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RADIATIVE CORRECTIONS TO DEEP INELASTIC MUON SCATTERING

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1. INTRODUCTION

The investigation of deep inelastic lepton-nucleon scattering is one of the most important methods to probe the nucleon structure. In the case of charged leptons which are regarded as pointlike Dirac particles the single-photon exchange part of a cross section of the process

$$\ell + N \to \ell + \text{hadrons} \qquad (1)$$

can be explicitly calculated in quantum electrodynamics and interpreted in terms of the structure of a target nucleon or nuclei.

In an inclusive experiment

$$\ell + N \to \ell + X \qquad (2)$$

when in the final state only the scattered lepton is measured and X is anything including hadrons, γ - quanta and additional leptons, the observed cross section contains, in addition to the single-photon, contributions from higher order electromagnetic and, for beam energies larger than 100 GeV, weak interactions too.

These, in the following loosely called radiative corrections, have to be calculated and subtracted from the observed cross section (2) before the determination of structure functions. Obviously, the attainable accuracy of the structure functions directly depends on the accuracy level of the radiative correction calculation.

Traditionally, the calculation of radiative corrections to deep inelastic ℓN scattering is based on the classical work by Mo and Tsai/1/ and contains α^3 leptonic electromagnetic processes to (2). Only soft-photon emission contributions are summed up over all orders in α. Higher-order electromagnetic contributions and radiative corrections to the hadronic vertex are believed to be comparatively small and negligible.

The majority of deep inelastic scattering experiments has been analysed at this level of radiative-correction treatment.

A revised calculation of electromagnetic radiative corrections was performed in refs./2,3/. Completely covariant formulae are obtained which contrary to those given by Mo and Tsai, do not contain the unphysical "soft-photon" parameter.

Furthermore, progress has been achieved in the treatment of hadronic-vertex corrections, α^4 electromagnetic and weak contributions.

In this paper, contributions of the electromagnetic radiative processes up to the order α^4 and weak interaction contribution in the lowest order to (2) have been taken into consideration for the analysis of a muon scattering experiment at CERN/4/. We give a brief survey of the formalisms and models used for its calculation and summarize the results for beam energies of 280 GeV and, with regard to Fermilab muon beam, 750 GeV.

2. HIGHER-ORDER ELECTROMAGNETIC PROCESSES

The radiative corrections resulting from each group "a" of processes from (2) are defined as

$$\delta^a(x,y) = \frac{d^2\sigma^a}{dx\,dy} \Big/ \frac{d^2\sigma^o}{dx\,dy}\ , \qquad (3)$$

where $d^2\sigma^a/dx\,dy$ and $d^2\sigma^o/dx\,dy$ denote the inclusive cross sections of higher-order processes of group "a" and the single-photon exchange part of the reaction (1), respectively; x and y are the scaling variables usually used. The overall radiative correction factor, i.e., the ratio of the measured cross section $d^2\sigma^{meas}/dx\,dy$ to the cross section $d^2\sigma^o/dx\,dy$, can be expressed as follows:

$$\Delta(x,y) \equiv \frac{d^2\sigma^{meas}}{dx\,dy} \Big/ \frac{d^2\sigma^o}{dx\,dy} = 1 + \sum_a \delta^a(x,y). \qquad (4)$$

Results are discussed for the case of μp scattering, but the majority of formulae can be conventionally transcribed also for the case of muon-nucleus scattering /1,5,6/.

Higher-order electromagnetic radiative processes are coarsely subdivided in lepton- and hadron-current contributions.

