СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 8506

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ON COHERENT STATES. 3. CLASSICAL FORM OF QUANTUM FIELD THEORY EQUATIONS (COHERENT STATE REPRESENTATION)

1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ON COHERENT STATES.
3. CLASSICAL FORM OF QUANTUM FIELD THEORY EQUATIONS (COHERENT STATE REPRESENTATION)

I. INTRODUCTION

It has been shown in refs.[1,2], that in theories of three kinds:

1) theory of a free quantized field,

2) theory of a quantized field, interacting with an external current, and

3) theory of a spinor (or any other) charged field, interacting with an external electromagnetic field, – transition from field operators, say, $\hat{\phi}(x)$ and $\hat{\varphi}(x)$ [x]) to coherent state expectation values $\phi(x) = \langle \phi | \hat{\phi}(x) | \phi \rangle$ and $\varphi(x) = \langle \varphi | \hat{\varphi}(x) | \varphi \rangle$ transforms the Kallen-Yang-Feldman equations for the field operators into equations of a type of classical ones.

The equations for the massless electromagnetic (scalar or any other) field in the theories 1) and 2) coincide exactly with classical ones. For a non-zero mass both in 1),2) and in 3) equations have been obtained, which are of a form of classical ones, except for the Planck constant $\hbar$ enters into the equations via the Compton wave length $\frac{\hbar}{mc}$ and in the case of 3) also via the fine structure constant. The causality such as in classical wave

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x) We take notations which somewhat differ from those in [1,2] (the latter are given in brackets for a translation):
$\hat{\phi}(x)$ $(\varphi(x))$ is an interacting (Heisenberg) field operator;
$\hat{\varphi}(x)$ $(\phi(x))$ is a free field operator, the zero approximation for $\hat{\phi}(x)$ in the Kallen- Yang- Feldman equations, the field operator in the interaction representation;
$\varphi(x)$ is an interacting classical field;
$\phi(x)$ $(\phi'(x))$ is a free classical field, the zero approximation for $\varphi(x)$ in classical equations of a form like the Kallen-Yang-Feldman equations;
$|\phi\rangle$ $(|\phi'\rangle)$ means the coherent state (see[1]), the more detailed writing of $|\phi\rangle$ would be $|\phi \dot{\phi}\rangle$.

theory (see /1/, Appendix Б) is characteristic for all these equations.

Main attractive features of the coherent states are in the following properties of the coherent state expectation values.

a) The expectation values of the quantum field coordinate and momentum $\varphi(\vec{x},t') = \langle\varphi|\hat{\varphi}(\vec{x},t')|\varphi\rangle$ and $\partial_4\varphi(\vec{x},t') = \langle\varphi|\partial_4\hat{\varphi}(\vec{x},t')|\varphi\rangle$ at an initial time t' are two completely arbitrary functions of $\vec{x}$ *) like initial values of a classical field.

b) The expectation value theorem for the coherent states. Any operator is determined by its coherent state expectation values (i.e., only by diagonal matrix elements!) **) . To reconstruct the operator we consider its coherent state expectation value as a functional of the initial values $\varphi(\vec{x},t')$ and $\dot{\varphi}(\vec{x},t')$, i.e., of two functions of the 3-argument $\vec{x}$, or, equivalently, as a functional of one function $J(x)$ of the 4-argument x_μ (see /1,2/). The latter is more convenient.

The expectation values $\varphi(x) = \langle\varphi|\hat{\varphi}(x)|\varphi\rangle$ and $\langle\varphi|\hat{\varphi}(x)|\varphi\rangle$ are such functionals. The first of them is linear functional in all the cases, the second one also is linear in the theories I)-3), but non-linear in this work. According to the above theorem we may turn back to the equations for field operators from the equations for the coherent state expectation values in 1)-3)/1,2/.

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*) Unlike the mean squared coordinate and momentum, which are subjected to the uncertainty relation.

**) The theorem is well known in quantum mechanics (see /I.2, I.3/, i.e., refs. /2,3/ in /I/). In relativistic quantum field theory in x-space it is shown in /2/ .

