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ΔΔ-PLOT

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$\Delta\Delta$-PLOT

A $\Delta\Delta$ -plot is a plot convenient for investigating the interference of identical particles. The closer the momenta of two particles are the stronger the interference between their two-particle states is /1/; it is natural to introduce the plot on which the dependence of the interference upon the difference $\Delta\vec{p} = \vec{p}_1 - \vec{p}_2$ is seen. We call it $\Delta\Delta$-plot.

Since momenta are vectors, the $\Delta\Delta$ -plot should have three or four dimensions. But as we can see, in fact two-dimensional plot is sufficient to show the effect of interference. We have chosen such components of 4-momenta:

1) $\Delta\omega = \omega_1 - \omega_2$ is the difference between the particle energies.

2) $\Delta\vec{p}_\perp = \vec{p}_{1\perp} - \vec{p}_{2\perp}$ is the difference between their "transversal" momenta. But we determine the last quantity not as usual. Find the direction of the sum of particle momenta

$$\vec{n} = (\vec{p}_1 + \vec{p}_2) / |\vec{p}_1 + \vec{p}_2| . \tag{1}$$

Project onto $\vec{n}$ the difference $\vec{q} = \Delta\vec{p} = \vec{p}_1 - \vec{p}_2$

$$q_{\parallel} = \vec{q} \cdot \vec{n} . \tag{2}$$

Then by definition

$$\Delta\vec{p}_\perp = \vec{q}_\perp = \vec{q} - q_{\parallel}\vec{n} . \tag{3}$$

It is easy to see that

$$\Delta\vec{p}_\perp^{\,2} = \vec{q}_\perp^{\,2} = \vec{q}^{\,2} - q_{\parallel}^2 = 4\,\frac{\vec{p}_1^{\,2}\,\vec{p}_2^{\,2} - (\vec{p}_1 \cdot \vec{p}_2)^2}{(\vec{p}_1 + \vec{p}_2)^2} \tag{4}$$

or in an invariant form

$$\Delta\vec{p}_\perp^{\,2} = -4\,\frac{\Delta_3(p, p_1, p_2)}{\Delta_2(p, p_1 + p_2)} , \quad \Delta\omega = \frac{p \cdot q}{(p^2)^{1/2}} . \tag{5}$$

Here Δ_j is the $j \times j$ Cayley determinant /2/; 4-vector p is the

4-momentum of a group of particles in whose CMS we want to determine $\Delta\omega$ and $\Delta p_\perp^2$.

The two-dimensional plot, which shows the dependence of the number of particle pairs upon $\Delta\omega$, $\Delta p_\perp^2$, is referred to as the $\Delta\Delta$ -plot (fig. 1). If there is no interference, this number should be distributed as the dotted line shows. The interference of two-particle states of spinless like bosons ($\pi^+\pi^+$, $\pi^-\pi^-$,etc.) should increase by a factor of two (and that of like nucleons should decrease by a factor of two) the number of pairs in the vicinity of the origin.

In this paper we show that the $\Delta\Delta$ -plot is useful for a search for the like particle interference. In Section 1 we show that for this aim the two-dimensional plot indeed suffices (and the one-dimensional plot may be not). In Section 2 we esteem the phase space distribution over the $\Delta\Delta$ -plot (the background of interference events).

1. THE CHOICE OF VARIABLES

We are not obliged to choose the pair $\Delta\omega$, $\Delta p_\perp^2$ as the only possible variables on the $\Delta\Delta$ -plane ; another pair, e.g., $\Delta p_\parallel$, $\Delta p_\perp^2$, could be chosen. The reason why the first pair has been chosen is that the simplest models of the processes, in which the interference can show itself, predict its dependence on $\Delta\omega$, $\Delta p_\perp^2$. Consider, e.g., the model in which several mesons are emitted from an excited volume of a radius R. All emitters are independent point oscillators which are switched on simultaneously and have the life time $\tau \gg R/c$. Then the possibility of observing the like meson pairs is /3,4/

$$p_\perp^{(2)} \sim 1 + \frac{I(|\Delta\vec{p}_\perp| R)}{1+(\Delta\omega\cdot\tau)^2} , \qquad (6)$$