2.1. Lepton-Current Radiative Processes

A substantial part of the lepton-current corrections of the order α,

$$\delta^{lept}(\alpha) = \frac{d^2\sigma^{lept}}{dx\,dy}(\alpha^3) \Big/ \frac{d^2\sigma^o}{dx\,dy}(\alpha^2), \qquad (5)$$

where the parentheses contain the order of the cross section and correction in the electromagnetic coupling constant α, re-

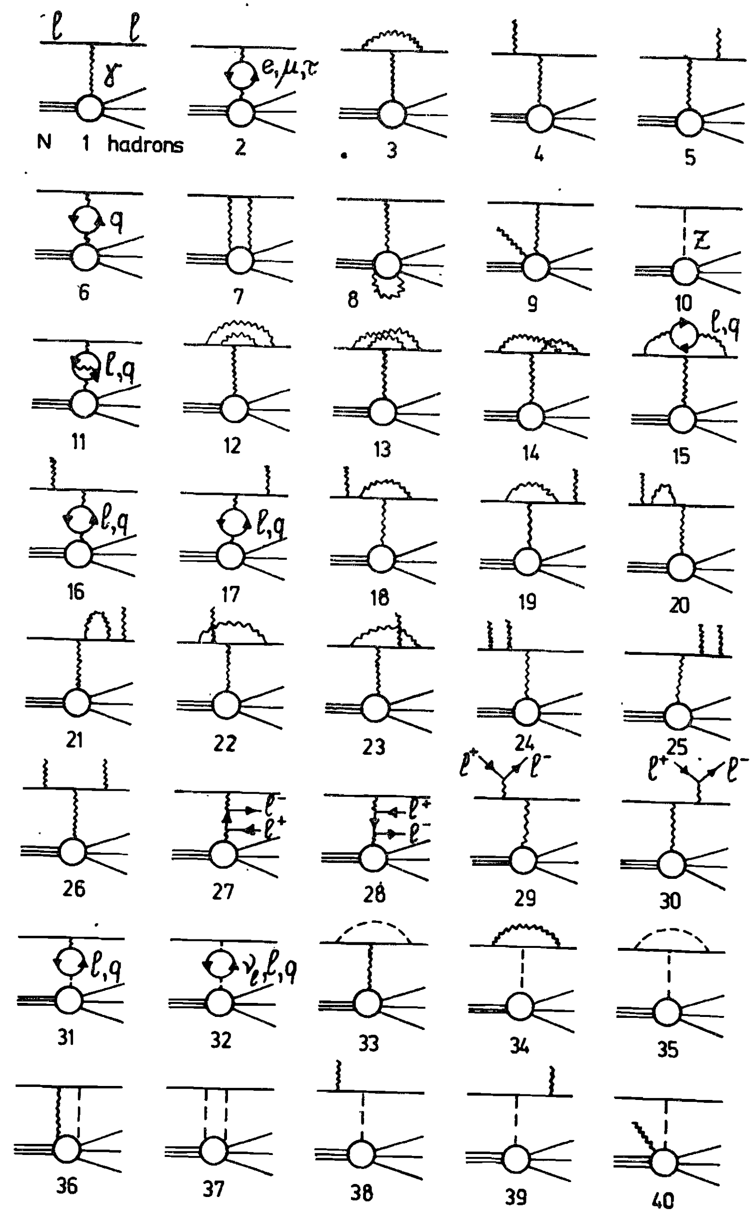

Fig.1. Lowest- and higher-order electroweak radiative processes contributing to the observed deep inelastic cross section.

sults from the bremsstrahlung diagrams 4 and 5 of Fig.1. The hadronic final state may be either a single proton (the so-called elastic radiative tail) or a hadron jet (the inelastic radiative tail) that forms together with diagram 3 the corrections to the continuous spectrum, i.e.,

$$\delta^{lept}(\alpha) = \delta^{lept}_{el}(\alpha) + \delta^{lept}_{inel}(\alpha). \qquad (6)$$

A detailed description of the calculation of $\delta^{lept}(\alpha)$ was given in ref.[2] Exact Lorentz-invariant formulae were obtained for the cross sections of the elastic and inelastic radiative tail, valid in the whole kinematic region of deep inelastic scattering. The hadronic vertex was treated in terms of experimentally measured form factors. The radiative tail of the elastic peak has been calculated by using the fit of the proton form factors $G_E(Q^2)$ and $G_M(Q^2)$ in the Q^2 range up to 25 GeV^2[7], whereas for the inelastic tail the published measurements on the structure function[5,8,9] are applied.