Hence, the same theory may equivalently be represented in a classical form or in a quantum one.

It must be stressed that the classical form of quantum theory is a new representation, namely, the coherent state representation (CSR); we include in this concept the prescription 1: by an operator in this representation we mean only the set of its diagonal matrix elements. (The others are superfluous)x). CSR is a new representation among the $\vec{x}$-representation, $\vec{p}$-representation, angular momentum representation (which are popular in quantum mechanics) and Fock representation, or occupation number representation (which is popular in quantum field theory and in quantum statistics). From CSR we can turn back to any other representation.

In the present work we consider closed systems of interacting fields. Here the situation is more complicated (than in 1)-3)) because of the non-linearity of equations for a field operator. We cannot identify the coherent state expectation value $\langle\varphi|\hat{\varphi}(x)|\varphi\rangle$ of the Heisenberg field operator $\hat{\varphi}(x)$ with a classical field $\varphi(x)$. As prescription 2 for definition of CSR we introduce a new operator $\hat{\varphi}'(x)$, which differs in general from $\hat{\varphi}(x)$ and has the coherent state expectation value, which we can identify with a classical field: $\varphi(x) = \langle\varphi|\hat{\varphi}'(x)|\varphi\rangle$.

There is some operator Λ such that

$$\langle\varphi|\hat{\varphi}(x)|\varphi\rangle = \Lambda\varphi(x) \equiv \Lambda\langle\varphi|\hat{\varphi}'(x)|\varphi\rangle$$

and it reduces to unity, when operating on a linear functional, as in theories 1)-3). Hence, according to the above theorem we can turn back to the initial operator form of quantum field theory (to the occupied number representation).

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x) See Appendix B here, and eq. (A.4) in /2/.

The transition to CSR (including both prescription 1 and 2) leads, in general, to equations of the type of classical field theory ones with inherent to this theory causality. The only difference is that they contain the Planck constant $\hbar$, which enters into them like a coupling constant (to some extent). Moreover, those equations are essentially more non-linear than usually used in classical theory.

Thus, the same theory may equivalently be represented in a classical form or in a quantum one in all the cases.

It is interesting to note that we may consider any classical field for given initial values as a coherent state expectation value and, further, as a functional of initial values. Then a classical integral equation for this functional (i.e., a set of the equations, which differ in initial values) may be rewritten in the form of a quantum field theory equation for a field operator. Even if the original classical equation does not contain the constant $\hbar$, the latter will appear, due to dimensionality considerations, in the commutation relation $[\hat{\varphi}(x), \partial_4 \hat{\varphi}(x')]_{t'=t} = \hbar c\, \delta(\vec{x}-\vec{x}')$ for the coordinate and momentum of the introduced field operator.

The author follows, in many aspects, to the interesting article by Bialynicki-Birula /I.8/ . In particular, equations for classical fields are the same. However in /I,2/ for the more simple theories it has turned out that the Heisenberg operator $\hat{\varphi}(x)$ may be expressed directly via the classical field $\varphi(x)$, i.e., $\varphi(x)$ via $\Phi_{cl}[x|\phi]$ in notation of /I.8/. Below we express $\langle\varphi|\hat{\varphi}(x)|\varphi\rangle$ via $\varphi(x)$ (i.e., $\Phi_q[x|\phi]$ via $\Phi_{cl}[x|\phi]$) and also $\hat{\varphi}(x)$ via $\langle\varphi|\hat{\varphi}(x)|\varphi\rangle$ and via $\varphi(x)$ for the closed systems of interacting fields. None of these relations have been given explicitly in /I.8/ , but only the relation between $\Phi_q[x|\phi]$ and $\Phi_{cl}[x|\phi]$ has been stated (eqs.(64),

(65) and (70)). The latter is beyond the framework of given theories, because $\phi(x)$ does not obey any equation, being an arbitrary function. Some of these relations we are interested in have arisen in /I.8/ only in the limit $\hbar \to 0$, which was taken inconsistently: $\hbar$ was kept fixed in some places (as in papers of many other authors).