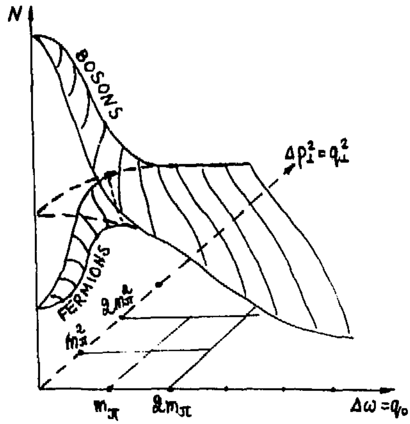

Fig. 1. Expected distribution of like bosons, like fermions and unlike particles on the $\Delta\Delta$ -plot. The approximate scale is indicated.

where $I(x)=[2J_1(x)/x]^2$, J_1 being the Bessel function. If one knew the coefficient of proportionality in this formula, one could obtain from the dependence of the effect upon $\Delta\omega$, $|\Delta\vec{p}_\perp|$ the parameters τ and R.

A more general consideration in /1/ permits one to substitute in (6) the sign $\sim$ by = . The right-hand side of (6) is, as it turns out, the most rapidly changing part of the expression for the probability of the effect of interest. Besides, the factors depending separately on p_1 and on p_2 can enter into this expression. But they vary much more smoothly. If there is no interference, there remain only these slowly varying factors, so the expression should be similar to the usual momentum distribution obtained from the multiple production matrix element. The formula was proposed in /5/, which includes the interference effects in the usual expression for the N-particle production cross section

$$d\sigma(p_1,\ldots,p_N)=|\mathfrak{M}|^2 P^{(2)}\delta^4(\Sigma p_i-P_o)\prod_1^N \frac{d^3\vec{p}_i}{2\omega_i}. \qquad (7)$$

Here $d\sigma$ is the differential cross section of production of N particles with momenta $p_1,\ldots,p_N$; $\mathfrak{M}$ is the amplitude of this process neglecting the interference effects (for example, $\mathfrak{M}$ is equal to the sum of squares of Feynmann graphs); P_o is the initial 4-momentum, and the factor $P^{(n)}$ takes into account the interference between n-particle states. For $n=2$ $P^{(2)}$ is given by the r.h.s. of (6). More generally,

$$P^{(2)}=1+\lambda\left|\frac{\mathcal{E}(\Delta\omega,\Delta\vec{p})}{\mathcal{E}(0,\ 0)}\right|^2 {}^{*)}, \qquad (8)$$

where $\mathcal{E}(\Delta\omega,\Delta\vec{p})$ is the mutual coherence function. The method of its obtaining was described in /1/. Using it, one can calculate

*) λ is arbitrary

$P^{(2)}$ for some other models. Consider, e.g., the sphere, which is homogeneously filled with meson sources. Then we should take in (6)

$$I(x) = \left[3(\sin x - x\cos x)x^{-3}\right]^2, \qquad (9)$$

where $x = |\Delta\vec{p}|R$. If the sources of mesons 1 and 2 are distributed in a Gauss-like manner

$$\rho(r_i) \sim \exp\left[-\frac{1}{2}\left(\frac{x_i^2}{A^2} + \frac{y_i^2}{B^2} + \frac{z_i^2}{C^2}\right)\right], \quad i = 1, 2, \qquad (10)$$

then[6]

$$I(x) = \exp\left(-A^2\Delta p_x^2 - B^2\Delta p_y^2 - C^2\Delta p_z^2\right). \qquad (11)$$

There appear the projections of $\Delta\vec{p}$ onto the principal axes of the distribution.

