}{2} B_{\mathbb{P}}^{0} \cdot \vec{q}^{2}\right] . \tag{3}
\end{equation*}
$$

We remind that the supercritical pomeron was first proposed 1) to fit a steep rise of the p-p total cross section encountered at ISR. A fit to the Serpukhov - Fermilab - ISR data on $\boldsymbol{\sigma}_{\text {tot }}(\mathrm{pp})$ and the diffraction cone slope $B_{p p}$ had resulted in $\Delta=0.07$. An extrapolation of that fit up to the $S p \bar{p} S$ collider energy $\sqrt{S}=540 \mathrm{GeV}$ and $\sqrt{\mathrm{s}}=900 \mathrm{GeV}$ gave $\boldsymbol{\sigma}_{\text {tot }}(\mathrm{pp})=57 \mathrm{mb}$ and 59 mb 1),2), well below the experimental data of $61.9 \pm 1.5 \mathrm{mb}$ 3) and $66 \pm 2 \mathrm{mb}$ 4), respec-
tively. In order to intercept the $S p \bar{p} S$ data points one needs $\Delta=$ 0.12 5) ( see below, Section 4) . A general trend is that the higher the energy $E$, the steeper the rise of $\sigma_{\text {tot }}^{(p)}$ ). Still more ${ }^{(p)}$ data on absorption of the cosmic ray protons in air.

The purpose of this paper is to demonstrate that such a behaviour of $\sigma_{\text {tot }}(\mathrm{pp})$ nicely fits the pattern suggested by Quantum ChromoDynamics. Our major task was an accurate determination of $\sigma_{t o t}(\mathrm{pp})$ from the extensive air shower data 6),7) . Our principal findings are:
i) The Akeno - Fly's Eye data imply, at $E=10^{8}-10^{9} \mathrm{GeV}$,

$$
\begin{equation*}
\sigma_{\mathrm{tot}}(\mathrm{pp})=160-200 \mathrm{mb} \tag{4}
\end{equation*}
$$

The of ten quoted determinations 7), 8), $\vec{\sigma}_{\text {tot }}(\mathrm{pp})=120 \mathrm{mb}$, from the Fly's Eye data 7) are erroneous and should be disoarded.
ii) Only the QCD scenario in which the two-gluon exohange, $\Delta=0$, dominance in $\vec{\sigma}_{\text {tot }}(\mathrm{pp})$ at moderate energies is superseded by a dominance of a series 9 ) of poles with $\Delta>0$, is consistent with the observed pattern of the rise of $\sigma_{\text {tot }}(\mathrm{pp})$. The result (4) corresponds to

$$
\begin{equation*}
\Delta_{\text {eff }}=0.25-0.35 \tag{5}
\end{equation*}
$$

at $\dot{E}=10^{8}-10^{9} \mathrm{GeV}$. If taken literally, the Akeno data $\left.{ }^{6}\right)^{\circ}$ do even suggest $\Delta_{\text {eff }}>0.35$.

It is worth while to recall that the first evidence for a rapid rise of $\sigma_{\text {tot }}(p p)$ was inferred from an analysis 10 ) of the cosmic ray data on $\boldsymbol{\sigma}_{a b s}$ (pAir).

Our further presentation will be as follows: In Section 2 we describe the QCD suggested model of the pomeron. In Section 3 we discuss the diffraction dissociation and inelastic shadowing. The QCD suggested asymptatics of the proton - proton scattering and fits to the accelerator data are presented in Section 4 . In Section 5 we discuss a calculation of the proton-nucleus cross sections and conver-
sion of $\sigma_{\text {abs }}(\mathrm{pAir})$ into $\boldsymbol{\sigma}_{\text {in }}(\mathrm{pp})$ and $\boldsymbol{\sigma}_{\text {tot }}(\mathrm{pp})$. In Section 6 we comment on. violations of the scaling of the fragmentation spectra in $p-p$ and p-air inelastic collisions. In the final section we summarize our principal conclusions.

## 2. THE POMERON IN QCD

In QCD the pomeron is generated by exchange by glueballs - the bound states of gIuons - in the $t$ - channel. The lowest order, two-gluon exchange diagram gives the constant cross section (fixed pole at $j=1$ ), which nicely reproduces the hadron - nucleon total cross sections at moderate energies 1l). The higher order perturbation theory diagrams give rise to a series of poles with
$1<j<1+\Delta$, which accumulate at $j=1$, rather than a simple isolated pole ?). At large $\vec{q}^{2}$, in the perturbative regime of $\alpha_{S}\left(\vec{q}^{2}\right) \ll I$ (hereafter $\alpha_{S}\left(\vec{q}^{2}\right)$ is the strong interaction coupling $)_{2} . j-1 \sim \alpha_{s}\left(\vec{q}^{2}\right)$. Anyway, the fixed pole at $j=1$ is expected to take over the moving poles at large $\vec{q}^{2}$, where $\alpha_{R}\left(q^{-2}\right)<1$, i.e., at

$$
\begin{equation*}
\vec{q}^{2} \geq \Delta / \alpha_{\mathbb{P}}^{\prime} \tag{6}
\end{equation*}
$$

This qualitative prediction 12) of $Q C D$ theory of the pomeron is nicaly confirmed by the experimental data on the meson-nucleon elastic scattering, which exhibit the conventional Regge shrinkage at low $|t|=\vec{q}^{2}$ (Fig.la), that disappears at $\mid t \geqslant 0.8$ (GeV/c) ${ }^{2}$ (Fig.lb). Besides, the $\pi N$ and $K N$ differential cross sections coincide at large $|t|$ ( the 2G-exchange contribution to the large- $\mid$ t| elastic scattering amplitude does not depend on the meson's form factor). Furthemore, the large- $|t|$ data on $\pi N$ and $K N$ elastic scattering do nicely follow the $q^{-8}$ law 14) suggested by the 2G-exchange (if the nucleon's difactor has a weak $\mathbb{q}^{2}$-dependence) (Fig.2).


Fig.l - The energy dependence of the differential cross section of $\pi N$ and KN elastic scattering at low (a) and high (b) momentum transfer 13). Notice the energy independence and equality of $\pi \mathbb{C N}$ and KN cross sections at large $\mid \mathrm{tl}$ •


Pig. 2 - The onset of the QCD law $q^{-8}$ in the $\pi N$ and KN elastic
, scattering at large momentum transfer ${ }^{14}$ )

Yet there are no simple prescriptions how to extrapolate the perturbation theory results from the perturbative region of $\vec{q}^{2}>1(\mathrm{GeV} / \mathrm{c})^{2}$, $\alpha_{s}\left(\vec{q}^{2}\right) \ll 1$, into the diffraction cone region of $\vec{q}^{2} \ll 1(\mathrm{GeV} / \mathrm{c})^{2}$. None the less, the fundamentals of the ultimate QCD phenomenology of the diffraction scattering of hadrons are fairly obvious.

Namely, as energy goes up, the higher and still higher order perturbation theory diagrams come in, so that a dominance of the 2Gexchange and the constant cross section are superseded by the rising contributions, $\sigma_{\text {tot }} \mathrm{E}^{\Delta}$, of poles to the right of $j=1$. The higher the energy, the larger the relative contribution of the rightmost pole, so that $\Delta_{\text {eff }}$ is predicted to rise with energy and then saturate.

A crude approximation to a complete QCD phenomenology of the hadronic high-energy scattering is a substitution of ( 1 ) by the two-pole amplitude

$$
\begin{equation*}
f(\vec{q})=i h_{2 G}(\vec{q})+i h_{\mathbb{p}}(\vec{q}) \cdot \exp \left[A \cdot \xi-\alpha_{\mathbb{p}}^{\prime} \cdot \xi \cdot \vec{q}^{2}\right] \tag{7}
\end{equation*}
$$

Here we have explicitly isolated the $2 G$-exchange contribution which dominates at moderate energies, whereas the second term is a representative of poles to the right of ' $j=1$.

