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I. Introduction

In models with dynamical symmetry the Goldstone and the gauge fields are on the distinct status. The invariance under some group of dynamical symmetry is achieved via appropriate interactions with these preferred fields.

It may happen that some Goldstone and gauge fields are, in fact, unimportant, superfluous for the content of theory in the sense that one may express them in terms of other preferred fields or remove them by a redefinition of the latter. For instance, in the case of spontaneously broken gauge symmetry it is possible to rule out all the Goldstone fields from an invariant Lagrangian by the gauge transformation (the Higgs phenomenon/1-3/).

We would like to call attention to the fact that in nonlinear realizations of symmetries /4-8/ there exists a possibility to eliminate superfluous Goldstone and gauge fields by imposing invariant conditions on the Gartan differential forms.

The crucial point is that any covariant Cartan form with the homogeneous group transformation law can be put equal to zero with preserving all the invariance properties of theory. If the resulting covariant equation is solvable relative to some preferred field one may express the latter in terms of remaining variables. Those of initial fields which cannot be eliminated with such a procedure have the meaning of the "true" Goldstone and "true" gauge ones.

The exclusion of unimportant variables by putting the Cartan forms zero we call the inverse Higgs phenomenon because, when applying to the gauge fields it is opposed with the usual Higgs phenomenon in three aspects.

First, they are opposed in essence.

The Higgs phenomenon is usually meant to be spontaneous generation of mass for the gauge field $\mathfrak{Z}_\mu^i$ associated with the transformations under which the vacuum is not invariant /1–3/. One may also treat it as the possibility to choose the "unitary" gauge in which the Goldstone field $\xi^i(x)$ disappears from an invariant Lagrangian (the gauge $\xi^i(x)=0$) and the field $\mathfrak{Z}_\mu^i$ satisfies the condition:

$$\mathfrak{Z}_\mu^i|_{\xi=0} = \frac{1}{f} \nabla_\mu \xi^i = \frac{1}{f} \partial_\mu \xi^i(x) + \mathfrak{Z}_\mu^i|_{\xi\neq 0} + O(\xi) , \qquad (I.1)$$

where $\nabla_\mu \xi^i$ is the covariant derivative of $\xi^i(x)$, f is a constant. Thus, in the case of the direct Higgs phenomenon, the Goldstone field $\xi^i(x)$ is eliminated, being absorbed by $\mathfrak{Z}_\mu^i$. On the contrary, in the case of the inverse Higgs phenomenon it is just the field $\mathfrak{Z}_\mu^i$ that is eliminated. One expresses $\mathfrak{Z}_\mu^i$ by means of the covariant equation

$$\nabla_\mu \xi^i = 0 \qquad (I.2)$$

in terms of both $\xi^i(x)$ and the true gauge field $\mathfrak{V}_\mu^a(x)$ which is associated with the algebraic subgroup (leaving the vacuum invariant).

Second, both the phenomena differ in range of applicability.

The direct one occurs only for invariant Lagrangians, when the gauge symmetry is broken only spontaneously. If the external symmetry breaking term $\sim \mathfrak{Z}_\mu^i \mathfrak{Z}_\mu^i + \mathfrak{V}_\mu^a \mathfrak{V}_\mu^a + \ldots$ is added to an invariant Lagrangian, the Goldstone fields become physical ones and cannot be ruled out.

At the same time, the inverse Higgs phenomenon turns out to be constructive just in the case of external gauge symmetry breaking. In this case the exclusion of field $\mathfrak{Z}_\mu^i$ results in nontrivial models where all the restrictions of underlying broken gauge symmetry hold and which deal with fields $\xi^i(x)$ and $\mathfrak{V}_\mu^a$ only unlike the conventional approach where the canonical gauge field $\mathfrak{Z}_\mu^i$ is present also *. If the gauge symmetry is broken in purely spontaneous way, condition (I.2)$_1$ means no more than the requirement that the field $Z_\mu|_{\xi=0}$ be zero that, of course, contradicts nothing but has no physical consequences.

At last, both the phenomena are opposed in the sense that the kinetic term of the Goldstone field $-\nabla_\mu \xi^i \nabla_\mu \xi^i$ in the case of direct phenomenon turns into the mass term of $Z^i_\mu|_{\xi=0}$ while on using the inverse one the mass term of field Z^i_μ gives the kinetic term of the Goldstone particles.

Besides applying the inverse Higgs phenomenon to gauge fields it can be used for eliminating superfluous Goldstone fields, for instance, in nonlinear realizations of space-time symmetries /10/. Its applicability in this case depends on the structure of given group.

Remind that nonlinear models with spontaneously broken symmetry are nonrenormalizable and pretend only to describing low energy phenomena.

The paper is planned as follows. In Section II, the general treatment of the inverse Higgs phenomenon for gauge fields is given and two examples are considered. At first, we briefly discuss the model of $SU(2)\times SU(2)$ -field algebra without canonical axial field /11/ (that is the chiral invariant theory of massive Yang-Mills field $\vec{\rho}_\mu$ interacting with conserved isospin current). Unfortunately, its predictions are less satisfactory than those of conventional approach with A_1 -meson. The second example concerns the nonlinear realizations of supersymmetry in the approach by D.V.Volkov et al. /12, 13/. The inverse Higgs phenomenon helps in eliminating the gauge fields connected with spinor translations /13/.

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* To come back to conventional approach one needs to replace (I.2) by other covariant condition

$$\nabla_\mu \xi^i(x) = f \tilde{Z}^i_\mu ,$$

where field $\tilde{Z}^i_\mu$ is defined to transform like $\nabla_\mu \xi^i$ (Such a condition has been used, in fact, by Kawarabayashi and Kitakado for the case of chiral $SU(2)\times SU(2)$ -symmetry /9/). It is the same to formulate the standard scheme in terms of $\tilde{Z}^i_\mu$ or in terms of Z^i_μ.