According to eqs. (9), (11) and to general eq. (8), the interference effect depends not only on $|\Delta\vec{p}_\perp|$ but also on all three components of $\Delta\vec{p}$. Nevertheless, one can manage with the two-dimensional $\Delta\Delta$-plot only. For that one can take into account the identity $\Delta p_\parallel = \Delta\omega/v$, where v is the pair velocity. Then the argument $I(x)$ in (9) can be written in the form

$$x^2 = (\Delta\vec{p})^2 R^2 = (\Delta\vec{p}_\perp)^2 R^2 + (\Delta\omega)^2 (R^2/v^2).$$

We see that the interference effect in fact depends upon two variables $\Delta\vec{p}_\perp^{\,2}$, $\Delta\omega$ only.

The situation with the model (10) is more complicated: to do with the two-dimensional plot, one should take into account the cylindrical symmetry of the excited volume (A = B) and fix the pair direction $\vec{n}$ along the interaction axis $(\vec{n}\,\|\,\vec{z})$; then in (10) there again remains the dependence on $\Delta\omega$ and $\Delta\vec{p}_\perp^{\,2}$. Mixing together the events observed in various directions, we should lose the possibility of measuring the parameters A, C, τ

separately, but the interference peak on the $\Delta\Delta$ -plot will still be seen.

Is it possible to manage with a one-dimentional plot? No, if it is a $\Delta\omega$- or $\Delta\rho_\perp^2$ -plot, because, integrating (6) over $\Delta\vec{p}_\perp^2$ or $\Delta\omega$, correspondingly, the interference term contribution is much less than that of the first term. One-dimensional plots hide the effect if there are no additional dynamics which increases the interference term.

However, let us replace two decreasing functions entering into the interference term in (6) approximately by the exponents

$$I(x) = I\left(|\Delta\vec{p}_\perp| R\right) \cong \exp\left(-\tfrac{1}{2}R^2\Delta\vec{p}_\perp^{\,2}\right),\ \frac{1}{1+(\tau\Delta\omega)^2} \cong \exp(-\tau^2\Delta\omega^2).$$

Let also $2\tau^2 \cong R^2$. Then we have approximately

$$P^{(2)} \cong 1 + e^{-\tau^2(\Delta\vec{p}_\perp^{\,2}+\Delta\omega^2)} \cong 1 + e^{-\tau^2\Delta\vec{p}^{\,2}}. \qquad (12)$$

The last substitution is valid for high energies when $\Delta\omega \cong \Delta p_\parallel$. Therefore the interference maximum sometimes can become apparent in one-dimensional spectra of a $\Delta\vec{p}^{\,2}$-type (or of a $\Delta m_{\pi\pi}^2$-type correlated with them). But the multiple production parameters get mixed and cannot be determined separately.

Now let us return to eq. (7). Substitute one of the integration variables (for $N>3$) by $\Delta\vec{p}$ and integrate it over all other variables. If the factor 1 stood instead of $P^{(2)}$, one would obtain after integration the phase space distribution on the $\Delta\Delta$ -plot: $W_{PS}(\Delta\omega, \Delta p_\perp^2)\,d\Delta\omega\, d\Delta p_\perp^2$. But except for the term 1, the interference term enters into $P^{(2)}$. As it depends only upon $\Delta\omega$, $\Delta p_\perp^2$, the integration does not change the density W_{PS}. Therefore the distributions on the $\Delta\Delta$ -plot with and without the interference are connected by a simple formula

$$W(\Delta\omega, \Delta\vec{p}_\perp^2) = P^{(2)}(\Delta\omega, \Delta\vec{p}_\perp^2)\, W_{PS}(\Delta\omega, \Delta\vec{p}_\perp^2). \quad (13)$$

The existence of such a simple formula is an extra reason to use the $\Delta\Delta$ -plot. In the next section we shall calculate W_{PS} . However, the background W_{PS} can be also obtained experimentally if the interference has been switched off. For example, it is widely believed that for $\bar{p}p$ -annihilation the statistical model is valid, which excludes the interference between unlike pions. In this case eq. (13) can be written in the form

$$W_{\pi^\pm\pi^\pm}(\Delta\omega, \Delta\vec{p}_\perp^2) = P^{(2)}(\Delta\omega, \Delta\vec{p}_\perp^2)\, W_{\pi^+\pi^-}(\Delta\omega, \Delta\vec{p}_\perp^2). \quad (14)$$

This relation holds not only for the phase space distribution but also for $\mathfrak{M} \neq 1$.