The residue of the $2 G-a m p l i t u d e ~ c a n ~ e x p l i c i t l y ~ b e ~ c o m p u t e d: ~$
$h_{2 G}(\vec{q})=\frac{16}{3} \alpha_{s}^{2} \int \frac{d^{2} \vec{k}\left[F(\vec{q})-D\left(\vec{k}^{2}+\frac{1}{4} \vec{q}^{2}\right)\right]}{\left[\left(\vec{k}+\frac{1}{2} \vec{q}\right)^{2}+m_{G}^{2}\right] \cdot\left[\left(\vec{k}-\frac{1}{2} \vec{q}\right)^{2}+m_{G}^{2}\right]}$
Here $F(\vec{q})$ and $D\left(\vec{R}^{2}+\frac{1}{4} \vec{q}^{2}\right)$ are the single-quark (the charge form factor ) and two-quark vertex functions of the proton; $m_{G}$ is an effective mass of the gluon. Eq. (8) gives the right value of the $p-p$ slope, for our purposes it is sufficient to use as an input the Gaussian approximation $h_{2 G}(\vec{q})=\sigma_{2 G} \cdot \exp \left[-\frac{1}{2} B_{0} \vec{q}^{2}\right]$, what greatly simplifies the further analysis.

In view of the conformal properties of the QCD pomeron all the residues are basically determined by the quark wave functions of the interacting hadrons, so that one should not expect $h_{2 G}(\vec{q})$ and $h_{p}(\vec{q})$ to exbibit significantly distinct $\vec{q}^{2}$-dependence. In view of that, and in order to reduce the number of the free parameters, we suppose that

$$
\begin{equation*}
R=h_{2 G}(\vec{q}) / h_{\mathbb{P}}(\vec{q})=\text { const } \tag{9}
\end{equation*}
$$

The experimental data suggest $R \gg 1$. We have no obvious arguments in favour of that apart from the above mentioned perturbation theory considerations. Lipatov's QCD pomeron possesses, in the weak coupling regime, a conformal symmetry in the impact parameter space ${ }^{9}$, so that naively all the poles might have comparable residues. Yet, accumulation of poles at $j=1$ suggests an enhancement of the $j=1$ component of the cross section. Anyway, we have to live to the fact that at moderate energies the $2 G-e x c h a n g e$ contribution perfectly reproduces the total cross sections li),15), the relative magnitude of the diffraction dissociation cross section and the diffraction cone slopes, including their t-dependence ${ }^{15}$ ).

Finally, in the impact parameter representation, the amplitude (7) is

$$
\begin{aligned}
f(\vec{b}) & =-\frac{i}{2} \int \frac{d^{2} \vec{q}}{(2 \pi)^{2}} \cdot \exp [-i \vec{q} \vec{b}] \cdot f(\vec{q})= \\
& =G_{2 G}\left\{\frac{1}{4 \pi B_{0}} \cdot \exp \left[-\frac{\vec{b}^{2}}{2 B_{0}}\right]+\frac{1}{R} \cdot \frac{1}{4 \pi B_{\mathbb{P}}} \cdot \exp \left[-\frac{\vec{b}^{2}}{2 B_{\mathbb{P}}}\right]\right\}
\end{aligned}
$$

where

$$
\begin{equation*}
B_{\mathbb{P}}=B_{D}+2 \alpha_{\mathbb{P}}^{\prime} \cdot \xi \tag{11}
\end{equation*}
$$

3. THE s-GHANEL UNITARIZATION, DIFFRACTION DISSOCIATION AND INELASTIC SHADOW NG

The amplitude (7), (10) as well as (1) overshoots, at high energy, the unitarity bound $f(\vec{b}) \leqslant 1$. The unitarity is restored after summing up the eikonal s-channel diagrams:

$$
\begin{equation*}
F(\vec{b})=1-\exp [-f(\vec{b})] \tag{12}
\end{equation*}
$$

The complete unitarized amplitude depends on the inelastic shadowing (IS) - the result of a presence of the diffraction dissociation (DD), $h \rightarrow h^{*}$, alongside the elastic scattering $h \rightarrow h$. For our purposes it is vital to have a unique description of IS both in the $p-p$ and $p-A$ scattering. We shall rely upon the method of the diffraction scattering eigenstates (DSE), first applied to the p-A total cross sections in Ref. ${ }^{16 \text { ) ( see also 17) ). }}$

Diffraction scattering eigenstates $|\alpha\rangle$ are the set of eigenstates of the matrix $\widehat{\mathrm{F}}$ of the diffraction amplitudes: $\langle\beta| \hat{F}|\alpha\rangle=$ $F_{\alpha} \cdot \delta_{\alpha \beta}$. A relationship (12) between the bare and unitarized amplitudes is, then, retained for the DSE amplitudes. The real hadrons are superpositions of DSE ,

$$
\begin{equation*}
|h\rangle=\sum_{\alpha} c_{\alpha}^{h}|\alpha\rangle \tag{13}
\end{equation*}
$$

and

$$
\begin{align*}
& F(\vec{b})=\langle h| \vec{F}|h\rangle=\sum_{\alpha}\left|c_{\alpha}^{h}\right|^{2} F_{\alpha}(\vec{b})= \\
& =\left\langle F_{\alpha}(\vec{b})\right\rangle=1-\left\langle\exp \left[-f_{\alpha}(\vec{b})\right]\right\rangle \tag{14}
\end{align*}
$$

Henceforth, the right way of unitarization is, first, to sum up all the eikonal diagrame and, then, to take the average (14), rather than to averace first the bare amplitude (eikonal) $f(\bar{b})=\left\langle f_{\alpha}(\bar{b})\right\rangle$, and, then, to sum up the eikonal diagrams with this $f(\vec{b})$ as an input. The difference between the naive amplitude (12) and (14) is called the inelastic shadowing correction.

The problem of finding an explicit set of DSE is not yet solved. We shall expose the role of IS using some models.

The stmplest option is the so-called quasieikonal model 1), 2),5) (QEM) of the two eigenstates of which one, with the weight $w_{0}=$ 1-1/C, has a vanishing cross section (t'ie passive state). Then

$$
\begin{equation*}
F(\vec{b})=\frac{1}{c}\{1-\exp [-c f(\vec{b})] \tag{15}
\end{equation*}
$$

and the opacity saturates at $f(\vec{b}) \gg 1$, but at the grey disc limit. In the original formulation of the model $C$ is an energyindependent constant, so that if taken ifterally, QEM is already in conflict with an evidence ${ }^{18 \text { ) for an onset of the black disk limit }}$ at $\mathrm{Sp} \overline{\mathrm{p}}$.

Another option is the QCD 2G-exchange amplitude. Here the relative separation of the constituent quarks, $\vec{\rho}$, is frozen in the scattering process and is the eigenvalue parameter. In view of the colour cancellations, the meson-nucleon scattering amplitude will be proportional to $\vec{\rho}^{2}{ }^{19)}$,

$$
\begin{equation*}
f_{\vec{\rho}}(\vec{b}) \simeq \frac{\vec{\rho}^{2}}{\left\langle\vec{\rho}^{2}\right\rangle} \cdot f(\vec{b}) \tag{16}
\end{equation*}
$$

with the meson's wave function being the weight function:

$$
\begin{equation*}
\left|c_{\vec{\rho}}^{h}\right|^{2}=\left|\psi_{h}(\vec{\rho})\right|^{2}=\frac{1}{\pi\left\langle\vec{\rho}^{2}\right\rangle} \cdot \exp \left[-\frac{\vec{\rho}^{2}}{\left\langle\vec{\rho}^{2}\right\rangle}\right] \tag{17}
\end{equation*}
$$

Then, the expectation value (14) is

$$
\begin{equation*}
F(\vec{b})=\frac{f(\vec{b})}{1+f(\vec{b})} \tag{18}
\end{equation*}
$$

Though being of different functional form, both (18) and (12) are broadly alike as they exhibit the black. disc limit at $f(\vec{b}) \gg 1$. Incidentally, the amplitude (16) does also have the passive component, $\vec{\beta}^{2} \simeq 0$, but its relative weight vanishes with energy.