The dependence of $\delta^{lept}(\alpha)$ on y for different values of x is shown in Fig.2 at 280 GeV beam energy. $\delta^{lept}(\alpha)$ is generally a smoothly growing function of y with a strong rise in the vicinity of the kinematic limit $y \simeq 1$ connected here with the preferable emission of hard photons.

The leptonic vacuum polarization correction $\delta^{vac}_{lept}(\alpha)$ (the contribution of the interference of diagram 2 with 1 in Fig.1) has been calculated first of all only for internal e and μ loops and is shown in Fig.3 as a function of Q^2 up to 1000 GeV^2. A contribution of $\simeq$ 0.5% from the τ internal loop and the hadron loops (see section 2.2) has finally been added to the total vacuum polarization contribution.

Lepton-current radiative corrections of the order α^2,

$$\delta^{lept}(\alpha^2) = \frac{d^2\sigma^{lept}}{dx\,dy}(\alpha^4) \Big/ \frac{d^2\sigma^{o}}{dx\,dy}(\alpha^2) \qquad (7)$$

result from diagrams 11-30 of Fig.1*.

An explicit calculation of the contributions resulting from diagrams 21-26 for the elastic radiative tail, $\delta^{lept}_{el}(\alpha^2)$, has been performed for the first time in ref.[10].

The formulae obtained may be applied to any discrete final hadronic state but, taking into account that the emission of one or more photons by the lepton should not depend on the nature of the final hadronic state, as a good approximation for the contribution of diagrams 11-26 to the corrections from the

* Only a representative set of α^4 diagrams is shown here. A complete collection can be found in ref.[10].

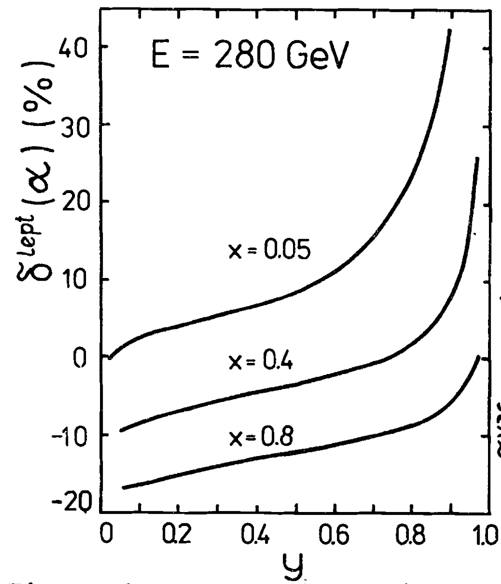

Fig.2. The lepton electromagnetic correction $\delta^{lept}(\alpha)$ to deep inelastic μp scattering as functions of y for several x at 280 GeV.

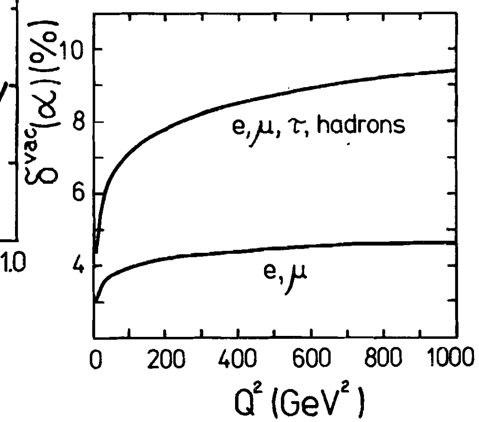

Fig.3. The vacuum polarization corrections $\delta^{vac}(\alpha)$ arising from (e, μ) and (e, μ, τ, hadrons) contributions as functions of Q^2.