In Section III, the inverse Higgs phenomenon for Goldstone fields in nonlinearly realized space-time symmetries is studied. We formulate general conditions under which some space-time symmetry group permits certain Goldstone fields to be eliminated. Two instructive examples are considered. These are the spontaneously broken conformal symmetry /5-7,10/ and the nonlinear realizations of projective group, isomorphic to the group SL (5,R). In the first case the inverse Higgs phenomenon is responsible for the known fact /14/ that conformal invariance can be ensured through interactions with the only dilaton $\sigma(x)$ which is just the true Goldstone field in this case. In the second example the only true Goldstone field is shown to be the tensor one $h_{\mu\nu}(x)$ associated with the proper affine transformations. Thus, the nonlinear realizations of projective group reduce effectively to those of affine subgroup $P_4 \subset\!\!\!\!\times GL(4,R)$ treated in detail in papers /6,10/.

II. The Inverse Higgs Phenomenon for Gauge Fields

Let G be a dynamical symmetry group with the following algebra of generators:

$$\begin{aligned} [Z_i, Z_k] &= iC^{ik\ell} Z_\ell + iC^{ik\alpha} V_\alpha \\ [V_\alpha, Z_i] &= iC^{\alpha ik} Z_k \\ [V_\alpha, V_\beta] &= iC^{\alpha\beta\rho} V_\rho \ , \end{aligned} \tag{II.1}$$

where C are the structure constants. The generators V_α form the algebra of the stability subgroup H. Note that the translation generator P_μ should be included into a set of Z_i in the case of space-time symmetries /7/. If G determines a supersymmetry some generators Z_i are meant to obey anticommutation relations /12/.

The group G is realized in the space of left cosets G/H (that is the quotient space over subgroup H) parametrized by the Goldstone fields $\xi^i(x)$ /4-8/

$$G(\xi) = e^{i\xi_k Z_k} \xrightarrow{g} gG(\xi) = e^{i\xi'_k Z_k} e^{iU_\alpha(\xi, g) V_\alpha} \qquad \text{(II.2)}$$

Shift (II.2) induces the nonlinear transformation of field $\xi^i(x)$:

$$\delta \xi^i(x) = \mathcal{F}^{ik}(\xi)\, \beta^k + \xi^m C^{m\rho i} \alpha^\rho, \qquad \text{(II.3)}$$

where β^k, α^ρ are the group transformation parameters and the matrix $\mathcal{F}^{ik}(\xi)$ is a nonsingular one.

An arbitrary field $\psi(x)$ and also covariant differentials of fields $\xi^i(x)$ and $\psi(x)$, $\omega^i(d)$ and $D\psi$, respectively, transform under the group G according to their representations of the subgroup H but with parameters - functions $U_\alpha(\xi . g)$.

We are interested in the case of gauge symmetry, where parameters β^k and α^ρ are space-time dependent. The covariant differentials $\omega^i(d)$ and $D\psi$ are introduced by standard formulae /4–8/ :

$$G^{-1}(\xi)\{ d + if(\mathcal{Z}_k Z_k + \mathcal{V}_\alpha V_\alpha)\} G(\xi) = i\omega^k(d) Z_k + i\theta^\alpha(d) V_\alpha , \qquad \text{(II.4)}$$

$$D\psi = d\psi + i\theta^\alpha(d)\bar{V}_\alpha \psi , \qquad \text{(II.5)}$$

where $\bar{V}_\alpha$ are generators of the subgroup H in the ψ-representation. The quantities $\mathcal{Z}_k$ and $\mathcal{V}_\alpha$ are related to gauge fields $\mathcal{Z}^k_\mu$ and $\mathcal{V}^\alpha_\mu$ as:

$$\mathcal{Z}_k = \mathcal{Z}^k_\mu dx^\mu$$
$$\mathcal{V}_\alpha = \mathcal{V}^\alpha_\mu dx^\mu \qquad \text{(II.6)}$$

and possess the following transformation properties:

$$\delta\mathcal{Z}_i = \mathcal{Z}_p \beta_k C^{pki} + \mathcal{V}_\alpha \beta_k C^{\alpha ki} + \mathcal{Z}_p \alpha_\rho C^{p\rho i} - \frac{1}{f} d\beta_i \qquad \text{(II.7)}$$

$$\delta\mathcal{V}_\rho = \mathcal{V}_\mu \alpha^\beta C^{\mu\beta\rho} + \mathcal{Z}_k \beta^\ell C^{k\ell\rho} - \frac{1}{f} d\alpha^\rho , \qquad \text{(II.8)}$$

where f is a constant. The quantities $\omega^k(d)$ and $\theta^a(d)$ in decomposition (II.4) are the Cartan differential forms. The covariant derivatives $\nabla_\mu \xi_i$ and $\nabla_\mu \psi$ are related to $\omega^i(d)$ and $D\psi$ by formulae:

$$\omega^i(d) = \nabla_\mu \xi_i \, \omega^P_\mu(d)$$

$$D\psi = \nabla_\mu \psi \, \omega^P_\mu(d).$$

Here $\omega^P_\mu(d)$ is the Cartan form associated with the generator of 4-translations P_μ *. It is crucial for our analysis that the Cartan form $\omega^i(d)$ (and, respectively, the covariant derivative $\nabla_\mu \xi_i$) transforms homogeneously under the group G.

We shall show that there exists the nonlinear function of fields $\xi^i(x)$ and $\mho^a_\mu(x)$ which transforms under the gauge group like the field Z^i_μ (law (II.7)). So only the fields $\xi_i(x)$ and $\mho^a_\mu(x)$ are of need to construct invariant Lagrangians.

Let us put the following condition

$$\omega^i(\xi, d\xi, Z, \mho) = 0 \qquad \text{(II.9)}$$

or

$$\nabla_\mu \xi^i = 0 \qquad (\omega^P_\mu(d) \neq 0). \qquad \text{(II.9')}$$

Since the Cartan form $\omega^i(d)$ transforms homogeneously, eqs. (II.9), (II.9') are covariant under an action of group. Equation (II.9) is solved in the Appendix:

$$\tilde{Z}^i(\xi, \mho) = -\frac{1}{f}(\mathcal{F}^{-1}(\xi))^{im}(d\xi_m + f\xi_p \mho_a C^{pam}). \qquad \text{(II.10)}$$

Using transformation laws (II.3) and (II.8) and also the Jacobi identities following from group properties of trans-

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* Hereafter the generator P_μ is assumed to transform under subgroup H independently of remaining generators Z_i for the form ω^P_μ be covariant. Note that in the case of internal symmetries

$$\omega^P_\mu = dx_\mu$$

formation (II.3) one may check that the function $\tilde{Z}^i(\xi, \mho)$ transforms according to law (II.7). One can easily be convinced also of that expression (II.10) is covariant with respect to an arbitrary canonical replacement of the field ξ_m.

