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A CLASSIFICATION OF MODEL PREDICTIONS ON ASYMPTOTIC BEHAVIOUR OF MULTIPLICITY MOMENTS

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1. Introduction

In the last few years a lot of experimental data on multiparticle production at high energies have become available from Serpukhov, ISR (CERN), and Fermilab (Batavia) /1/. It is relatively easy to measure the average multiplicity and higher moments, although usually only the charged multiplicities are determined. Parallelly, the dependence of the average multiplicity (and higher moments) on primary energy is intensively studied theoretically. However, there are unknown any rigorous results on the behaviour of the average multiplicity excepting the trivial kinematical bound, of course. Till now it is not clear if the general principles (like unitarity and analyticity) could control the average multiplicity behaviour /2/. In a recent paper, with some general assumptions and the Jin-Martin bound /3/, Khuro /2/ finds a logarithm of s upper bound for the average multiplicity $\langle n \rangle$, where s is the squared center of mass energy. We remember the main assumptions of ref. /2/: A. The convergence of the perturbation series of the n-particle production cross sections in the (unphysical) neighbourhood of the renormalized coupling constant. B. The temperedness of the n-particle production cross sections. Pointing out that both assumptions have not been rigorously proven, Khuri shows that the bound $\langle n \rangle \lesssim \ln s$ is, in fact, specific to multiperipheral-type models, independent of their dynamical details. Similar results are obtained in refs. /4/ for general classes of multiperipheral models. However, although both hypotheses A and B are accepted by many models, especially hypothesis A is very likely to be not true in realistic field models (see refs. included in /2/). So far, it seems that other (upper or lower) bounds are not excluded by the present status of the theory.

In this paper we present a method to classify various asymptotic behaviours of the average multiplicities and higher moments. The method is similar to that used to deduce the Froissart bound (see, e.g., /5/) for the total cross section (Sect. 2). Section 3 deals with an outline of the main model predictions for $<n>$ and their realization in the presented scheme. Finally, we briefly discuss the classification of models against experimental data (Sect. 4).

2. Bounds on Multiplicity Moments

We start with some definitions and notations. Let us denote by σ_n the (exclusive) cross section of a hadronic ab collision with n particles in the final state and by σ_{tot} the total cross section for this collision. We have obviously the relation σ_{tot}

$$\sigma_{tot} = \sum_{n=2}^{\sqrt{s}} \sigma_n . \tag{1}$$

In fact, in relation (1) n runs till the integer part of $(\sqrt{s}-m_a-m_b)/m_\pi$, but we take $\sqrt{s}$ in pionic masses and m_a, m_b are negligible because we shall be concerned with asymptotic behaviours. Defining the probabilities of n - particle production $p_n = \sigma_n / \sigma_{tot}$,eq. (1) can be written

$$1 = \sum_{n=2}^{\sqrt{s}} p_n . \tag{1'}$$

Let $<n>$ denotes the average multiplicity

$$<n> = \sum_{n=2}^{\sqrt{s}} n p_n , \tag{2}$$

and $<n^p>$ the multiplicity moments

$$<n^p> = \sum_{n=2}^{\sqrt{s}} n^p p_n , \quad p = 2,3,\ldots \quad . \tag{3}$$

We introduce also the generating multiplicity function /6/ :

$$E(z+1) = \sum_{n=2}^{\sqrt{s}} (z+1)^n p_n , \tag{4}$$

where z is a real number. The function E has evidently the normalization $E(\ell) = \ell$. Let us consider that

$$E(z+1) \leq T(z,s) , \tag{5}$$

and for simplicity we shall denote by T the function $T(z,s)$.

Further, we shall deduce upper bounds for $\langle n^p \rangle (p=1,2,..)$ (eqs. (2), (3)) using the generating function (4), the normalization (1') and supposing various behaviours for T. The idea of the proof is similar to that used for the Froissart bound, where it is supposed the temperedness of the amplitude and the Martin-Lehmann analyticity domain (see, e.g., /5/).