inelastic radiative tail, $\delta^{lept}_{inel}(\alpha^2)$, we consider

$$\frac{\delta^{lept}_{inel}(\alpha^2)}{\delta^{lept}_{inel}(\alpha)} = \frac{\delta^{lept}_{el}(\alpha^2)}{\delta^{lept}_{el}(\alpha)}. \qquad (8)$$

The contributions from electromagnetic production of e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$ pairs (diagrams 27-30) to the measured cross section of the deep inelastic μp scattering were calculated in ref[11]. It was found that the contribution of diagrams 27-30 to (7) is less than 0.4% in the major part of the kinematic region. Furthermore, an upper limit of 1% was estimated in the framework of the simple quark-parton model for contributions resulting from diagrams 27-30 when instead of leptons hadrons were produced. We consider these contributions to (7) as being negligible.

The dependence of

$$\delta^{lept}(\alpha^2) = \delta^{lept}_{el}(\alpha^2) + \delta^{lept}_{inel}(\alpha^2) \qquad (9)$$

on y for several x is shown in Fig.4 for beam energies of 280 and 750 GeV. $\delta^{lept}(\alpha^2)$ amounts to few per cent, in particular, at low x and large y, almost energy independent.

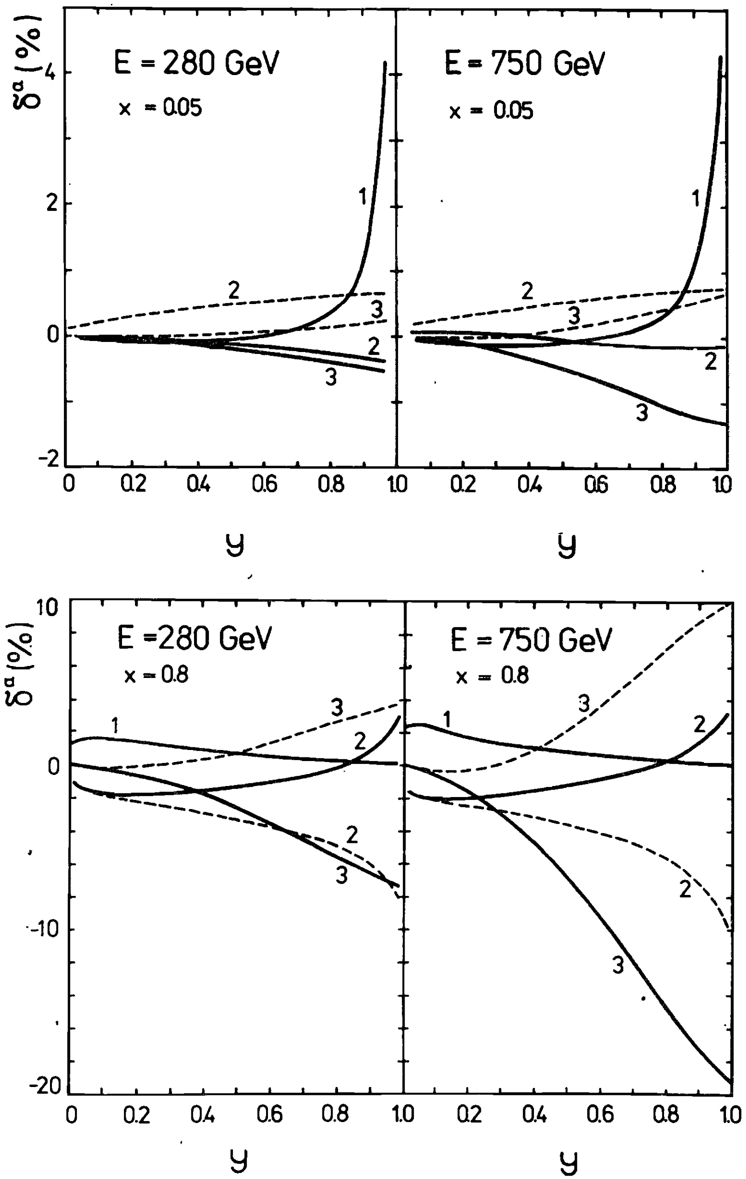

Fig.4. The electromagnetic corrections $\delta^{\rm lept}(\alpha^2)$ (1), $\delta_\pm^{\rm hard}(\alpha)$ (2) and the weak correction $\delta_{\pm\lambda}^{\rm weak}(G_F\alpha^{-1})$ (3) as functions of y for two values of x at 280 and 750 GeV. Solid lines correspond to μ^+ ($\lambda = -0.8$) and dotted lines, to μ^- ($\lambda = +0.8$).

Taking into account the uncertainties due to the approximations made in the calculation of $\delta^{\rm lept}(\alpha^2)$ and the neglection of electromagnetic lepton pair production, the accuracy of the total leptonic correction

$$\delta^{\rm lept} = \delta^{\rm lept}(\alpha) + \delta^{\rm lept}(\alpha^2) + \ldots \qquad (10)$$

has been estimated as

$$\Delta\delta^{\rm lept} \simeq 0.1\delta^{\rm lept}(\alpha^2). \qquad (11)$$

2.2. Hadron-Current Radiative Processes

The processes which have to be taken into consideration for these corrections correspond to diagrams 6,7,8 and 9 of Fig.1. Because of the interference of diagram 7 with 1 and 9 with 4 and 5, their cross section depends on the lepton charge, i.e.,