It should be stressed that if some function $\tilde{Z}^i(\xi, \mho)$ possesses transformation law (II.7) and one substitutes it into the Cartan form $\omega(\xi, d\xi, Z, \mho)$ for the field Z_i, condition (II.9) is satisfied identically. That follows from the evident properties:

$$\tilde{Z}^m(\xi, d\xi, \mho)\,|_{\xi=0} = 0$$

$$\omega^i(\xi, d\xi, \tilde{Z}, \mho)|_{\xi=0} = f\,\tilde{Z}^m|_{\xi=0} = 0$$

and from existing of the gauge transformation g_0 such that

$$\omega^i(\xi, d\xi, \tilde{Z}, \mho) \xrightarrow{g_0} \omega\,(0,0,0,\mho_0).$$

Thus we have proved the following Theorem.

Theorem 1. In nonlinear realizations of gauge symmetries it is always possible to construct the function $\tilde{Z}^i(\xi, \mho)$ with transformation properties of the gauge field Z_i by putting the covariant Cartan form $\omega^i(\xi, d\xi, Z, \mho)$ be zero. Inversely, if such a function exists, its substitution into the Cartan form $\omega^i(d)$ for the field Z_i converts this form into zero identically.

It is important that the gauge field $\mho_\mu^a$ cannot be omitted because there are no Cartan forms with the homogeneous transformation law which equating to zero would result in equations solvable relative to $\mho_\mu^a$. The field $\mho_\mu^a$ is the true gauge one and has to be introduced as a canonical field. It is worth noting that now the transformation law of field $\mho_\mu^a$ is nonlinear in the Goldstone field. The unessential field Z_μ^i may not be referred to at all.

Note that components of covariant derivative $\nabla_\mu \xi_i$ which are irreducible with respect to the subgroup H transform under the group G independently. Therefore it is possible to put conditions of the type of (II.9') separately for each irreducible H-representation contained in $\nabla_\mu \xi_i$. In other words, one may eliminate not all fields $\mathcal{Z}^i_\mu$ but only those which belong to a given representation of the subgroup H. In this case, generally speaking, it may happen that solutions to equations of type of (II.9), (II.9') will be dependent on the remaining fields $\mathcal{Z}^i_\mu$. Such a situation, for instance, occurs in nonlinear realizations of space-time symmetries where it makes no sense to put the Cartan form $\omega^P_\mu(d)$ zero because this form determines the invariant space-time volume element. Therefore the gauge field associated with the translation subgroup is the true gauge one and should not be eliminated.

When eliminating not all the fields $\mathcal{Z}^i_\mu$ one obtains in general more complicated expressions than (II.10). There is a simple case where formula (II.10) is still valid.

Let $Z_{i'}$ generate an invariant subgroup of G:

$[Z, Z_{i'}] \sim Z_{k'}$, $[V, Z_{i'}] \sim Z_{k'}$.

Denote the Goldstone and gauge fields associated with other generators of nonlinear transformations by $\xi_{k''}$ and $\mathcal{Z}^{k''}_\mu$. Then the following Theorem holds:

Theorem II. It is possible to express the gauge field $\mathcal{Z}^{k''}_\mu$ in terms of $\xi_{k''}$ and $\mathcal{C}^\rho_\mu$ as follows

$$\mathcal{Z}^{k''}_\mu = -\frac{1}{f} (\mathcal{F}^{-1}(\xi''))^{k''\ell''} (d\xi_{\ell''} + f\xi_{s''} \mathcal{C}^\rho_\mu C^{s''\rho\ell''}), \quad (II.11)$$

where $\mathcal{F}^{k''\ell''}(\xi'')$ is the matrix of a nonlinear transformation of the field $\xi_{k''}$:

$$\delta\xi_{k''} = \mathcal{F}^{k''\ell''}(\xi'')\beta^{\ell''}.$$

Taking into account that $\xi_{k''}$ transforms under G independently of $\xi_{k'}$ formula (II.11) can be derived in the way similar to the derivation of (II.10).

The exclusion of gauge fields by imposing the invariant conditions like (II.9) is opposed with the well-known Higgs phenomenon /1-3/ and therefore can be called the inverse Higgs one. Indeed, in the present case one eliminates the field $\mathcal{Z}^i_\mu$ expressing it through $\xi^i(x)$ and $\mathcal{V}^a_\mu(x)$ while the Higgs phenomenon leads to the elimination of the Goldstone field $\xi^i(x)$ which is absorbed by $\mathcal{Z}^i_\mu$ (the choice of the gauge $\xi_i(x)=0$ in an invariant Lagrangian).

The inverse Higgs phenomenon, contrary to the direct one, leads to nontrivial results when the gauge symmetry breaking term is added to an invariant Lagrangian. In this case the inverse Higgs phenomenon makes it possible both to maintain all the restrictions imposed on the field couplings by broken gauge invariance under algebraic subgroup H and to achieve the invariance under constant parameter transformations of the whole group G, without including $\mathcal{Z}^i_\mu$. The latter invariance is achieved now via an appropriate nonminimal interactions of fields $\xi_i(x)$ and $\mathcal{V}^a_\mu(x)$.

Note that the kinetic term of Goldstone fields now can appear only from the bilinear in $\mathcal{Z}^i_\mu(\xi,\mathcal{V})$ part of gauge symmetry breaking:

$$\tilde{\mathcal{Z}}^i_\mu(\xi,\mathcal{V})\,\tilde{\mathcal{Z}}^i_\mu(\xi,\mathcal{V}) = \frac{1}{f^2}\partial_\mu\xi_i\,\partial_\mu\xi_i + \ldots$$

The invariant kinetic term of the true gauge field $\mathcal{V}^a_\mu$ can be constructed in a standard way using the covariant curl $R^a_{\mu\nu}$ defined as /13/

$$\frac{1}{2}\left(\omega^P_\mu(d_1)\,\omega^P_\nu(d_2) - \omega^P_\mu(d_2)\,\omega^P_\nu(d_1)\right)R^a_{\mu\nu} =$$

$$= d_1\theta^a(d_2) - d_2\theta^a(d_1) - C^{a\beta\gamma}\theta^\beta(d_1)\,\theta^\gamma(d_2),$$

where the Cartan form $\theta^a(d)$ is defined by decomposition (II.4).