Eqs. (4) and (5) imply

or $p_n (1+z)^n < T , \quad n=2,3,\ldots$

$$p_n \leq e^{\ell n T - n\gamma} , \quad n = 2,3,\ldots, \tag{6}$$

where

$$\gamma = \ell n (z+1) . \tag{7}$$

It is clear that for $n > N$, where

$$N = C \ell n T / \gamma , \tag{8}$$

and C=constant$\gg 1$, we get

$$p_n < e^{\ell n T (1-C)} = 1/T^{C-1} . \tag{9}$$

With the notation

$$I_p = \sum_{N+1}^{\sqrt{s}} n^p p_n , \quad p = 0,1,2,\ldots, \tag{10}$$

the relations (2) and (3) take the form

$$\sum_{n=2}^{N} p_n + I_0 = 1 \ , \tag{11}$$

$$\sum_{n=2}^{N} n^p p_n + I_p = \langle n^p \rangle \ , \quad p = 1,2,\ldots \quad . \tag{12}$$

Introducing eq. (11) in eq. (12) we obtain.

$$\langle n^p \rangle < N^p (1 - I_0) + I_p \ , \quad p = 1,2,\ldots \tag{13}$$

In particular, if $I_p \to 0 \, (p=0,1,2,\ldots)$ as $s \to \infty$, then

$$\langle n^p \rangle \leq N^p \ , \quad p = 1,2,\ldots \quad . \tag{14}$$

If in eq. (8) we take $\gamma \to 0$ and choose various behaviours for T such that $I_p \to 0$ $(p = 0,1,2,\ldots)$, then we get asymptotically the bounds (14) for the multiplicity moments. This is our main result. Moreover, we shall prove that if T is asymptotically unbounded, a sufficient condition for $I_p \to 0 \, (p = 1,2,\ldots)$ is that $I_0 \to 0$.

We now proceed to evaluate the asymptotic behaviour of $I_p \, (p=0,1,2,\ldots)$. We write

$$I_0 = \sum_{n=N+1}^{\sqrt{s}} p_n = \int_{N=C\sqrt{s}\,\ln T/\gamma}^{\sqrt{s}} p(n)\, dn .$$

Here $p(n) = p[n]$, where n is a real number and $[n]$ is the smallest integer greater than n. We obtain easily

$$I_0 \leq 1/\gamma \, T^{c-1} . \tag{15}$$

For $I_p \, (p=1,2,\ldots)$ we get readily the recurrence relation

$$I_p \leq N^p I_0 + p I_{p-1}/\gamma \ , \quad p = 1,2,\ldots ,$$

which explicated leads to the inequality

$$I_p \leq I_0 \left(\sum_{j=0}^{p} \frac{N^{p-j}}{\gamma^j} \frac{p!}{(p-j)!} \right) , \quad p = 1,2,\ldots$$

With the definition (8) we arrive at

$$I_p < I_0 N^p [1+O((1/C\ell n T)^p)],\ T\to\infty \text{ for } s\to\infty,\ p=1,2,\ldots \quad (16)$$

Introducing ineq. (16) in ineq. (13) it may be seen that if $I_0 \to 0$, then we get the bounds (14). So far, with this method, if N is an upper bound for $<n>$, then eq. (14) furnishes automatically bounds for $<n^p>$ $(p=2,3,\ldots)$. Note that in this proof it is not needed the Jin-Martin lower bound on σ_{tot}.

3. Model Predictions

In order to get various possible bounds for $<n>$, we shall particularize the results from Sect. 2 and after this we briefly show how the models realize these behaviours.

Supposing different asymptotic behavious for T (eq. (5)) and γ (eq. (7)) such that $I_0 \to 0$ (eq. (15)), we get various possible bounds for $<n>$ and $<n^p>$ (eq. (14), resp. eq. (8)). Some typical situations are illustrated in Table 1. In fact, any possible asymptotic behaviour for $<n>$ can be reproduced. The case γ = ct. (eqs. (17)-(19)) has been considered in ref. /2/. In eq. (17) the polynomial boundness is supposed for E (eq. (4)), and the case (18) corresponds to the Cheng-Wu model /7/ which violates the logarithmic bound from ref. /2/. An interesting case is obtained in eq. (19) where a constant bound is predicted for $<n>$. Eqs. (17'), (18') repeat the behaviour $N \sim \ell n s$, s^a from eq. (17), respectively (18), but $T(\gamma)$ is asymptotically unbounded (resp. 0). The cases (19a,b,c) particularize eqs. (19d,e) and are presented only to point out that different behaviours of T and γ could give the same bound for $<n>$. Note that all the bounds from eqs. (17)-(19) are of the type $s^{\alpha}(\ell n s)^{\beta}$, $0 \le \alpha \le 1/2$. Eqs. (20), (21) represent other possible behaviours for $<n>$ claimed by some models.