May be we are not sure that there is no interference between $\pi^+\pi^-$-states. In this case M.I.Podgoretsky suggests to take background pion pairs from different many-particle events.

§ 2. PHASE SPACE DISTRIBUTION

Now we want to calculate the phase space distribution $W_{PS}(\Delta\omega, \Delta\vec{p}_\perp^2)$ over the $\Delta\Delta$ -plot. We designate for brevity $\Delta p = q = \{q_0, \vec{q}\}$, where $q_0 = \Delta\omega$, $\vec{q} = \Delta\vec{p}$. The bulk of relativistic pions, which makes a contribution to the effect at the origin, are "V-events": pion pairs with $\vec{p}_1 \approx \vec{p}_2$, $|\vec{q}| \ll |\vec{p}_1|$. It is natural to introduce for them the direction $\vec{p}_1 \approx \vec{p}_2 \approx \vec{p}$ as a polar axis; thus we come to cylindrical coordinates with the elementary volume $d^3\vec{q} = \pi\, dq_\perp^2\, dq_\parallel = \frac{\pi}{v} dq_\perp^2\, dq_0$. We see that at the origin ($q_0 = q_\perp = 0$) the pair density does not vanish. It would vanish if the variables $\Delta\omega, |\Delta\vec{p}_\perp|$ were plotted on the $\Delta\Delta$ -plot. This explains unsymmetrical, at first glance, choice of the variables on the $\Delta\Delta$ -plot.

Now we introduce $\vec{p}_1+\vec{p}_2=\vec{P}$, $\omega_1+\omega_2=E$, $M^2=E^2-P^2$ and multiply (7) by

$$1=\int d^4P\,\delta^4(p_1+p_2-P)\,dM^2\,\delta(E^2-\vec{P}^2-M^2).$$

After integrating over $\vec{p}_1,\dots,\vec{p}_N$, we obtain

$$\sigma=\int dM^2\,\frac{d^3\vec{P}}{2E}\int\frac{d^3\vec{p}_1}{2\omega_1}\,\frac{d^3\vec{p}_2}{2\omega_2}\,\delta^4(p_1+p_2-P)\times \tag{15}$$

$$\times\int\prod_3^N\frac{d^3\vec{p}_k}{2\omega_k}\,\delta^4(P+p_3+\dots+p_N-P_0).$$

The last integral is the phase space of particles 3,4,..., N with the total momentum P_0-P. We designate it as $S_{N-2}(\mu^2)$, where $\mu^2=(P_0-P)^2=M_0^2+M^2-2M_0E$. Let a minimal value of μ be $\bar{\mu}$ (the sum of the masses of particles 3,4,...,N).

Then we change the variables in the second integral. New variables are $q_0, q_\perp^2$. It follows from the identities $p_1^2-p_2^2=0$ and $(p_1+p_2)^2+(p_1-p_2)^2=4m^2$ that $q_0=q_\parallel P/E$ and $q_\perp^2+q_\parallel^2-q_0^2=M^2-4m^2$. Excluding $q_\parallel$, we see that all possible decays of a "particle" with momentum P and mass M lie on the arc of the ellipse on the plane $(q_0, q_\perp)$

$$\frac{q_0^2}{\left(\frac{P}{M}\sqrt{M^2-4m^2}\right)^2}+\frac{q_\perp^2}{\left(\sqrt{M^2-4m^2}\right)^2}=1,\ (q_\perp>0) \tag{16}$$