Consider now two examples of the inverse Higgs phenomenon in gauge theories.

1. The model of SU(2) $\times$SU(2) -field algebra without A_1-meson

The model of SU(2)$\times$SU(2)-field algebra with an appropriate function $\tilde{A}_\mu(\pi,\rho)$ instead of the A_1-meson field has been discussed first by Gasiorowitcz and Geffen in review /11/.

In parametrization of nonlinear σ-model general formula (II.10) gives

$$\vec{A}_{\mu}(\pi,\rho) = -\frac{Z_{\rho}^{-1/2}}{g_{\rho}} \frac{1}{\sqrt{f_{\pi}^{2}-\vec{\pi}^{2}}} (\partial_{\mu}\vec{\pi} - g_{\rho}\vec{\rho}_{\mu}\times\vec{\pi}), \quad \text{(II.12)}$$

where g_{ρ} and Z_{ρ} are, respectively, the universal coupling constant and a renormalization constant of ρ-meson, $f_{\pi} \approx 94$ MeV is the pion decay constant.

The invariant Lagrangian is of the form:

$$L_{inv} = -\frac{1}{4} \frac{f_{\pi}^{2}}{f_{\pi}^{2} - \vec{\pi}^{2}} \{\vec{\rho}_{\mu\nu}\vec{\rho}_{\mu\nu} - \frac{1}{f_{\pi}^{2}}(\vec{\rho}_{\mu\nu}\vec{\pi})(\vec{\rho}_{\mu\nu}\vec{\pi})\}, \quad \text{(II.13)}$$

where

$$\vec{\rho}_{\mu\nu} = \partial_{\mu}\vec{\rho}_{\nu} - \partial_{\nu}\vec{\rho}_{\mu} - g_{\rho}\vec{\rho}_{\mu}\times\vec{\rho}_{\nu} - g_{\rho}Z_{\rho}\vec{A}_{\mu}(\pi,\rho)\times\vec{A}_{\nu}(\pi,\rho). \quad \text{(II.14)}$$

The ρ-meson mass term is determined uniquely by chiral invariance and by requiring the $\vec{\rho}$-field be connected with conserved current:

$$L_{Br} = -\frac{1}{2} m_{\rho}^{2} (Z_{\rho}\vec{A}_{\lambda}(\pi,\rho)\vec{A}_{\lambda}(\pi,\rho) + \vec{\rho}_{\mu}\vec{\rho}_{\mu}). \quad \text{(II.15)}$$

Note that conserved vector and axial currents calculated by making use of Gell-Mann-Levy method obey the standard commutation relations of $SU(2)\times SU(2)$-field algebra /15/ despite the absence of the A_1-field in the axial current.

Thus, the ρ-universality and the chiral invariance can be combined consistently without including the conserved source axial field. The only way to choose between the model in question and the conventional approach with A_1-meson is to compare their predictions with experimental data.

For the correct normalization of the kinetic term of pions in (II.15) it is necessary that the sum rule

$$m_{\rho}^{2} = g_{\rho}^{2} f_{\pi}^{2} \qquad \text{(II.16)}$$

be valid /11/. This sum rule is inconsistent with empirical KSRF-relation /16/

$$m_{\rho}^{2} = 2 g_{\rho}^{2} f_{\pi}^{2}. \qquad \text{(II.17)}$$

Mainly due to this discrepancy, the given model leads to ρ -meson widths too small as compared with those of conventional approach /11/.

It seems to us that a more serious defect of the model in question is the impossibility, due to vanishing of the pion covariant derivative, of constructing a gauge - invariant πN -interaction involving the pseudovector coupling $-\bar{N}\gamma_5\gamma_\mu \vec{\tau} N \partial_\mu \vec{\pi}$ which is needed for description of the p -wave part of πN -scattering /17/.

Let us make one comment concerning KSRF-relation (II.17). It is known that one cannot obtain it within the framework of ρ -universality and current algebra only /18/. On the other hand, both the assumptions that the pion covariant derivative is zero and that the symmetry breaking is of form (II.15) together lead necessarily to sum rule (II.16) which is incompatible with (II.17). Therefore for the KSRF-relation be valid it is necessary that the covariant derivative of pion be not zero, i.e., that the part of axial current with the quantum numbers 1^+ be dominated by A_1-meson (this condition, of course, is not sufficient).

It is interesting that the model with $\vec{A}_\mu(\pi,\rho)$ (II.12) can be achieved by taking the limit $m_{A_1} \to \infty$ in conventional model with A_1 -meson.

2. Elimination of gauge fields in the supersymmetry case

In papers /13/ D.V.Volkov and V.A.Soroka have shown how to introduce the gauge fields into nonlinear realizations of supersymmetry [12]. It has been argued there that gauge fields which correspond to spinor translations are, in fact, unessential ones and can be replaced by functions of other fields.

Let us show that the result of papers [13] is reproduced simply by applying Theorem II to the present case.

In this case, the quotient space is parametrized by the Goldstone spinor fields ψ_a, ψ_a^+ related to generators of spinor translations (subscript a denotes an index of some internal linear symmetry) and by the space-time coordinate x_μ serving as the "Goldstone field" corresponding to the usual translation subgroup [7]. The gauge fields associated with the spinor translation generators are $\phi^\mu_b(x)$, $\phi^\mu_b(x)^+$ [13].

The space-time translations are an invariant subgroup of the supersymmetry group, therefore Theorem II can be used. Applying general formula (II.11) to the fields ϕ^μ_b one finds:

$$\phi^\mu_b(x) = -\frac{1}{f}\{\partial_\mu \psi_b + i\frac{1}{2}\Omega^\mu_{\rho\gamma} L^\psi_{\rho\gamma}\psi_b + iV^\mu_a I^a_{bc}\psi_c\},$$

where $\Omega^\mu_{\rho\gamma}$ and V^μ_a are the true gauge fields associated with the homogeneous Lorentz subgroup and the internal symmetry subgroup, respectively. The matrices $L^\psi_{\rho\gamma}$, I^a are generators of these algebraic subgroups in the representation by which the field ψ_a transforms (i.e., they are the structure constants in the commutator of the spinor translation generators with those of the Lorentz and internal symmetry subgroups)

III. The Inverse Higgs Phenomenon for the Goldstone Fields

Equation (II.9') is solvable relative to $\mathcal{Z}^i_\mu$ because this field enters into the covariant derivative $\nabla_\mu \xi_i$ linearly and additively.