A more realistic model should allow for $D D$ of both the target and projectile. In view of the factorization property of the residues
one can write

$$
\begin{equation*}
f_{\alpha \beta}(\vec{b})=\alpha \cdot \beta \cdot f(\vec{b}) \tag{19}
\end{equation*}
$$

Here the subscripts $\alpha$ and $\beta$ in the l.h.s. of Eq.(19) refer to the eigenstates $|\alpha\rangle$ and $|\beta\rangle$ of the projectile and target, whereas the coefficients $\alpha$ and $\beta$ in the r.h.s. of Eq. (19) stand for the relative residues with the normalization $\langle\alpha\rangle=\langle\beta\rangle=1$, so that $\ll f_{\alpha \beta}(\vec{b}) \gg=f(\vec{b})$. We have neglected here variations of the slope $B_{\alpha \beta}$. Then the $\nu$-fold scattering amplitude will be proportional to $\left\langle\alpha^{\nu}\right\rangle\left\langle\beta^{\nu}\right\rangle$, where

$$
\begin{align*}
& \left\langle\alpha^{\nu}\right\rangle=K_{\nu}=\left\langle(1+\Delta \alpha)^{\nu}\right\rangle= \\
& =1+\frac{1}{2!} \nu(\nu-1) \cdot\left\langle\Delta \alpha^{2}\right\rangle+\frac{1}{3!} \nu(\nu-1)(\nu-2)\left\langle\Delta \alpha^{3}\right\rangle+\ldots \tag{20}
\end{align*}
$$

An expansion for the $D D$ cross section $\vec{\sigma}_{D D}$ starts with the $\left\langle\Delta \alpha^{2}\right\rangle$ term. Since $\sigma_{\mathrm{DD}} \ll \sigma_{e l}$, then $\left\langle\Delta \alpha^{2}\right\rangle$ is a small parameter and, for begtnners, it is sufficient to only keep terms $\left.n<\Delta \alpha^{2}\right\rangle$. Then one can evaluate $\left\langle\Delta \alpha^{2}\right\rangle$ from the inclusive forward diffraction dissociation of the projectile, which can be estimated 17) putting

$$
\begin{aligned}
& \beta=1: \\
& \left.\frac{d \sigma_{D D}}{d t}\right|_{t=0}=\int d M^{2} \frac{d \sigma_{D D}}{d t d M^{2}}=\frac{\left\langle\sigma_{\alpha}^{2}\left(1+\rho_{\alpha}^{2}\right)\right\rangle-\left.\left|<\sigma_{\alpha}\left(1-i \rho_{\alpha}\right)\right\rangle\right|^{2}}{16 \pi},
\end{aligned}
$$

where

$$
\begin{equation*}
\vec{\sigma}_{\alpha}\left(1-i \rho_{\alpha}\right)=2 \int d^{2} \vec{b} F_{\alpha}(\vec{b}) \tag{22}
\end{equation*}
$$

and $\vec{\sigma}_{\alpha}$ and $\rho_{\alpha}$ are the total cross section and $R e / I m$ for interaction of the eigenstate $|\alpha\rangle$ with the target.

One also could evaluate the total DD cross section:

$$
\begin{equation*}
\left.\vec{G}_{D D}=\int d^{2} \vec{b}\left\{\left.\langle | F_{\alpha}(\vec{b})\right|^{2}\right\rangle-\left.\left|<F_{\alpha}(\vec{b})\right\rangle\right|^{2}\right\} \tag{23}
\end{equation*}
$$

However, at moderate energies variations of the slope $B_{\alpha \beta} \beta$ account for about half of $\vec{\sigma}_{D D}$, so that putting in (19) $B_{\alpha} \beta=$ const would underestinate $G_{D D}{ }^{20}$ ). To the contrary, the forward DD cross section (21) is insensitive to varlations of $B_{\alpha \beta}$ and, hericelorth, gives a reliable estimation of $\left\langle\Delta \alpha^{2}\right\rangle$

The residues do not depend on energy and the moments $\left\langle\alpha^{\nu}\right\rangle$,
which are controlled by the wave functions of hadrons, are, thus, energy-independent. In the QCD model of the pomeron of our interest we observe that, in view of the conformal properties of the QCD pomeron, the residues of both components in the decomposition (7), (10) should vary in parallel, i.e., the ratio $R$ does not depend on $\alpha$ and $\beta$. Hence; once $\left\langle\Delta \alpha^{2}\right\rangle$ wefe evaluated using (21) at some moderate energy, it can be used for a reliable extrapolation of IS corrections to both $p-p$ and $p-A$ scattering beyond the accelerator energies.

In what follows, we shall use $\left\langle\Delta \alpha^{2}\right\rangle=0.35$, inferred from the Dakhno's calculation 2l) of the inelastic shadowing enhancement coefficient $K_{2}$ in the proton-deuteron scattering at $E=300 \mathrm{GeV}$.
4. THE ACCELERATOR DATA ON FROTON-FROTON SCATTERING AND EXTRAPOLATION OF $\stackrel{\sigma}{t o t}^{(p p)}$ BEYOND THE ACCELERATOR ENERGIES

We have fitted the data 22 ) on $\sigma_{\text {tot }}(\mathrm{pp})$ above $E=100 \mathrm{GeV}$, the Sp $\mathrm{p} S$ data 3$), 4$ ) on $\sigma_{\text {tot }}(\mathrm{p} \overline{\mathrm{p}})$ assuming that at such a high energy $p-p$ and $\bar{p}-p$ cross sections are identical, and the data 23) on the $p-p$ scattering slope $B_{p p}$ at $|t|=0.02(\mathrm{GeV} / \mathrm{c})^{2}$ (a very usefur compilation and analysis of the data on $B_{p p}$ can be found in Thesis 24), On top of the pomeron exchange cross section we have added in the Regge term $\sigma_{R} / S^{1-\alpha_{R}(o)}$. The fits are fairly insensitive to the Regge intercept $\alpha_{R}(0)$, and we have put $\alpha_{R}(0)$ $=1 / 2$.

By virtue of the derivative analiticity 25) the phase of the forward scattering amplitude is uniquely related to the energy dependence of the total cross section, so that we did not include


Fig. 3 - A dependence of the intercept $\Delta$ on $R$.
into the fit the data on $\rho_{\mathrm{pp}}$.

There are five free parameters in the QCD amplitude (10): $\vec{\sigma}_{2 G}$, $\Delta, R, B_{0}$ and $\alpha_{\mathbb{P}}^{\prime}$ : One mare parameter is the Regge residue $\overrightarrow{\boldsymbol{\sigma}} \mathrm{R}^{\circ}$ However, it turnes out that $R$ and $\Delta$ are very strongly correlated and cannot be fixed uniquely. This $R-\Delta$ correlation is shown in Fig. 3 and the typtcal sets of paxameters are quoted in Table 1 .

Table l : Sets of fitted parameters of the bare QCD at different choices of the ratio $R$ and intercept $\Delta$.

| R | $\Delta$ | $\begin{aligned} & \sigma_{2 G} \\ & \mathrm{mb} \end{aligned}$ | $\begin{gathered} \mathrm{B}_{\mathrm{o}} \\ (\mathrm{GeV} / \mathrm{c})^{-2} \end{gathered}$ | $\begin{gathered} d_{\mathbb{P}}^{\prime} \\ (\mathrm{GeV} / \mathrm{c})^{-2} \end{gathered}$ | $\sigma_{R}$ $\mathrm{mb}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 0.18 | 31.3 | 9.68 | 0.138 | 40.9 |
| 8 | 0.22 | 37.9 | 9.88 | 0.132 | 31.5 |
| 12 | 0.24 | 41.3 | 9.98 | 0.127 | 25.4 |
| 20 | 0.28 | 45.0 | 10.09 | 0.118 | 17.6 |
| 36 | 0.32 | 48.7 | 10.15 | 0.105 | 8.3 |
| 0 | 0.097 | $28.7^{34}$ | 8.87 | 0.141 | 65.0 |
| QE-1 | 0.091 | $27.6^{\text {3F) }}$ | 9.05 | 0.167 | 65.0 |
| QE-2 | 0.12 | $\left.22.3^{36}\right)$ | 9.86 | 0.089 | 87.0 |

These entries for the single-pole fits are the residue of the pole at $j$.