In Table 2 we display some model predictions for multiplicity moments. f_k denote the multiplicity correlation moments

Table 1

Various upper bounds N (eq. (8)) for the average multiplicity $<n>$ corresponding to different asymptotic behaviours of T (eq. (5)) and γ (eq. (7)). In all cases the condition $I_0 \to 0$ is fulfilled (eq. (15)). See the text.

N ($\leq$)	T ($\sim$)	γ ($\sim$)	Conditions	Eq.
$\ln s$	s^a	ct	$a>0$	(17)
s^a	$\exp(s^a)$	ct	$a>0$	(18)
ct	ct	ct		(19)
$\ln s$	$\ln s$	$\ln\ln s/\ln s$		(17')
s^{α}	$s^{(\alpha+\beta)/2}$	$s^{(\beta-\alpha)/2}$	$\alpha>\beta>0$	(18')
$s^{\beta}\ln s$	s^{α}	$s^{-\beta}$	$\alpha,\beta>0$	(19a)
$s^{\beta}(\ln s)^{\alpha}$	$\exp(\ln s)^{\alpha}$	$s^{-\beta}$	$\alpha>1,\beta>0$	(19b)
$(\ln s)^{\alpha}$	$\exp(\ln s)^{(\alpha+\beta)/2}$	$(\ln s)^{(\alpha-\beta)/2}$	$\alpha<\beta,\alpha+\beta>2$	(19c)
$s^{\beta}(\ln s)^{\alpha}$	$\exp(s^{\beta})$	$(\ln s)^{-\alpha}$	$\alpha,\beta>0$	(19d)
$s^{\beta}(\ln s)^{-\alpha}$	$\exp(s^{\beta}(\ln s)^{-2\alpha})$	$(\ln s)^{-\alpha}$	$\alpha,\beta>0$	(19e)
$\ln\ln s$	$\exp(\frac{\ln\ln s}{\ln\ln\ln s})$	$(\ln\ln\ln s)^{-1}$		(20)
$\exp(\ln s)^{1/2}$	$\exp\exp(\ln s)^{1/2}$	$\exp(-(\ln s)^{1/2})$		(21)

$$f_k = \int \prod_{i=1}^{k} \frac{d\vec{p}_i}{p_i^0} C_{1\ldots k}(\vec{p}_1,\ldots,\vec{p}_k) , \qquad (22)$$

and F_k are the binomial moments /6/

$$F_k = <n(n-1)\ldots(n-k+1)> . \qquad (23)$$

The models are classified in diffractive (D), multiperipheral (M) and mixed (D-M). Also considered are statistical (S) and hybrid models (H) (in H models the clusters arise unstatistically and decay statistically /8/). A review could be found, e.g., in /9/.

It may be remarked that all possible dependences of $<n>$ from Table 1 are indeed predicted by various models (Table 2). Different power predictions are obtained by taking particular values a, α, β in Table 1.

Table 2

Summary of model predictions for the asymptotic behaviour of the average multiplicity and higher moments. f_k and F_k are defined by eq. (22), respectively (23). Here V is the "interaction volume", V_0 is a constant "interaction volume", γ is the usual kinematical factor, $p(\epsilon)$ denotes the "preasure" (energy) density, $f(E)$ represents the energy distribution function, and $\gamma = \eta$ $(-\eta)$ in Gribov Reggeon calculus (respectively in absorbtive model)/33/.