If some covariant derivative contains a term linear and additive with respect to some Goldstone field one also may exclude this field with the use of equation similar to eqs. (II.9) or (II.9'). Such a possibility exists, for instance, in nonlinear realizations of a number of space-time symmetries. It is natural to call the exclusion of superfluous Goldstone fields, by analogy with that of gauge ones, the inverse Higgs phenomenon.

This Section deals with space-time symmetries.

Let relations (II.1) determine some space-time symmetry. Then subgroup H contains the homogeneous Lorentz group. We denote generators but P_μ and V_α by Z_i' and corresponding Goldstone fields by ξ_i. Thus, the quotient space G/H is parametrized by coordinates x_μ, $\xi_i(x)$ /7,8/.

We confine ourselves to symmetries which satisfy the following conditions.

i. The product of any representation of the subgroup H is fully reducible with respect to H.

ii. The generators P_μ and Z_i' transform under the subgroup H independently.

The condition ii has been meant throughout Section II. It ensures that x_μ and ξ_i transform under subgroup H by different representations. Let the field ξ_i form basis of some representation $D(h)\,(h \in H)$ which can be expanded into a sum of irreducible representations $R_N(h)$. Then ξ_i breaks up into a set of components ξ_{iN} which are irreducible with respect to H. The derivative ∂_μ^x transforms under H by the representation $T^P(h)$ as well as the generator P_μ itself does.

The quantity $G(\xi)$ defined by (II.2) can be written in the present case as

$$G(x,\xi) = e^{ix_\mu P_\mu} e^{i\xi_i Z_i'} . \qquad \text{(III.1)}$$

The Cartan forms $\omega^i(d)$, $\omega_\mu^P(d)$ are introduced through the decomposition:

$$G^{-1}dG = e^{-i\xi_i Z_i'} iP_\mu dx_\mu e^{i\xi_i Z_i'} + e^{-i\xi_i Z_i'} d e^{i\xi_i Z_i'} =$$

(III.2)

$$= i\omega^i(d) Z_i' + i\omega_\mu^P(d) P_\mu + i\theta^\alpha(d) V_\alpha .$$

Let us now formulate a Theorem which enables one to determine whether a given space-time symmetry permits some H -multiplet ξ_{i_N} of Goldstone fields to be eliminated by using the inverse Higgs phenomenon. It turns out that it is crucial to know the structure of commutator of Z'_{i_N} and P_μ:

$$[Z'_{i_N}, P_\mu] = iC^{i_N \mu \iota} Z'_\iota + \dots \qquad \text{(III.3)}$$

the possible terms $\sim V_a$ and $\sim P_\mu$ in the right-hand side of (III.3) being unessential.

Theorem III. It is possible to express some field ξ_{i_N} in terms of remaining Goldstone fields if and only if:

a) The product $T^P(h) \otimes D(h)$ contains the irreducible representation $R_N(h)$.

b) There is an index ι for which the structure constants $C^{i_N \{\mu\iota\}_N}$ are not zero (the symbol $\{\mu\iota\}_N$ denotes a combination of indices μ and ι which corresponds to the irreducible representation $R_N(h)$).

It follows from definition (III.2) that under these conditions the covariant derivative $\nabla_\mu \xi_\iota$ includes a term bilinear and additive in ξ_{i_N}. Then, taking into account that $C^{i_N \{\mu\iota\}_N} = \beta \delta^{i_N \{\mu\iota\}_N}$ by the Shur lemma and $\beta \neq 0$ because of condition b), one may express ξ_{i_N}, solving the covariant equation

$$\nabla_{\{\mu} \xi_{\iota\}_N} = 0 \,, \qquad \text{(III.4)}$$

in terms of true Goldstone fields:

$$\xi_{i_N} = -\frac{1}{\beta} \partial^{\{\mu} \xi^{\iota\}_N} + O(\xi, x)$$

The necessity of conditions a), b) is proved in Appendix.

Note, if several different combinations $\{\mu\iota\}_N$, $\{\mu\iota\}'_N$, $\{\mu\iota\}''_N$.. correspond to the same representation $R_N(h)$ the most general covariant equation is the following one

$$a_N \nabla_{\{\mu} \xi_{\iota\}_N} + a'_N \nabla_{\{\mu} \xi_{\iota\}'_N} + a''_N \nabla_{\{\mu} \xi_{\iota\}''_N} + \dots = 0$$

whence

$$\xi_{i_N} = -\frac{1}{\beta}\partial^{\{\mu}\xi^{\iota\}_N} - \frac{1}{\beta a_N}(a'_N\partial^{\{\mu}\xi^{\iota\}_{N'}} + a''_N\partial^{\{\mu}\xi^{\iota\}_{N''}} + \dots)\}$$

where $a_N, a'_N, a''_N \dots$ are some numbers. If there are several different irreducible components ξ_{i_N}, $\xi_{i'_N}$, $\xi_{i''_N} \dots$ which transform by the same representation $R_N(h)$, the structure constants $C^{i'_N\{\mu\iota\}_N}$, $C^{i''_N\{\mu\iota\}_N}, \dots$ being not zero, the excluded field ξ_{i_N} contains $\xi_{i'_N}, \xi_{i''_N} \dots$ additively:

$$\xi_{i_N} = -\frac{1}{\beta}\partial^{\{\mu}\xi^{\iota\}_N} - \frac{1}{\beta}\xi_{i'_N}C^{i'_N\{\mu\iota\}_N} + \dots$$

On imposing condition (III.4) not all the invariant kinetic terms of fields $\xi_{i_N}, \xi_{i_M} \dots$ are independent. To make up shortage of these terms one may use covariant differential forms of the second order in field derivatives. Summing these forms over different pairs of indices one gets terms $\sim \partial_\mu \xi_\iota \, \partial_\mu \xi_\iota$ and $\sim \xi_\iota \, \Box \xi_\iota$ /10/.