Fig. 4 - The energy dependence of the $\mathrm{p}-\mathrm{p}$ total cross section versus $\Delta$. For the legend of curves see Fig. 6 . Shown also are the values of the $\mathrm{p}-\mathrm{p}$ total cross section determined from the Akeno - Fly's Eye data on $\overrightarrow{\mathrm{O}}_{\mathrm{abs}}$ (pAir) (see Section 5 ) and the fitted accelerator data on $\vec{\sigma}_{\text {tot }}(\mathrm{pp})$.

The resulting extrapolation of the $p-p$ total cross section beyond the accelerator energies is shown in Fig. 4 .

Apart from the $Q C D$ fits with the bare amplitude (10) we have also tried the quasieikonal model fits. (QN) with $C=1.4$. The global unconstrained.fit $Q E-1$ gave $\Delta=0.091$ compared to the pre-Spps intercept. $\Delta=0.07^{17,2)}$, still the fitted curve passes well below the Sppis data points, as the major constraint on $\Delta$ is imposed by the high-precision Fermilab - ISR data. (In fact, such a fit is
basically identical to that of ReIs.1),2) : our intercept $\Delta$ is bigger, but of $\mathbb{P}$ is found to be smaller than that used in 1), 2), $\left.\alpha_{p}^{\prime}=0.25(\mathrm{GeV} / \mathrm{c})^{-2}\right)$. If the fit is constrained to intercept the Spps point at $\sqrt{S}=540 \mathrm{GeV}(Q \mathbb{R}-2)$, then $\Delta=0.12$, but this fit is very poor at the Fermilab energies.

All the single-pole fits are alike in that they grossly under estimate $\vec{\sigma}_{\text {tot }}(\mathrm{Pp})$ in the Akeno - Fly's Eye energy range (Fig.4).

Since experimentally $R \gg 1$ ( see Table 1 ), the effective intercept $\Delta_{\text {eff }}=d \ln \hat{f}(\xi) / d \xi$ rises rapidly with energy,

$$
\begin{equation*}
\Delta_{e f f}=\frac{\Delta}{1+R \cdot \exp [-\Delta \cdot \xi]} \tag{24}
\end{equation*}
$$

from
$=\Delta \Delta_{\text {eff }}=\Delta /(1+R) \ll \Delta$ at moderate energy up to $\Delta_{\text {eff }}$ $=\Delta$ at the truly asymptotic energy - a steep rise of the total cross section thereof. As was discussed above, a certain evidence for that already comes from the accelerator data, and is futher corroborated by the Akeno - Fly's Eye data.


Fig. 5 - The energy dependence of the diffraction cone
slope $B_{p p}$ versus $\Delta$. For the logend of curves see Fig.6. Shown also are the fitted experimental data on $B_{p p}$.

The energy dependence of the diffraction cone slope $B_{p p}$ is shown in Fig. 5 . A rapid rise of the slope for fits with large beyond $10^{7}$ is noticeable, and can easily be understood. An opacity of the p-p amplitude satuxates at the black disc limit and the further rise of $\hat{\sigma}_{\text {tot }}(p p)$ and $B_{p p}$ up with enorgy would mainly come from an expansion of the central black disc area. A counterpart of Eq. (24) holds for the effective slope of the bare pomeron too, but at high energy the rise of $B_{p p}$ is mostly enforced by the


Fig. 6 - The diffraction cone slope $\mathrm{B}_{\mathrm{pp}}$ versus the total cross section $\sigma_{\text {tot }}(\mathrm{pp})$ for the different QCD and QEM fits. Typical values of energy $10^{8}$ and $10^{10} \mathrm{GeV}$. are indicated by dashes normal to the curves.

8 - channei unitarity, rather than the input slope $\alpha_{\mathbb{P}}^{\prime}$. The larger $\Delta$, the steeper ia the rise of $\sigma_{\text {tot }}(p p)$ and the sooner is an onset of the $s$ - channel unitarity imposed rise of the slope, which will be correlated with the rise of the total cross section (see Fig.6).

Specifically, in the black disc limit $\boldsymbol{\sigma}_{\text {tot }}(\mathrm{pp})=4 \mathbb{R} \mathrm{~B}_{\mathrm{pp}}$, but in all the fits this limit is elusive. In the grey disc limit of qZM

$$
\begin{equation*}
\text { . } \sigma_{\text {tot }}(p p)=\frac{1}{c} 4 \pi B_{p p} \tag{25}
\end{equation*}
$$

so that at the same value of


Figal - Predictions of the $\bar{p}-p$ total cross section at the Fernilab collider ( $\sqrt{\mathrm{S}}=1600 \mathrm{GeV}$ )
$\sigma_{\text {tot }}(\mathrm{pp})$ the QEM slope is larger than that of the QCD fits (see Fig.6). Though the exact geometrical scaling is lacking, the above $\sigma_{\text {tot }}(\mathrm{pp})-\mathrm{B}_{\mathrm{pp}}$ corrleation will be of paramount importance for conversion of the data on $\boldsymbol{\sigma}_{a b s}$ (pAir) into $\sigma_{\text {tot }}(p p)$.

The accuracy of the accelerator data on $\mathrm{p}-\mathrm{p}$ scattering is not sufficient to constrain $\Delta$ and $R$. One can well await for the Fermilab collịder data at $\sqrt{S}=1600 \mathrm{GeV}$ a fow mb's accuracy in $\boldsymbol{\sigma}_{\text {tot }}(\mathrm{p} \overline{\mathrm{p}})$ at this energy could put a stringent constraint on $\Delta$ (Fig.7). In the
meantime we shall discuss what could one learn from the existing experimental data on the very high energy air showers, in the $10^{7}-$ $10^{9}$ energy range.
5. A RELIABLE DETERMINATION OF THE PROTON-PROTON TOTAL CROSS SECTION FROM THE EXPERIMENTAL DATA ON ABSORPTION OF THE COSMIC RAY PROTONS IN AIR

We remind that in the cosmic ray experiments one only could measure the absorption cross section

$$
\begin{equation*}
\sigma_{a b s}=\sigma_{t o t}-\vec{\sigma}_{e l}-\sigma_{Q e l} \tag{26}
\end{equation*}
$$

where $\dot{\sigma}_{Q e l}$ is a cross section of the quasielastic scattering on the target nucleus, $p A \rightarrow p A^{*}$, not followed by a production of the new particles, what leaves the incident proton still in the incident flux.

We shall use the standard Glauber - Sitenko - Gribov 26) theory of the multiple scattering. At moderate energies to a good accuracy (for a review see Rer. 27),

$$
\begin{equation*}
\vec{\sigma}_{a b s}(p A)=\int d^{2} \vec{b}\left\{1-\left[1-\frac{1}{A} \vec{\sigma}_{i n}(p p) T(\vec{b})\right]^{A}\right\} \tag{27}
\end{equation*}
$$

where

$$
\begin{equation*}
\Gamma(\vec{b})=\int d z d^{2} \vec{c} \rho_{A}(z, \vec{c}) \cdot \frac{1}{2 \pi B_{p p}} \exp \left[-\frac{(\vec{b}-\vec{c})^{2}}{2 B_{p p}}\right] \tag{28}
\end{equation*}
$$

and $\rho_{A}(\vec{r})$ is the nuclear matter density.