On the contrary, the same equation on the plane of the variables (P,M) gives the curve

$$\frac{q_0^2}{P^2}+\frac{\tilde{q}^2}{M^2}=1,\quad \tilde{q}^2=q_\perp^2+4m^2,\ m=m_\pi, \tag{17}$$

drawn in fig.2. The physical values of (P,M) lie on the curve (KLK') inside the region OAB

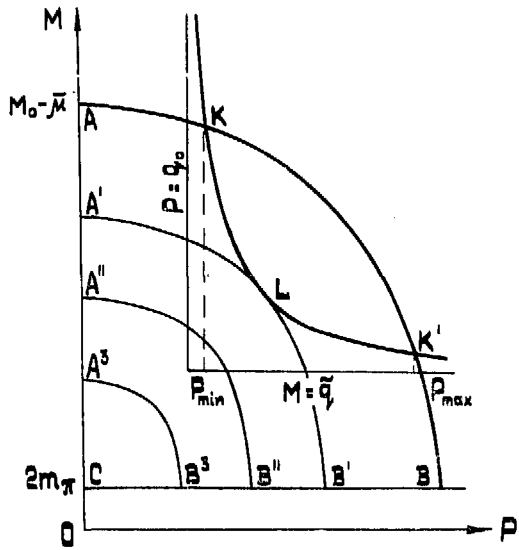

Fig. 2. Physical region of (P, M). For any $q_0, \tilde{q}$ this is the region ABC, for given $q_0, \tilde{q}$ — KLK'. The arcs AB, A'B', A''B'',etc. are the curves of constant μ^2 (or constant S_{N-2}). For $q_0 = 0$ the arc KLK' turns into the segment of the straight line parallel to the axis P.

$$2m \leq M \leq M_0 - \bar{\mu} \quad ; \quad 0 \leq P \leq \sqrt{\left(\frac{M_0^2 + M^2 - \bar{\mu}^2}{2M_0}\right)^2 - M^2} \, . \qquad (18)$$

Replacing in the integral (15) the variables $\vec{p}_1, \vec{p}_2$ by $q_0, q_\perp^2$ and holding (P, M) fixed, we obtain

$$\sigma = \int \frac{4\pi P^2 dP dM^2}{2E} \int \frac{2\pi dq_\perp^2 dq_0}{E^2 - q_0^2} p^{(2)} \delta(\omega_1 + \omega_2 - E) S_{N-2}(M_0^2 + M^2 - 2M_0 E) \qquad (19)$$

Now we can change the order of integration and integrate the δ -function. The final expression for the phase space distribution in the integral form is

$$W_{PS}(q_0, q_\perp^2) = 8\pi^2 \int_{P_{min}}^{P_{max}} dP \, \frac{P^2 S_{N-2}(M_0^2 + M^2 - 2M_0 E)}{\left[(P^2 - q_0^2)(P^2 - q_0^2 + \tilde{q}^2)\right]^{1/2}} \, . \qquad (20)$$

Here the values M and E should be taken along the curve KLK'. This means that

$$M = P \frac{\tilde{q}}{(P^2 - q_0^2)^{1/2}} \, , \quad E = P \frac{(P^2 - q_0^2 + \tilde{q}^2)^{1/2}}{(P^2 - q_0^2)^{1/2}} \, . \qquad (21)$$

The integration limits P_{min}, P_{max} in (20) are the abscissas of the points where the curve KLK$'$ intersects the physical boundary ABC. Eq.(20) must be substituted in the r.h.s. of eq.(13) to obtain on the $\Delta\Delta$ -plot the distribution of interfering two-particle states. The background W_{PS} is normalized to the total phase space of all N particles.