Let us give two simple examples of the inverse Higgs phenomenon in space-time symmetries.

1. Spontaneously broken conformal symmetry

Nonlinear realizations of the conformal symmetry with linearization on the Lorentz subgroup have been discussed in papers /5-7,10/. The general theory in this case prescribes the Goldstone fields $\phi_\mu(x)$ and $\sigma(x)$ to be introduced. These correspond to the generators of the special conformal and scale transformations, K_λ and D, respectively.

The commutator of P_μ and K_λ contains the dilatation generator D in the right-hand side

$$[P_\mu, K_\nu] = 2i\,(\delta_{\mu\nu} D - L_{\mu\nu}),$$

where $L_{\mu\nu}$ are the Lorentz group generators. Therefore, by Theorem III, the field $\phi_\mu(x)$ can be expressed through the true Goldstone field $\sigma(x)$. It has been emphasized in papers /7/ and /10/ that the field $\phi_\mu(x)$ is unessential.

The covariant derivative of "dilaton" $\sigma(x)$ takes the form /5–7,10/:

$$\nabla_\mu \sigma(x) = e^{-\sigma(x)} \{ \partial_\mu \sigma(x) - 2\phi_\mu(x) \} .$$

Putting it zero one finds /10/

$$\phi_\mu(x) = \frac{1}{2} \partial_\mu \sigma(x) \cdot \qquad \text{(III.5)}$$

Note that with condition (III.5) taken into account the invariant action part depending only on the field $\sigma(x)$ can be constructed coinciding with such a part of the action for the case of nonlinear realization of scale invariance alone.

The interaction of dilaton with an arbitrary field $\psi(x)$ is determined by the form of the covariant derivative $\nabla_\mu \psi(x)$ /10/

$$\nabla_\mu \psi(x) = e^{-\sigma(x)} \partial_\mu \psi(x) + i e^{-\sigma(x)} \partial_\nu \sigma(x) L^\psi_{\mu\nu} \psi , \qquad \text{(III.6)}$$

where $L^\psi_{\mu\nu}$ are the Lorentz group generators in the representation to which the field ψ belongs. In expression (III.6), the first term, minimal with respect to nonlinear realizations of scale symmetry, and the second, nonminimal one are related (in the case of scale invariance alone they are covariant separately). This connection is the sole trace of the dynamical conformal symmetry that remains after eliminating the field $\phi_\mu(x)$.

2. Nonlinear realizations of projective group

The projective group is isomorphic to the group SL(5,R). The action of the latter on space-time coordinates x_μ is determined by means of the following identification:

$$x_\mu = \frac{y_\mu}{y_5} \ (\mu = 1,2,3,4), \quad x_4 = i x_0 , \quad y_4 = i y_0 ,$$

where y_μ, y_5 are coordinates of 5-dimensional space on which the group SL(5,R) acts linearly:

$$\delta y_i = a_{ik}\, y_k \; (i = 1,2,3,4,5), \quad a_{ii} = 0 .$$

Then

$$\delta x_\mu = a_{\mu\nu} x_\nu - a_{5\nu} x_\mu x_\nu - a_{55} x_\mu + a_{\mu 5}, \; a_{\mu\mu} = -a_{55}$$

where parameters $a_{\mu\nu}$, $a_{\mu 5}$, $a_{5\nu}$ correspond to the linear subgroup GL (4, R), to the translation subgroup P_4 and to the projective transformations, respectively.

The algebra of the projective group includes 24 generators which obey the relations

$$\frac{1}{i}[R_{\mu\nu}, R_{\rho\gamma}] = \delta_{\mu\rho} L_{\gamma\nu} + \delta_{\mu\gamma} L_{\rho\nu} + (\mu \to \nu) \qquad (a)$$

$$\frac{1}{i}[R_{\mu\nu}, P_\rho] = \delta_{\mu\rho} P_\nu + (\mu \to \nu) \qquad (b)$$

$$\frac{1}{i}[R_{\mu\nu}, F_\rho] = -\delta_{\mu\rho} F_\nu + (\mu \to \nu) \qquad (c)$$

$$[F_\mu, F_\nu] = 0 \qquad (d)$$

$$\frac{1}{i}[F_\rho, P_\lambda] = -\frac{1}{2}(\delta_{\rho\lambda} R_{\mu\mu} + R_{\rho\lambda} + L_{\rho\lambda}), \qquad (e)$$

(III.7)

where one has omitted trivial commutators with the Lorentz generators $L_{\mu\nu}$ in the left-hand side. The generators $R_{\mu\nu}$, $L_{\mu\gamma}$, P_λ form the algebra of the affine sibgroup $P_4 \otimes GL(4,R)$ (20 generators). The projective transformations are generated by F_λ.

Let us consider nonlinear realizations of the projective group when the algebraic subgroup is the homogeneous Lorentz one.

The quantity G (II.2) in this case can be represented as follows:

$$G(x, \xi) = e^{i x_\mu P_\mu}\, e^{i \frac{1}{2} h_{\mu\nu} R_{\mu\nu}}\, e^{i q_\rho F_\rho}, \qquad \text{(III.8)}$$

where $h_{\mu\nu}(x)$, $q_\lambda(x)$ are the Goldstone fields.

One may check that the Cartan forms $\tilde{\omega}^P_\mu$, $\tilde{\omega}^R_{\mu\nu}$, $\tilde{\omega}^F_\lambda$, $\tilde{\omega}^L_{\rho\gamma}$ are related to the Cartan forms of nonlinear realizations of the affine group ω^P_μ, $\omega^R_{\mu\nu}$, $\omega^L_{\mu\nu}$ /10/ as

$$\tilde{\omega}^P_\mu = \omega^P_\mu \qquad \text{(a)}$$

$$\tilde{\omega}^R_{\mu\nu} = \omega^R_{\mu\nu} - (q_\nu \omega^P_\mu + q_\mu \omega^P_\nu) - 2\delta_{\mu\nu} q_\lambda \omega^P_\lambda \qquad \text{(b)}$$

$$\tilde{\omega}^F_\mu = dq_\mu + (q_\rho \omega^R_{\rho\mu} - q_\rho \omega^L_{\rho\mu}) - q_\mu (q_\nu \omega^L_\nu) \qquad \text{(c)}$$

$$\tilde{\omega}^L_{\mu\nu} = \omega^L_{\mu\nu} - (q_\mu \omega^P_\nu - q_\nu \omega^P_\mu) \qquad \text{(d)}$$

(III.9)

The covariant derivative of the field $h_{\mu\nu}(x)$ takes the form:

$$\tilde{\nabla}_\lambda h_{\mu\nu}(x) = \nabla_\lambda h_{\mu\nu}(x) - q_\nu \delta_{\mu\lambda} - q_\mu \delta_{\nu\lambda} - 2q_\lambda \delta_{\mu\nu}, \qquad \text{(III.10)}$$

where $\nabla_\lambda h_{\mu\nu}(x)$ is the covariant derivative in nonlinearly realized affine symmetry itself /10/.