> We have to derive IS correction to (27) with allowance for DD of the projectile and target's nucleons. We shall follow the general method of Ref. 17). Let us start with the total cross section. For interaction of the DSE $|\alpha\rangle$ one finds in a usual way

$$
\begin{equation*}
\left.\sigma_{\text {tot }}(\alpha A)=2 \int d^{2} \vec{b}\left\{1-<1, \operatorname{in}|<\exp [-\alpha \beta f(\vec{b}-\vec{c})]\rangle_{\beta} \mid 1, \text { in }\right\rangle^{A}\right\} \tag{29}
\end{equation*}
$$

what can be derived as follows: Start with the state of the nucleus in which the nucleons $\mu_{1}$ are in DSE $\left.\left|\beta_{1}\right\rangle, \ldots, 1 \beta_{A}\right\rangle$. The averaging over $\beta_{i}$, i. $\theta_{0}$, correcting the $\alpha N_{i}$ ampiltude for DD of the target nucleon $N_{i}$, can be carried out separately for different nucleone. Namely, one has to calculate the s-matrix element over the single-nucleon wave function of the ground state of the nucleus,
$\langle 1$, in $\mid<\exp [-\alpha \beta f(\vec{b}-\vec{c})]\rangle \beta \mid 1$, in $\rangle$, then take their product over all the targets nucleons and, finally, take the average over the eigenstates of the projectile:
$\sigma_{\text {tot }}(p A)=2 \int d^{2} \vec{b}\left\{1-\left\langle\left[\langle 1, i n|\langle\exp [-\alpha \beta f(\vec{b}-\vec{c})]\rangle_{\beta}|1, i n\rangle\right]^{A}\right\rangle_{\alpha}\right\}$.
A similar derivation of the absorption cross section results in $\sigma_{a b s}(p A)=\int d^{2} \vec{b}\{1-$
$\left.-\left\langle\left\langle\left[\langle 1, \operatorname{in}|\left\langle\left\langle\exp \left[-\alpha_{1} \beta_{1} f(\vec{b}-\vec{c})-\alpha_{2} \beta_{2} f^{*}(\vec{b}-\vec{c})\right]\right\rangle\right\rangle_{\beta_{1} \beta_{2}}|1, i n\rangle\right]^{A}\right\rangle\right\rangle_{\alpha_{1} \alpha_{2}}\right\}$.
It is worth while to emphasize that is eorrection for $D D$ of both the projectile and the target's nucleons is computed in (30), (31) in a unique way.

In the QCD fits we retain in (31), as well as in computing the $p-p$ amplitude, only terms $\sim\left\langle\Delta \alpha^{2}\right\rangle$. We have used the Gaussian paramerrization of the nuclear charge density with the charge radii $\left\langle R_{C h}^{2}\right\rangle^{1 / 2}=2.54 \mathrm{~F}$ and 2.72 F for the nitrogen and oxygen nuclei, respectively ${ }^{28)}$. In order to calculate the nuclear matter density one has to subtract from the charge radii of nuclei a contribution of the proton's charge radius 27 ). Otherwise $\boldsymbol{\sigma}_{\text {abs }}$ (pAir) will erroneously be overestimated by about $7 \%$ :

At moderate energies the total cross section $\sigma_{\text {tot }}(\mathrm{pp})$ and the slope $\mathrm{B}_{\mathrm{pp}}$ are the two independent parameters, and $\sigma_{\text {abs }}$ (pAir) does depend on both (see Eqs.(27), (28)). However, at high energies, with an onset of the black disc regime, the absorption on nuclei will also approach the black disc limit

$$
\begin{equation*}
\sigma_{a b s}(p A) \simeq \pi \cdot\left(R_{A}+\sqrt{2 B_{p p}}\right)^{2} \tag{32}
\end{equation*}
$$

and will mainly depend on the slope $B_{p p}$ rather than $\sigma_{\text {tot }}(p p)$. Still, by virtue of the above discussed the $s$ - channel unitarity imposed correlation between $G_{\text {tot }}(\mathrm{pp})$ and $\mathrm{B}_{\mathrm{pp}}$, Eq.(32) is paramaount to $\sigma_{a b s}$ (pAir) being only a universal function of the $\mathrm{p}-\mathrm{p}$ total cross section.

In Fig. 8 we show our predictions for $\sigma_{a b s}$ (pAir) at different intercepts $\triangle$. The single-pole QCD and QEM fits do grossly underestimate $\sigma_{\text {abs }}$ (pAir) at $E>10^{6} \mathrm{GeV}$. Different fits give rapidly divergent curves for $\sigma_{a b s}$ (pAir), following the general




Fig. 8 - Predicted energy-dependence -of the cross section of absorption of protons in air versus $\Delta$. Shown are the AkenoFly's Eye data on $\sigma_{a b s}$ (pAir) - For the legend of curves see Fig. 6 -

Fig. 9 - The plot of $\sigma_{\mathrm{abs}}$ (pAir) versus $\sigma_{\text {tot }}(p p)$ in the different QCD and QEM fits. For the legend of curves see Fig. 6 .

Fig. 10 - The plot of $6_{\text {abs }}$ (pAir) versus $\sigma_{\text {in }}(\mathrm{pp})$ in the different $Q C D$ and Qum fitb. For the legend of curves see Fig. 6 .
pattern of $\sigma_{\text {tot }}(\mathrm{pp})$ in Fig. 4 . In Figs. 9 and 10 we demonstrate how $\vec{\sigma}_{\text {abs }}$ (pAir) is correlated with $\sigma_{\text {tot }}(\mathrm{pp})$ and $\boldsymbol{\sigma}_{\text {in }}(\mathrm{pp})$. The $\sigma_{\text {abs }}$ (pAir) $-\vec{\sigma}_{\text {in }}$ (pp) plot in Fig. 10 exhibits a remarkable stability of conversion of $\sigma_{\text {abs }}$ (pAir) into $\sigma_{\text {in }}$ (pp) . Specifically, in the region of interest, $\sigma_{a b s}$ (pAir) $\sim 600 \mathrm{mb}$, the theoretical systematic error of this conversion is well below 10 mb in $\boldsymbol{\sigma}_{\text {in }}(\mathrm{pp})$ : Furthermore, as it only does make sence to compare the curves for
$\Delta=0.24$ and $\Delta=0.32$, the ones which broadly reproduce the Akeno - Fly's Eye data on $\boldsymbol{\sigma}_{\text {abs }}$ (pAir), then the theoretical uncertainty of the so determined $\left.\sigma_{\text {in }}(p)^{\prime}\right)$ is at most few mbis.


Fig. 11 - Predictions for the high-energy behaviour of the $\mathrm{p}-\mathrm{p}$ inelastic cross section versus $\Delta$ and our determinations of $\sigma_{\text {in }}(\mathrm{pp})$ from the Akeno Fly's Eye data on $\sigma_{a b s}$ (pAir). For the legend of curves see Fig. 6 .

Our predictions for $\vec{\sigma}_{\text {in }}(\mathrm{pp})$ at high energies versus the intercept $\Delta$ and the values of $\sigma_{\text {in }}(\mathrm{pp})$ determined this way are shown in Fig.11. We emphasize once more their model independence. Its origin can be traced back to a $\vec{\sigma}_{\text {in }}(\mathrm{pp})-\mathrm{B}_{\mathrm{pp}}$ correlation being even stronger than that between $\sigma_{\text {tot }}(\mathrm{pp})$ and $\mathrm{B}_{\mathrm{pp}}$ (Fig.12).

Of the three quantities - $\boldsymbol{\sigma}_{\text {tot }}(\mathrm{pp}), \sigma_{\text {in }}(\mathrm{pp})$ and $\sigma_{\mathrm{el}}(\mathrm{pp})$ - the latter is the most semsitive one to the value of the slope $B_{p p}$. Hence, the major source of the theoretical uncertainty in determination of $\quad \sigma_{\text {tot }}(\mathrm{pp})$ from $\sigma_{\text {abs }}$ (pAir) is an uncertainty of evaluation of $\sigma_{e l}^{\text {el }}(\mathrm{pp})$ once $\sigma_{\text {in }}(\mathrm{pp})$ is known. Yet, from a comparison of curves for $\Delta=0.24$ and $\Delta=0.32$ in Fig. 9 we can conclude that the theoretical systematic error of determination of $\quad \sigma_{\text {tot }}(\mathrm{pp})$ does not exceed $10-15 \mathrm{mb}$. The Akeno - Fly's Eye values of $G_{\text {tot }}(\mathrm{pp})$ plotted in Fig. 4 do correspond to the $\Delta=0.32$ curve in Fig. 9 , the best approximation to the experimental data points.


Fig. 12 - The diffraction cone slope ${ }^{\mathrm{B}} \mathrm{pp}$ versus $\boldsymbol{\sigma}_{\text {in }}(\mathrm{pp}) \mathrm{pp}_{\mathrm{in}}$ the QCD and QEM fits with different intercept $\Delta$. All the notations are the same as in Fig. 6 .