The analytic estimates of eq. (20) are based on the assumption that a main contribution to the integral (20) (for $q_0 \neq 0$) is made from the parts of the curve KLK$'$ lying near the point L. It is the point where this curve is tangent to the curve A$'$B$'$ corresponding to the largest value of missing mass which is permitted by given $(q_0, q_\perp^2)$. These formulas are cumbersome and we do not write them here. The results of the nume-

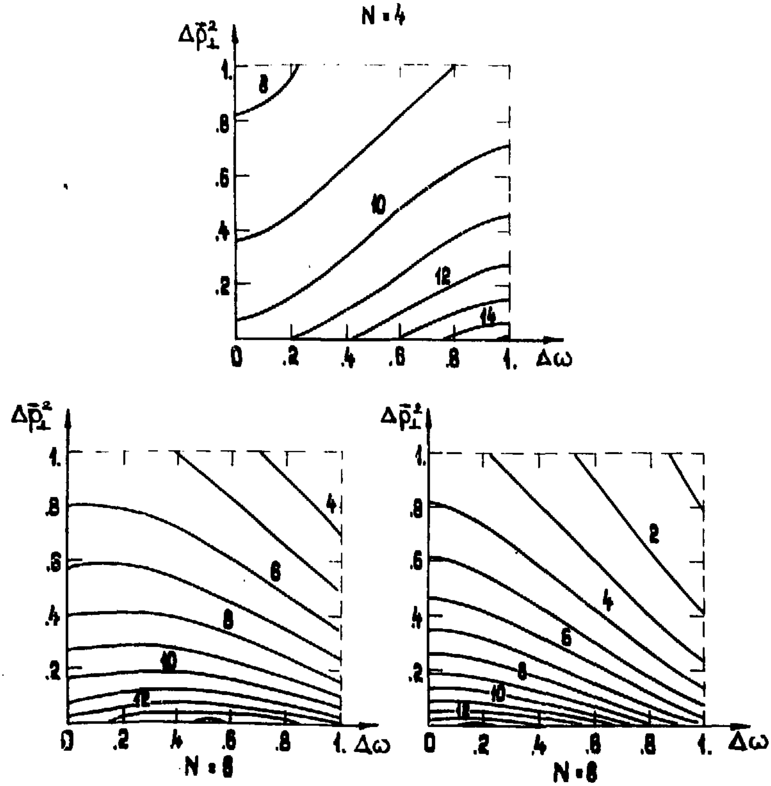

Fig. 3. Relief of the phase space distribution on the $\Delta\Delta$-plot for various N. The lines of constant density go through each fourteenth part of the total height.

rical calculations of $W_{PS}(q_0, q_\perp^2)$ for the extremely relativistic system of particles having M_0 = 6 GeV, N = 4, 6, 8 are shown in fig.3. For N = 4 the factor $S_{N-2}(\mu^2)$ does not enter into the integral and does not influence W_r ; it is seen that with increasing N the maximum of the density W_{PS} displaces toward the origin; the distribution of the background becomes more steeper; only the region near the origin is of interest - its dimension is a few (2 - 3) m_π (fig.1). In any case there is no sharp peak near the origin so we can substitute the nonanalytic surfaces of fig. 3 by convenient analytic ones, e.g., by a Gauss-like or bell-like background.

In order to obtain the function $P^{(2)}$ from the experiment, it is convenient to give the weight $1/W_{PS}(\Delta\omega, \Delta p_\perp^2)$ to each pion pair with momenta $\vec{p}_1, \vec{p}_2$.

§ 3. CONCLUSIONS

As the statistics in the interference experiment will become larger and larger, the interference peak on the $\Delta\Delta$-plot will begin to be more apparent. We call this construction, built on the bins of the $\Delta\Delta$ -plot, "the Babel tower effect". As the very Babel tower had fallen when the languages of its builders ceased to be identical, the peak on the $\Delta\Delta$ -plot holds only due to the identity of bosons and should be destructed for nonidentical particles. The measurement of the Babel tower effect can provide us the space distribution of pion sources and their life time. This can help to investigate how the mesons can be fruitful and multiply.

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