Since commutator (III.7e) contains the generator $R_{\mu\nu}$ in the right-hand side, it follows from Theorem III that the field $q_\lambda(x)$ is unessential and can be expressed in terms of the true Goldstone field $h_{\mu\nu}(x)$. Solving the most general covariant equation

$$\tilde{\nabla}_\lambda h_{\mu\mu} + b\, \tilde{\nabla}_\mu h_{\mu\lambda} = 0 \qquad \text{(III.11)}$$

relative to $q_\lambda(x)$ we find the one-parameter set of solutions:

$$q_\lambda(x) = \frac{1}{10+7b}(\nabla_\lambda h_{\mu\mu} + b\, \nabla_\rho h_{\rho\lambda})\quad b \neq -\frac{10}{7}. \qquad \text{(III.12)}$$

Note that using condition (III.12), one may represent the covariant derivative of an arbitrary field $\psi(x)$ as follows

$$\tilde{\nabla}_\lambda \psi \, \omega^P_\lambda = d\psi + \frac{i}{2}(\tilde{\omega}^L_{\mu\nu} + \tilde{V}_{\mu\nu}) L^\psi_{\mu\nu} \psi \;, \qquad \text{(III.13)}$$

where

$$\tilde{V}_{\mu\nu} = [a_1 (\tilde{\nabla}_\mu h_{\nu\lambda} - \tilde{\nabla}_\nu h_{\mu\lambda}) + a_2 (\delta_{\mu\lambda} \tilde{\nabla}_\tau h_{\tau\nu} - \delta_{\nu\lambda} \tilde{\nabla}_\tau h_{\tau\mu})] \omega^P_\lambda \qquad \text{(III.14)}$$

and a_1, a_2 are arbitrary parameters. At the same time in nonlinear realizations of the affine symmetry alone the general expression for the covariant derivative $\nabla_\lambda \psi$ includes three arbitrary parameters

$$\nabla_\lambda \psi \, \omega^P_\lambda = d\psi + \frac{i}{2}(\omega^L_{\mu\nu} + V_{\mu\nu}) L^\psi_{\mu\nu} \psi \qquad \text{(III.15)}$$

$$V_{\mu\nu} = [c_1 (\nabla_\mu h_{\nu\lambda} - \nabla_\nu h_{\mu\lambda}) + c_2 (\delta_{\mu\lambda} \nabla_\nu h_{\rho\rho} - \delta_{\nu\lambda} \nabla_\lambda h_{\rho\rho}) +$$

$$+ c_3 (\delta_{\mu\lambda} \nabla_\rho h_{\rho\nu} - \delta_{\nu\lambda} \nabla_\rho h_{\mu\rho})] \, \omega^P_\lambda \qquad \text{(III.16)}$$

Taking into account formulae (III.9,d), (III.10), (III.12) and comparing (III.13) with (III.15) one may show that for $\nabla_\lambda \psi$ be covariant with respect to the projective group it is necessary and sufficient that the following relation

$$1 + c_1 - 7c_2 + 10 c_3 = 0 \qquad \text{(III.17)}$$

hold *.

Thus, on using the inverse Higgs phenomenon, the dynamical restrictions of the projective symmetry reduce to relation (III.17) between constants of minimal and non-

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*It has been shown in paper /10/ that for $c_2 = c_3 = 0, c_1 = -1$, $\nabla_\mu \psi$ is covariant under the conformal symmetry and, at the same time, under the general covariance group. This choice of parameters c_1, c_2, c_3 is consistent with condition (III.17) that is natural since the projective group is a subgroup of the general covariance group.

minimal couplings of the true Goldstone field $h_{\mu\nu}(x)$ with a field $\psi(x)$ in nonlinear realizations of the affine subgroup.

IV. Conclusion

Throughout the above consideration we concentrated on those aspects of the inverse Higgs phenomenon in which it differed from the direct one. It should be pointed out, however, that there exists one analogy between these phenomena. Both of them lead to a reduction of initial symmetry to lower one. As seen from examples considered, on using the inverse Higgs phenomenon the massive Yang-Mills theory of gauge fields $\mathfrak{Z}_\mu^i$, $\mathfrak{V}_\mu^a$ reduces to that of field $\mathfrak{V}_\mu^a$ alone, the conformal symmetry reduces to the scale invariance, the projective group symmetry reduces to the affine one. In all the cases, the only trace of higher symmetry is relations between minimal and nonminimal coupling constants of true gauge and true Goldstone fields of a type of sum rules (II.16), (III.17). Analogously, after eliminating the Goldstone fields from some Lagrangian by the usual Higgs phenomenon the only manifest invariance which remains is the invariance under a subgroup of stability of the vacuum. In the "unitary gauge" the whole symmetry one begins with manifests itself through relations between parameters of Lagrangians (i.e., masses, coupling constants).

We would like to note that the main purpose of present paper was to treat general aspects of the inverse Higgs phenomenon. The examples should be regarded mainly as methodical ones. The applications of the phenomenon in question to more realistic models will be presented elsewhere.

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Appendix

Let us solve eq. (II.9).

We define first the nonsingular matrix $A_{nt}(\xi)$

$$e^{-i\xi_i Z_i}\frac{\delta}{\delta\xi_t}e^{i\xi_i Z_i} = i Z_m A_{mt}(\xi) + \ldots \qquad (A.1)$$

In (A.1) and subsequent formulae we are interested only in coefficients for generators Z_i.