Regarding the role of the inelastic shadowing corrections, we restrict ourselves to a statement that uncertainties with extrapolating Is correction to $\boldsymbol{\sigma}_{\text {abs }}$ (pAir) up in energy are virtually negligible at high energy. Indeed, IS correction to $\boldsymbol{\sigma}_{\text {abs }}$ (pAir) is known to be numerically small 17). Its relative magnitude does gradually decrease with energy, as in the black dise regime $D D$ only could take place on the edge of the nucleus. Moreover, for the same reason the ratio $\sigma_{D D}(\mathrm{pp}) / \boldsymbol{\sigma} \sigma_{\text {tot }}(\mathrm{pp})$ should decrease with energy already in $p-p$ scattering, and there is a positive evidence for that in the recent $S p \bar{p} S$ data ${ }^{4)}$.

One more technical comment. The proton-proton profile function $F(\vec{b})$ becomes significantly different from the naive Gaussian one already at Spps energy. At still higher energies it is absolutely wrong to evaluate the absorption cross section from Eq. (27) using in it $T(\vec{b})$ evaluated in the Gaussian approximation (28) only in terms of $\sigma_{\text {tot }}(\mathrm{pp})$ and $\mathrm{B}_{\mathrm{pp}}$. An accurate evaluation of the nuclear cross section demands for a much more detailed calculation of the $\mathrm{p}-\mathrm{p}$ profile function, complying with the s - channel unitarity constraints, and with the proper allowance for the inelastic shadowing, like we have done above. Otherwige an arbitrary choice of the high-energy parametrization of the $\mathrm{p}-\mathrm{p}$ total cross section and slope and, on top of that, making use of the Gaussian approximation (28), could drive to uncontrollable errors in the high-energy behaviour of $\sigma_{\text {abs }}$ (pAir) and, vice versa, of determination of $\sigma_{\text {tot }}(\mathrm{pp})$ from the data on the nuslear cross section.

Pricisely that kind of orror slipped in into previous detormina-
tions 7), 8), $\quad \widehat{\sigma}_{\text {tot }}(\mathrm{pp})=120 \pm 10 \mathrm{mb}$. from the Fly's Eye result $\sigma_{\mathrm{abs}}$ (pAir) $=540 \pm 40 \mathrm{mb}$ at $E=4.5 \cdot 10^{8} \mathrm{GeV}$. The right numbers with the right error bars are:

$$
\begin{aligned}
& \sigma_{\text {in }}(\mathrm{pp})=113 \pm 16 \text { (stat.) } \pm 5 \text { (syst.) } \mathrm{mb} \text {, } \\
& \sigma_{\text {tot }}(\mathrm{pp})=164 \pm 32 \text { (stat.) } \pm 10 \text { (syst.) } \mathrm{mb} .
\end{aligned}
$$

Similar numbers for the Akeno result $\quad \vec{\sigma}_{\text {abs }}(\text { pAir })^{\prime}=570 \pm 12 \mathrm{mb}$ at $\mathrm{E}=4.8 \cdot 10^{7} \mathrm{GeV}$ (read from the plot in the Conference paper ${ }^{6}$ ), are: $\quad \sigma_{\text {in }}(\mathrm{pp})=226 \pm 5 \mathrm{mb}$ and $\sigma_{\text {tot }}(\mathrm{pp})=188 \pm 10 \mathrm{mb}$. Apparently, the major


Fig. 13 - The ratio $\sigma_{\text {el }} / \sigma_{\text {in }}$ as a function of $\sigma_{\text {in }}(\mathrm{pp})$ for the different high-energy extrapolations of the $\mathrm{p}-\mathrm{p}$ scattering amplitude. For the legend of curves see Fig. 6 . source of the above underestimation of the $p-p$ total cross section in Refs. 7 , 8 ) was an imprudent extension of the geometric scaling up to the superbigh energies, though it is badly broken already at the Spp̄s energy 18). The geometric scaling does not hold to any sensible accuracy (see Figs. 6 and 12), and onforcing it one would underestimate $\sigma_{1 n}(p p)$ by about 15-20 \% . This estimation follows from comparison of Takagi's 8) determination of $\sigma_{\text {in }}(p p)$ with ours. Secondly, one has to add $\sigma_{e l}(p p)$ on top of $\hat{\sigma}_{\text {in }}(p p)$. To this end, the geometric scaling implies $\boldsymbol{\sigma}_{\text {el }} / \sigma_{\text {in }}=$ const, and Takagi had put this ratio to be 0.196 at all the energies. However, in theories with the rising cross section, an onset of the $s$ - channel unitarity enforced black disc regime leads to a rapid rise of $\sigma_{\text {el }}(\mathrm{pp}) / \sigma_{\text {in }}(\mathrm{pp})$ with energy. In Fig. 13 we show how this ratio depends on $\sigma_{\text {in }}(p p)$ at different intercepts $\Delta$.
6. SCALING VIOLATIONS AND MEASUREMENTS OF $\sigma_{a b s}$ (pAir).

Measurements of the depth of absorption of the cosmic ray protons would have given a straightforward determination of $\sigma_{\text {abs }}$ (pAir). In fact, one rather determines experimentally a depth of the maximum
of the electromagnetio oomponent of the extensive air shower, so that a quantity of prime interest is a rate of the transfer of energy from the incident proton to the secondary particles ( $\pi^{\circ} \mathrm{s}$ ).

An important observation is that the $s$ - channel unitarity enforces strong violation of the scaling of the fragmentation spectra and that a magnitude of the scaling violations is basically controlled by the magnitude of the p-air absorption cross section. Were this important correlation included into an analysis of the experimental data on the longituatinal deveiopment of extensive air showers, the accuracy of determination of $\boldsymbol{\sigma}_{\text {abs }}$ (pAir) would have been much thigher. Below we shall diecuss the scaling violations in a simple model.

Ignoring IS corrections, which are unimportant for the scaling violation analysis, we obtain

$$
\begin{equation*}
\vec{\sigma}_{i n}(p p)=\int d^{2} \vec{b}\{1-\exp [-2 \operatorname{Ref}(\vec{b})] \tag{33}
\end{equation*}
$$

and the $\nu$-fold interaction cross sections

$$
\begin{equation*}
\vec{G}_{\nu}=\int d^{2} \vec{b} \cdot \frac{[2 \operatorname{Re} f(\vec{b})]^{\nu}}{\nu!} \exp [-2 \operatorname{Re} f(\vec{b})] \tag{34}
\end{equation*}
$$

(A probabilistic derivation of (34) is since long widely used in analysis of the cosmic ray data 29), for a modern derivation based on the Abramovsiri - Gribov - Kancheli cutting rules ${ }^{30)}$ see Refs. ${ }^{2)}$,5) and the review 31),

Obviously, the larger $\nu$ the softer is the spectrum of the leading particles (protons . . .). If $L_{1}(x)=\left(x / \sigma_{1}\right)\left(d \sigma_{1} / d x\right)$ is a leading particle spectrum for orie cut pomeron $(\nu=1$ ) (here $x$ is Feynman's scaling variable), then

$$
\begin{equation*}
L_{v+1}(x)=\int_{x}^{1} \frac{d z}{z} L_{i}(z) L_{v}\left(\frac{x}{z}\right) \tag{35}
\end{equation*}
$$

and the final spectrum $\mathrm{L}_{\mathrm{pp}}(\mathrm{x})$ will be given by

$$
\begin{equation*}
L_{p p}(x)=\sum_{\nu} w_{\mathcal{V}} \cdot I_{\nu}(x) \tag{36}
\end{equation*}
$$

where $\quad w_{v}=\sigma_{v} / \sigma_{i n}(p p)$.
The above prescriptions (34) and (35) do correspond to the leading particle cascade model, familiar to the cosmic ray physicists.

A similar, but tedious calculations of the leading particle spectrum can be repeated for pAir collisions too. Obviously, $W_{\nu}(\mathrm{pp})$ and $\mathcal{W}_{\mathcal{V}}$ (pair) are almost uniquely determined by the magnitude of $\tilde{\sigma}_{\text {in }}(\mathrm{pp})$ and $\sigma_{a b s}$ (pAir), what implies that such important quantities like the elasticity coefficient $K_{e l}=\langle x\rangle$, only do depend on $\sigma_{\text {in }}(p p)$ and $\sigma_{a b s}$ (pAir).