$$\delta_\pm^{\rm hadr}(\alpha) = \frac{d^2\sigma_\pm^{\rm hadr}}{dx\,dy}(\alpha^3) \Big/ \frac{d^2\sigma^\circ}{dx\,dy}(\alpha^2). \qquad (12)$$

The practical calculation of $\delta_\pm^{\rm hadr}(\alpha)$ requires, contrary to $\delta^{\rm lept}(\alpha)$ and $\delta^{\rm lept}(\alpha^2)$ a model to describe the scattering on the hadron. Such a calculation was performed in ref./12/ in the framework of the simple quark-parton model. Nucleon constituents are assumed not to interact with each other, and the lepton interacts directly with one of the partons. Partons are identified with SU(3) quarks and antiquarks. Quark distribution functions have been taken from ref./13/

Figure 4 shows the behaviour of $\delta_\pm^{\rm hadr}(\alpha)$ at different x-values as a function of y. $\delta_\pm^{\rm hadr}(\alpha)$ is less than 1% at small x but may reach few per cent at large x and grows slightly with energy.

The reliability of the calculated $\delta_\pm^{\rm hadr}(\alpha)$ has been proved as a byproduct in a measurement of the charge asymmetry in deep inelastic μC scattering/14/ at 200 GeV. Though the experimental errors are rather large (~30%) the measurement can be considered as being consistent/15/ with the calculation/12/, and permits a rough estimation of the uncertainties of $\delta_\pm^{\rm hadr}(\alpha)$ as

$$\Delta\delta_\pm^{\rm hadr} \simeq 0.3\,\delta_\pm^{\rm hadr}(\alpha). \qquad (13)$$

The contribution of the vacuum polarization by hadrons $\delta_{\rm hadr}^{\rm vac}(\alpha)$ (diagram 6) can be obtained from the total cross section of the process $e^+e^- \to$ hadrons in the one-photon exchange approximation through a dispersion relation. Taking the evaluation given in ref./16/ for the spacelike region, a total vacuum polarization contribution of first order in α

$$\delta^{vac}(\alpha) = \delta^{vac}_{lept}(\alpha) + \delta^{vac}_{hadr}(\alpha) \qquad (14)$$

is obtained and is shown in Fig.3 as a function of Q^2_{vac}. As can be seen, $\delta^{vac}_{hadr}(\alpha)$ is nearly as large as $\delta^{vac}_{lept}(\alpha)$ and $\delta^{vac}(\alpha)$ reaches a value of $\simeq 8\%$ in the Q^2 region around 300 GeV^2.