Using the basic law of nonlinear realizations (II.2) it is easy to find

$$e^{-i\xi_k Z_k} Z_t e^{i\xi_k Z_k} = Z_m A_{mn}(\xi)\mathcal{F}_{nt}(\xi) + \ldots \qquad (A.2)$$

$$e^{-i\xi_k Z_k} V_\alpha e^{i\xi_k Z_k} = Z_m A_{mn}(\xi) C^{p\alpha n}\xi_p + \ldots \qquad (A.3)$$

Taking into account relations (A.1-3) one may represent the Cartan form $\omega^i(d)$ (II.4) as

$$\omega^i(d) = A_{is}(\xi)(d\xi_s + f\mathcal{F}_{sp}(\xi)\mathcal{Z}_p + fC^{p\alpha s}\xi_p(\mho_\alpha). \qquad (A.4)$$

As matrices A_{is}, $\mathcal{F}_{tp}$ are nonsingular, eq. (II.9) is solvable relative to $\mathcal{Z}_i$. As a result, solution (II.10) is arrived at.

Let us prove that conditions a) and b) of Theorem III are necessary ones.

Suppose that the analytical function $f_{i_N}(x,\xi_t,\partial\xi_t)\,(t\neq i_N)$ with transformation properties of field ξ_{i_N}

$$\delta f_{i_N} = \beta_{i_N} + O(\xi_t, x_\mu, \partial_\nu\xi_t) \qquad (A.5)$$

exists, where β_{i_N} is the parameter of the group transformation with generator Z_{i_N}. It follows from law (A.5) that expansion of f_{i_N} in power series of $\xi_t, x_\mu, \partial_\rho\xi_t$ starts from terms of the first order in the fields. It is not possible for additive term β_{i_N} in (A.5) to appear from terms of the first order in x_μ or in ξ_t because infinitesimal group transformations of x_μ and ξ_t contain parameter β_{i_N} in higher orders in x_μ and the Goldstone fields. Hence, the addition β_{i_N} may appear only from terms of the first order in the derivative $\partial_\mu \xi_\iota$. The function f_{i_N} transforms under the subgroup H by the representation $R_N(h)$ therefore the linear in $\partial_\mu \xi_\iota$ term in this function has to transform by $R_N(h)$ too, that is possible only if the product $T^P(h)\otimes D(h)$ contains the representation $R_N(h)$. The necessity of condition a) is proved.

Thus we may write:

$$f_{i_N} = \lambda\, \partial^{\{\mu}\, \xi^{\iota\}_N} + \dots$$

Using the basic law of nonlinear realization (II.2) and commutation relations (II.1) one easily finds the infinitesimal form of transformation of the field ξ_{i_N} in the lowest order in parameter β_{i_N} and in the first non-vanishing order in the Goldstone fields ξ_ι and coordinate x_μ:

$$\delta \xi_\iota = -\beta_{i_N} x_\rho C^{i_N \rho \iota} - \frac{1}{2} \beta_{i_N} \xi_k C^{i_N k \iota} + O(\xi, x, \beta) \quad (\iota \neq i_N)$$

that results in the following transformation law for the function f_{i_N}

$$\delta f_{i_N} = -\lambda \beta_{i_N} C^{i_N \{\mu \iota\}_N} + O(\xi, x, \partial_\nu \xi) . \qquad \text{(A.6)}$$

It follows from the Shur lemma that

$$C^{i_N \{\mu \iota\}_N} = \beta \delta^{i_N \{\mu \iota\}_N} .$$

Comparing (A.5) with (A.6) one finds then that $\beta \neq 0$ and $\lambda = -\frac{1}{\beta}$ that proves the necessity of condition b).

References

1. P.W.Higgs. Phys.Rev., 145, 1156 (1966).
2. А.А.Мигдал, А.М.Поляков. ЖЭТФ, 51, 135 /1966/.
3. T.W.Kibble. Phys.Rev., 155, 1554 (1967).
4. S.Coleman, J.Wess, B.Zumino. Phys.Rev., 177, 2239, (1969); C.L.Callan, Jr. and S.Coleman, J.Wess and B.Zumino. Phys.Rev., 177, 2247 (1969).
   Д.В.Волков. Препринт ИТФ 69-75, Киев /1969/.
   C.Isham. Nuovo Cim., v. LIXA, No. 3, 356 (1969).

5. A.Salam, J.Strathdee. Phys.Rev., 184, 1750 (1969).
6. C.Isham, A.Salam, J.Strathdee. Ann.Phys., (N.Y.), 62, 98 (1973).
7. Д.В.Волков. ЭЧАЯ, 4, 3 /1973/.
8. V.I.Ogievetsky. Proceedings of X-th Winter School of Theoretical Physics in Karpacz, v. 1, p. 117, Wroclaw, 1974.
9. K.Kawarabayashi, S.Kitakado. Preprint, Kyoto (1968). K.Kawarabayashi. Lectures in Theor. Phys., v. XI-A, pt. 1, p. 227, Interscience, New York, 1969.
10. A.B.Borisov,V.I.Ogievetsky. Preprint JINR, E2-7684, Dubna, 1974.
А.Б.Борисов, В.И.Огиевецкий. ТМФ 21, 329 /1974/.
11. S.Gasiorowicz, D.A.Geffen. Rev. of Mod. Phys., 41, 531 (1969).
12. Д.В.Волков, В.П.Акулов. Письма в ЖЭТФ, 16, 621 /1972/. Препринт ИТФ 73-51Р, Киев /1973/; ТМФ, 18, 39 /1974/.
D.V.Volkov, V.P.Akulov. Phys.Lett., 46B, 109 (1973).
13. Д.В.Волков, В.А.Сорока. Письма в ЖЭТФ, 18, 529 /1973/, ТМФ, 20, 291 /1974/.
14. C.G.Callan, S.Coleman, R.Jackiw. Ann.Phys., 59, 42 (1970).
15. T.D.Lee, S.Weinberg, B.Zumino. Phys.Rev.Lett., 18, 1029 (1967).
16. K.Kawarabayashi, M.Suzuki. Phys.Rev.Lett., 16, 255 (1966).
Riazuddin, Fayyazuddin. Phys.Rev., 147, 1071 (1966).
17. С.Адлер, Р.Дашен. Алгебры токов и их применение в физике частиц. "Мир", 1970.
18. D.A.Geffen. Phys.Rev.Lett., 19, 770 (1967).

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