Simple calculations show that for the input spectrum of the form

$$
\begin{equation*}
I_{I}(x)=\beta \cdot x^{\beta} \tag{37}
\end{equation*}
$$

the leading particle spectra in the $p-p$ and $p-a i r$ collisions too are very well approximated by equation (37) with the exponents

$$
\begin{align*}
& \beta_{\mathrm{pp}}=\beta_{\mathrm{pp}}(E=100 \mathrm{GeV}) \exp \left[-\frac{\sigma_{i n}(\mathrm{pp})-32}{120}\right],  \tag{38}\\
& \beta_{\mathrm{pAir}}=0.3 \cdot \beta_{\mathrm{pp}}(E=100 \mathrm{GeV}) \exp \left[-\frac{\sigma_{\mathrm{abs}}(\mathrm{pAir})-500}{250}\right] \tag{39}
\end{align*}
$$

where $\sigma_{\text {in }}(p p)$ and $\sigma_{\text {abs }}$ (pAir) are in millibarns. The parametrization (39) holds to a five per cent accuracy at $\sigma_{a b s}$ (pAir) > 350 mb . A similar parametrization of the elaticity coefficient is

$$
\begin{equation*}
K_{e I}(\text { pAir })=K_{e l}(\mathrm{pp}, E=100 \mathrm{GeV}) \cdot 0.45 \cdot \exp \left[-\frac{\dot{\sigma}_{a b s}(\mathrm{pAir})-500}{300}\right] . \tag{40}
\end{equation*}
$$



Fig. 14 - The energy dependence ( $x=E / \mathrm{F}_{\text {max }}$ ) of the leading particle spectrum in p-air collisions at different values of $\vec{\sigma}_{a b s}$ (pAir). Shown is the ratio of the p-air spectrum to the $p-p$ spectrum at $\mathrm{E}=100 \mathrm{GeV}$.

A pattern of violation of the scaling of the leading particle spectrum in the p-air collisions is shown in Fig. 14 . The ratio $R_{x}$
would have been unity were it not for the intranuclear absorption of the leading particle and the increase of the absorption probability, i.e., a decrease of $\omega_{l}$, with the rise of ${\overrightarrow{\sigma_{i n}}}^{(p p)}$ and $\vec{\sigma}_{\text {abs }}$ (pAir).

For instance, starting with $K_{i n}(\mathrm{pp}, \mathrm{E}=100 \mathrm{GeV})=0.5$ we wind up with $K_{\text {in }}$ (pAir) $=0.78$ if $\vec{\sigma}_{\text {abs }}($ pAir $)=500 \mathrm{mb}$. Therefore, in the range of the p-air absorption cross sections of our interest the scaling violation is quite significant. Specifically, the rate of dissipation of the primary proton's energy is much higher than the naive scaling model would suggest. It is very likely that incorporation of the above scaling violation into the Monte-Carlo simulation of the longitudinal development of extensive air showers would eventually result in a somewhat lower values of $\sigma$ abs (pAir) than those cited in Refs. 6),7).

The Additive Quark Model 3l) and the Dual Unitarization Model (Refs. 2).5), would predict weaker scaling violations that the leading particle cascade model. The latter model differs just in prescriptions for constructing $I_{V}(x)$, whereas in the former the rules of computing $\omega_{\nu}$ differ too. Yet, the results would not differ much from (38) - (40).

The extensive air shower data will remain the sole source of information on the proton-proton total cross section at superhigh energies up to $10^{9} \mathrm{GeV}$ for at least a decade, until SSC becomes operative. In view of the immense importance of learning of the asymptotic properties of the hadronic interactions in view of the recent developements in $Q C D$ theory of the pomeron, a novel analysis of the Akeno - Fly's Eye data with allowance for the above scaling violations is eagerly awaited.

## 7. CONCLUSIONS

The quantum chromodynamics predicts a specific family of the $j-$ plane singularities of the hadron-hadron scattering amplitude ${ }^{9}$ ). Namely, the pomeron is shown to be a series of poles at $1<j<l+\Delta$. The perturbative calculations of the intercept of the leading pole lead to a rather big $\Delta$ at $|t|=0{ }^{9}$ ):

$$
\Delta \simeq \frac{12 \sqrt{2}}{\pi} \alpha_{s} \sim 1
$$

The perturbative calculations also predict the flattening off of the effective trajectory, $\alpha_{\mathbb{P}}\left(\vec{q}^{2}\right) \rightarrow 1$, at large $\left.\vec{q}^{2} 12,9\right)$. There is
a goad evidence for that $Q C D$ flattening of the pomeron trajectory from the large-t meson nucleon elastic scattering 12).

A phenomenological analysis of the experimental data suggests the following hierarchy of the $j$ - plane singularities in the total cross section: A dominance of the two-gluon exchange $(\underset{J}{f}=I, \Delta=$ 0 ) at the Serpukhov - Fermilab energies is superseded by a dominance of the contribution of poles with $\Delta>0$, the higher the energy the bigger $\Delta$ of the dominant pole, and so forth till the dominance of the rightmost singularity. Such a rise of $\Delta_{\text {eff }}$ with energy is already corroborated by a steep rise of the protonproton total cross section from Fermilab to ISR, and still steeper rise further up from ISR to Spps. The recent Akeno 6) and Fly's Eye 7) data on absorption of the cosmic ray protons in air at $E=10^{7}-10^{9} \mathrm{GeV}$ demand for $\Delta=0.25-0.35$, what nicely fits the QCD pattern of the high-energy scattering of hadrons.

The forthcoming data on $\vec{\sigma}_{\text {tot }}(\overline{\mathrm{p}} \mathrm{p})$ from the Fermilab collider at $\quad \sqrt{S}=1600 \mathrm{GeV}$ could significantly improve our understanding of the asymptotic properties of $p-p$ scattering.

Regarding the value of the cosmic ray data, our principal finding is that there exists a reliable, model-independent conversion of the experimental data on $\vec{\sigma}_{a b s}($ pAir $)$ into $\sigma_{i n}(p p)$. Adding up $\sigma_{\text {el }}(\mathrm{pp})$ to the so determined $\sigma_{\text {in }}(\mathrm{pp})$ introduces some uncertainty into such evaluation of $\sigma_{\text {tot }}(\mathrm{pp})$, but this uncertainty is well below 10 - 15 mb .

Another important observation is that one could greatly improve an accuracy of measurements of the proton absorption cross section $\sigma_{a b s}(p A i r)$ in the extensive air shower experiments, provided that the scaling violations, which are almost uniquely controlled by the magnitude of $\vec{\sigma}_{a b s}$ (pAir), were properly taken into account.

Our optimistic conclusion is that one can learn a lot on the proton-proton interaction cross section up to the end-point energy of $7 \cdot 10^{10}$ GeV from experiments on observation of the extensive air showers.

The authors are grateful to L.I.Lapidus for reading the manuscript and vaiuable suggestions, and to V.R.Zoller for useful comments.