[illegible]	Model	$\langle n \rangle \sim$	Observations	Author
	Bremsstrahlung	$s^{1/2}$	$\sigma_{Tot} \sim (\ln s)^2$	Heisenberg/10/
	Perfect gas	$s^{1/4}$	$V = V_0/\gamma$	Fermi/11/
	Perfect gas	$s^{3/8}$	$V = ct$	Fermi/12/
	Statistic covariant	$s^{1/3}$		Fermi, Satz/12/
	Statistic Pomeranchuk	$s^{1/2}$	$V = \ell n V_0$	Pomeranchuk/13/
S	Hydrodynamic	$s^{1/4}$	$p = 1/3\varepsilon$	Landau/14/
	Hydrodynamic generalized	$s^{\frac{1-H}{2(1+H)}}$	$p = H\varepsilon$	Suhonen et al/15/
	Uncorrelated jet	$\ln s$	$\ell = 1$ } $f(E) = 1/E^{\ell}$	Van Hove/16/,
		$s^{(\ell-1)/2}$	$1 < \ell < 2$ } $f(E) = 1/E^{\ell}$	Krzywicki/17/
		$s^{1/2}(\ln s)^{-2}$	$\ell = 2$ } $f(E) = 1/E^{\ell}$	
	Statistical bootstrap	$\sqrt{s}$		Montvay/18/
	Thermodynamical	$\geq \ln s$		Hagedorn/19/
	Nuclear cascade	$\exp(\ln s)^{1/2}$		Arnold/20/
	Two fireballs	$s^{3/8}$		Takagi/21/
Σ	Baryon Isobar	$a + bs^{\rho}$	$\rho = 1/2$	Pal, Peters/22/
	Narayan	$a + bs^{1/2}$		Narayan/23/
	Statistical diffractive	$\ln s$	$f_2 \leq \sqrt{s}, \sigma_n \leq 1/n^2$	Hwa/24/
	Nova	$\ln s$	$\langle n^2 \rangle \sim \sqrt{s}$	Jacob, Slansky/25/
Θ	Limiting fragmentation	$\ln s$	$\langle n^p \rangle \sim s^{(p-1)/2}$	Yang/26/
	Multiperipheral	$\ln s$	$f_n \sim a_n \ln s + b_n$	AFS/27/
	Short range correlations	$\ln s$	$F_n \sim f_1^n$ $(n > n_0)$	Willson/6/,/28/
Σ	Mueller-Regge	$\ln s$	$f_n \sim (C \ln s)^n$	Mueller/29/
	Multiperipheral	$\ln s$	$f_2 \sim 0$	Chew Pignotti/30/

Table II (continued)

X	Classes of multiperipheral models	lns	$f_n \leq C_n(\ln s)^n$, $C_n = C_1^n$	Bassetto /4/, Bassetto, Burras /4/
XI	Two components	lns	$f_2 \sim (\ln s)^2$	Willson /28/, /31/
	Field	$s^{\alpha}(\ln s)^{\beta}$	$.07 \leq \alpha \leq .14$, $-2.72 \leq \beta \leq -2.41$	Cheng Wu /7/
	Automodelity	ct		Matveev et al /32/
	Interacting Pomerons		$\langle n^n \rangle \sim \ln s^{n(1+\ )}$	Caneschi, Jengo /33/

4. Discussion

From Table 2 it may be observed that completely different models have the same predictions for $\langle n \rangle$, e.g., both M and D models could predict a $\ln s$ behaviour. This means that n is not enough to distinguish between the model predictions, such that higher moments are necessary to be taken into account. Unfortunately, our scheme from Sect. 2 establishes for the higher multiplicity moments a bound which is always a power of the bound of $\langle n \rangle$. This is the case only in some models, e.g., Mueller-Regge and some generalized classes of multiperipheral models /4,6/.

It is also interesting to note (Table 2) that the bremsstrahlung, hydrodynamical, generalized, Pomeranchuk statistical, bootstrap and Narayan models predict the saturation of the kinematical bound for $\langle n \rangle$. We remember that new data on average charged multiplicity from Fermilab are described by a logarithm of s /1/, but on the whole interval from accelerator till cosmic ray energies the power 1/4 /34/ (predicted by the hydrodynamical model /14/) or greater /15/ describes the data better. Also a dominant $(\ln s)^2$ dependence seems acceptable for f_2 /1/, ruling out for the moment the use of diffractive models in the initial formulation at available energies /24–26/ and giving apparent support for two-component models /28,29,31/.

However, because the theory (and also some models) does not exclude the asymptotic saturation of the kinematical bound of $<n>$, it is important to establish the conditions for this saturation. This will be done elsewhere [35].

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