Contributions of order higher than $\delta^{vac}(\alpha)$ are conventionally added to (14) in the leading log approximation resulting in the total vacuum polarization correction δ^{vac}. Because of a 10% uncertainty of the evaluated $\delta^{vac}_{hadr}(\alpha)$ we estimate the accuracy of the total vacuum polarization correction as

$$\Delta\delta^{vac} \simeq 0.1\,\delta^{vac}_{hadr}(\alpha). \qquad (15)$$

3. WEAK INTERACTION

As long as the Q^2 region under consideration is small in comparison to the squared mass of the intermediate vector boson, M^2_Z, the weak interaction contribution to the observed deep inelastic scattering cross section (2) can be treated as a correction.

The first contribution we will discuss is the γZ interference resulting from diagrams 1 and 10 in Fig.1

$$\delta^{weak}_{\pm\lambda}(G_F\alpha^{-1}) = \frac{d^2\sigma^{weak}_{\pm\lambda}}{dx\,dy}(\alpha G_F) \Big/ \frac{d^2\sigma^{o}}{dx\,dy}(\alpha^2), \qquad (16)$$

which depends on the charge and the longitudinal polarization λ of the initial lepton. It was explicitly calculated in ref.[12] in the framework of the standard theory (the hadronic vertex treated as in section 2.2).

The dependence of $\delta^{weak}_{\pm\lambda}(G_F\alpha^{-1})$ on y for different x and energy values is shown in Fig.4. $\delta^{weak}_{\pm\lambda}(G_F\alpha^{-1})$ contributes mainly at large x and y with values up to $\simeq 8\%$ at 280 GeV and $\simeq 20\%$ at 750 GeV beam energy, respectively, for pisitively charged muons.

Taking into account an estimation of the neglected Z^2 contribution the accuracy of the calculated $\delta^{weak}_{\pm\lambda}(G_F\alpha^{-1})$ is determined in the Q^2 range up to 300 GeV^2 as

$$\Delta\delta^{weak}_{\pm\lambda} \simeq 0.1\,\delta^{weak}_{\pm\lambda}(G_F\alpha^{-1}). \qquad (17)$$

A rough estimation reveals that higher order contributions like the Z^2 terms from diagram 10 and the one-loop diagrams like 31-40 may well reach several per cent at $Q^2 \simeq 800\ GeV^2$ and have, consequently, to be taken into account at Fermilab

beam energies. A set of formulae for its calculation is given in ref.[17]

4. SUMMARY

The electroweak radiative corrections to deep inelastic muon-proton scattering have been calculated including for the first time α^4 contributions to the lepton current, vacuum polarization by hadrons, hadron current and weak interaction contributions in the lowest order. The total radiative correction with account of all considered here contributions

$$\delta^{tot}_{\pm\lambda} = \delta^{vac} + \delta^{lept} + \delta^{hadr}_{\pm} + \delta^{weak}_{\pm\lambda} \qquad (18)$$

differs from earlier calculated[2,3] in the absolute units by several per cent at 100 GeV and up to about 10÷20 per cent at 750 GeV beam energy.