## ROIOEOnOe日

1. Kopeliovioh B.Z. and L.I.Lapidus - ZhETF 71, 61 (1976) Dubovikov M.S., B.Z.Kopeliovich, L.I.Tapidus and K.A.Ter-Martirosyan - Nuol. Phys. Bl23, 147 (1977)
2. Kaidalov A.B. - In: Elementary Particles. Proc. of XI-th ITEP Wintor Sohool in Particle Physics, Moscow, Energcizdat, 1984, v. 4
3. Bozzo M. et al. - Phys. Lett. B147, 392 (1984)
4. Rushbrooke J.G. - Rapporteurs Talk at the Bari Conference. CERN preprint EP/85-124 (1985)
5. Kaidalov A.B. - Lectures at A11-Union School on Inelastic Interactions Beyond Accelerator Energies. Nor-Hamberd, September 1985.
6. Hara T. et al. - Phys. Rev. Lett. 50, 2058 (1983) Hara T. et al. - In: Proc. Intern. Symp. Cosmic Rays and Particle Physics. Tokyo, INS, March 19-23, 1984
7. Baltrusaitis R.M. et al. - Phys. Rev. Lett. 52, 1380 (1984) Baltrusaitis R.M. et al. - In: Proc. 19-th Intern. Cosmic Ray Gonference. La Jolla, August 11-23, 1985, v. 6
8. Linsley J. - Lettere il Nuovo Cimento 42, 403 (1985) Gaisser T. and F.Halzen - Madison preprint MAD/PH/248 (1985) Takagi F. - Tohoku University preprint TU/83/265 (1983)
9. Lipatov L.N. - Gatchina preprint LINP-1137 (1985) Kuraev E.A., L.N.Lipatov and V.S.Fadin - ZhETF 29, 377 (1977)
10. Yodh G.B., Y.Pal and J.S.Trefil - Phys. Rev. Lett. 28, 1005 (1972)
11. Low F.E. - Phys. Rev. D12, 163 (1975)

Nussinov S. - Phys. Rev. Lett. 34, 1286 (1975) Gunion J.F. and D.E.Soper - Phys. Rev. Bl. (1977)
12. Kopeliovich B.Z. - In: High Energy Physics. Proc. of XX-th Winter School of LINP, Leningrad, 1985, p. 140
13. Akerlof C.W. et al. - Phys. Rev. D14, 2864 (1976)
14. Barger V., J.Luthe and P.J.N.Philiips - Nucl. Phys. B88, 237 (1975)
15. Levin E.M. and M.G.Ryskin - Yadernaya Fizika 34, 1114 (1981)
16. Kopeliovich B.Z. and L.I.Lapidus - Pisma v ZhETF 28, 664 (1978) Kopeliovich B.Z. and L.I.Iapidus - In: Multiple Production and Limiting Fragmentation of Nuclei. JINR-Dl2-12306, Dubna, 1978, p. 469
17. Nikolaev N.N. - ZhFif 81, 814 (1981)
18. Fiearnley T. - CERN preprint EP/85-137
19. Zamolodchikov A.B., B.Z.Kopeliovich and L.I.Lapidus - Pisma v ZhEITF 33, 612 (i981)
20. Miettinen H.I. and J.Pumplin - Phys. Rev. D18, 1469 (1978)
21. Dakhno E.G. - Yadernaya Fizika 32, 993. (1983)
22. Caroll A.S. et al. - Phys. Lett. B61, 303 (1976) ; Phys. Lett. B80, 423 (1979)
Ayres D.S et al. - Phys. Rev. D15, 3105 (1977)
Amaldi U. et al. - Nucl. Phys. Bl66, 301 (1980)
Baksay L. et aI. - Nuc1. Phys. B1.41, 1 (1978)
Ambrosio L. et al. - Phys. Lett. Bl15, 495 (1982)
23. Burq J.P. et al. - Nucl. Phys. B217, 285 (1983) Amaldi U. et al. - Phys. Lett. B36, 504 (1971); Phys. Lett. B66, 390 (1977)
Baksay L. et al. - Nucl. Phys. B141, 1 (1978)
Fajardo L.A. - Ph.D.Thesis, Yale University (1980)
Bozzo M. et al. - Phys. Lett. BI47, 385 (1984)
24. Martin J.P. - Ph.D.Thesis. Universite Claude Bernard, Lyon - 1, Institut de Physique Nucleaire (1981)
25. Bronzan J.B., G.L.Kane and U.P.Sukhatme - Pbys. Lett. B49, 272 (1974)
26. Glauber R.J. - In: Lectures in Theoretical Physics. Eds. W.E.Brittin and L.G.Dunham, Interscience, N-Y., 1959, v.l Sitenko A.G. - Ukrainian Journal of Physics 4, 152 (1959) Gribov V.N. - 2heTF 56, 892 (1969)
27. Nikolaev N.N. - Tokyo University preprint INS-Rep.-538 (1985)
28. Barrett R.C. and D.F.Jackson - Nuclear Sizes and Structure, Clarend̄̀n Press, Oxford, 1977
De Yaeger C.W., H.De Vries and C.De Viries - Atomic Data and

- Nuclear Data Tables 14, 479 (1974)

29. Azaryan M.O., S.R.Gevorkyan and E.A.Mamidzhanyan - Yadernaya Fizika 20, 398 (1974)
Lebedev A.M., S.A.Slavatinskii and B.V.Tolkachev - 2hETF 46, 2161 (1964)
30. Abramovskii V.A., V.N.Gribov and O.V.Kancheli - Yadernaya, Fizika 18, 595 (1973)
31. N.N.Nikolaev, Uspekhi Fizicheskikh Nauk 134, 369 (1981)

Копелиовии Б.З., Николавд Н.Н., Поташникова И.К.
E2-86-125
Дифракционноо рассопние адронов и ядер
в рамках продетоолений КХД
Обсущдоотея попрос об асимптотике нуклон-нуклонных сечений в рамках предстадлоний пертурбатияной квантовой хромодинамики. При умеренно высоких энергиях пертурбативный двухглюонный обмен удовлетворительно воспроиз водит постояннуп часть полного сечения. С роттом энертий начинает доминировать вклад серии полюсов с $\Delta=j-1>0$, причем $\Delta_{\text {eff }}$ растет с ростом знергии. Показано, что данные по поглощению космических лучей в атмосфере в области энергий $10^{5}-10^{6}$ ТэВ требуэт $\sigma_{\text {ioi }}^{\text {pp }}=160-200$ мбн, что удается воспроизвести количественно, только если в асимптотике $\Delta \approx 0,25 \div 0,35$. Стандартное однополюсное описание дает при этих энергиях существенно меньшее сечение, -100 мбн, и не воспроизводит данные космических лучей. Цитируемые в Һитературе пересчеты от $\sigma_{\text {аия }}$ (рАіг) к $\sigma_{\text {tot }}$ ( p ) - ошибочны. Сделано важное наблэдение, что нарушение скейлинга фрагментационных спектров, существенное при моделировании ШАЛ, жестко с'вязано с величиной $\sigma_{\text {abs }}$ ( pAir ). Учет этой зависимости позволил бы существенно повысить точность определения $\sigma_{\mathrm{zbs}}$ (pAit).

Работа выполнена в Лаборатории ядерных проблем ОИяи.
Препринт Объединенного института ядерных исследований. Дубна 1986

Kopeliovich B.Z., Nikolaev N.N., Potashnikova I.K.
E2-86-125
QCD Suggested High-Energy Asymptocics
of the Diffraction Proton-Protoin Scattering
and the Cosmic Ray Data
Asymptotics of nucleon-nucleon cross sections is discussed within the perturbation quantum chromodyriamics representations. At moderately high enefgies the perturbative twongluon exchange satisfactorily reproduces the constant part of the total cross section. As the energy goes up, a series of the $j$-plane poles at $\Delta=j-1>0$, dominates, the higher the energy, the bigger $\Delta_{\text {eff }}$. It is shown that the data on absorption of cosmlc rays in atmósphere within the $10^{5}-10 \mathrm{TeV}$ energy range need $\sigma_{\text {tot }}^{p \mathrm{p}}=160-200 \mathrm{mbn}$ which could be reproduced quantitatively, if only in asymptotics $\Delta=0,25 \div 0,35$. Standard one-pole description gives at these energies a sufficiently smal ler cross section, -100 mbn , and does not reproduce the cosmic ray data. The quoted in literature determinations from $\sigma_{2 b s}$ (pAir) to $\sigma_{\text {tot }}(p p)$ are erroneous. An important observiotion is that violation of the scaling of the fragmentation spectra is strongly correlated wlth the value of $\sigma_{\text {ais }}$ ( pAir ). Making allowance for thls dependence should essentially increase the rellability of $\sigma_{\text {abi }}$ ( $\mathrm{P} A$ it) determination.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Inslitute for Nuclear Reparch. Dubas 1986


[^0]:    * L.D.Landau Institute for Theoretical

    Physics of the Academy of Sciences of the USSR, 142432 Chernogolovka, Moscow Region