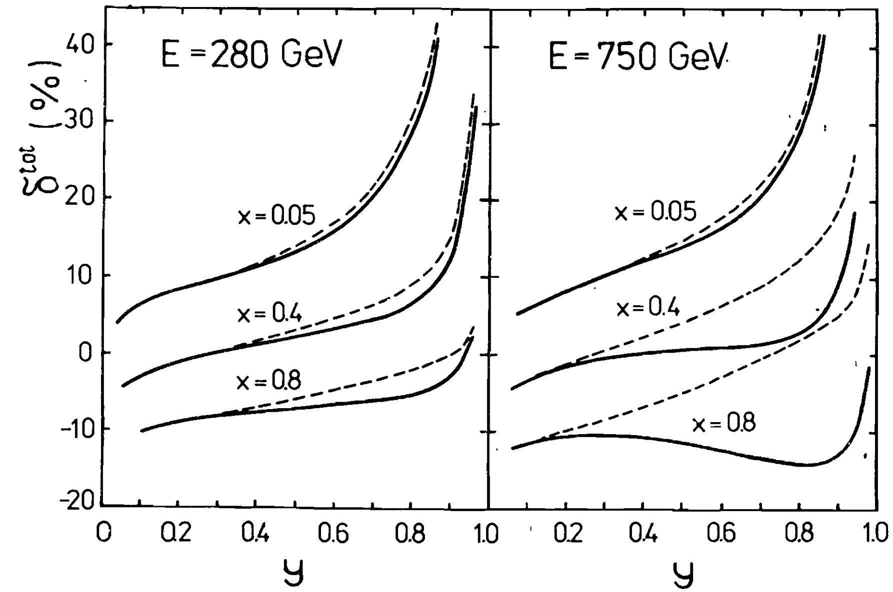

Fig.5. The total electroweak corrections $\delta^{tot}_{\pm\lambda}$ to deep inelastic $\mu^{\pm}p$ scattering as functions of y for several x at 280 and 750 GeV. Solid lines correspond to $\mu^{+}(\lambda = -0.8)$ and dotted lines, to $\mu^{-}(\lambda = +0.8)$.

The dependence of $\delta^{tot}_{\pm\lambda}$ on y for several x is shown in Fig.5 for $\mu^{\pm}p$ deep inelastic scattering at 280 and 750 GeV. For the positive and negative muon beams with natural polarization we observe a considerable difference between δ_+^{tot} and δ_-^{tot} amounting to few per cent at 280 GeV and growing up to $\simeq 20\%$ at 750 GeV.

In conclusion, we note that limitation $\delta^{lept} \lesssim 30\%$ commonly used in the analysis of deep inelastic muon scattering [2,9] due to unknown $\delta^{lept}(a^2)$ contribution are no longer persisting. The smallness of $\delta^{lept}(a^2)$ and estimated small uncertainties of other contributions imply that from the point of view of the radiative correction calculation deep inelastic scattering data can be interpreted in the whole kinematic domain.

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Ахундов А.А., Бардин Д.Ю., Ломанн В. E2-86-104
Радиационные поправки к глубоконеупругому рассеянию мюонов

Кратко описаны самые последние результаты расчета радиационных поправок к глубоконеупругому мюон-нуклонному рассеянию. Учтены вклады от лептонных электромагнитных процессов вплоть до α^4 порядка, поляризации вакуума лептонами и адронами, от адронных электромагнитных процессов $\sim\alpha^3$ и γZ -интерференции. Изучается зависимость отдельных вкладов от кинематических переменных. Вклады, не принимавшиеся во внимание в прежних вычислениях радиационных поправок, достигают в определенных кинематических областях нескольких процентов при энергиях свыше 100 ГэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ

Сообщение Объединенного института ядерных исследований. Дубна 1986

Akhundov A.A., Bardin D.Yu., Lohmann W. E2-86-104
Radiative Corrections to Deep Inelastic Muon Scattering

A summary is given of the most recent results for the calculation of radiative corrections to deep inelastic muon-nucleon scattering. Contributions from leptonic electromagnetic processes up to the order α^4, vacuum polarization by leptons and hadrons, hadronic electromagnetic processes $\sim\alpha^3$ and γZ interference have been taken into account. The dependence of the individual contributions on kinematical variables is studied. Contributions, not considered in earlier calculations of radiative corrections, reach in certain kinematical regions several per cent at energies above 100 GeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1986