Here we make an attempt to generalize the results of refs. /I,2/ and to obtain these lacking in /I.8/ relations, i.e., to introduce CSR for closed systems of fields (Secs. 2 and 3) and in quantum mechanics with one degree of freedom (Sec. 4). As in refs. /I,2/ the constant $\hbar$ remains arbitrary here. In Sec. 5 the relationship between the obtained equations of classical form and those of the Feynman theory /3/ is discussed. Appendix A contains an example to Sec. 2 and application of the Bialynicki-Birula formula for decomposition of the Heisenberg field operator into N-products of the free ones. Moreover, the commutativity of two local Heisenberg operators for space-like separations is demonstrated using the Kallen-Yang-Feldman equations (as mentioned in /I,2/). In Appendices B and C some formulas, used in Secs. 4 and 5, are given.

2. TRANSITION TO A CLASSICAL FIELD (TO CSR) IN THE CASE OF A SELF-INTERACTING SCALAR FIELD

As an equation of motion for a scalar field we take the following integral equation

$$\hat{\varphi}(x) = \hat{\phi}(x) + \int_{t'} d^4y\, \Delta_{ret}(x-y)\, j(\hat{\varphi}(y)) , \qquad (I)$$

where t' is an initial time of evolution, and[x)]

$$\hat{\phi}(x) = i\int d^3x' \left[\partial_4' \Delta(x-x')\hat{\varphi}(x') - \Delta(x-x')\partial_4'\hat{\varphi}(x')\right] = i\int d^3x'\, \Delta(x-x')\overleftrightarrow{\partial_4'}\hat{\varphi}(x') . \qquad (2)$$

[x)] Thus, one introduces its own free operator, interaction representation, integral equation (I) and coherent states for each time t'.

When iterating (I) infinitely many times, we obtain

$$\hat{\varphi}(x) = \sum_{n=0}^{\infty} \int_{t'} d^4x_1 \dots d^4x_n K_{ret}(x, x_1 \dots x_n) \{\hat{\varphi}(x_1) \dots \hat{\varphi}(x_n)\} = \qquad (3.a)$$

$$= \sum_{n=0}^{\infty} \int_{t'} d^4x_1 \dots d^4x_n K(x, x_1 \dots x_n) : \hat{\varphi}(x_1) \dots \hat{\varphi}(x_n): \quad , \qquad (3.b)$$

where $\{\hat{\varphi}(x_1) \dots \hat{\varphi}(x_n)\}$ is the symmetrized product

$$\{\hat{\varphi}(x_1) \dots \hat{\varphi}(x_n)\} = \frac{1}{n!} \sum_{\substack{\text{over all } n! \text{ permutations} \\ \text{of indices } I \dots n}} \hat{\varphi}(x_1) \dots \hat{\varphi}(x_n) \qquad (4)$$

and coefficient functions $K_{ret}(x, x_I \dots x_n)$ are constructed from the $\Delta_{ret}(x-y)$- and $\delta^4(x-y)$-functions and are represented by tree graphs. Expression (3.b) is obtained by a decomposition of the symmetrized products into the N-products

$$\{\hat{\varphi}(x_1) \dots \hat{\varphi}(x_n)\} = :\hat{\varphi}(x_1) \dots \hat{\varphi}(x_n): + \sum_{\substack{\text{one} \\ \text{pairing}}} \frac{1}{2} \Delta^{(1)}(x_1 - x_2) :\hat{\varphi}(x_3) \dots \hat{\varphi}(x_n): + \sum_{\substack{\text{two} \\ \text{pairings}}} \quad + \dots \qquad (5)$$

where $\frac{I}{2} \Delta^{(I)}(x_I - x_2)$-function corresponds to pairing of $\hat{\varphi}(x_1)$ and $\hat{\varphi}(x_2)$.

For a practical decomposition into the N-products there may be used either direct (algebraic) methods due to Wick/4/, Dyson/5/ and Caianiello/6/ or the symbolic methods of the external sources due to Schwinger/I.9, I.I4/ and Symanzik/I.I5/ or of the external fields due to Hori/7/ and Bialynicki-Birula/I.8/. The symbolic methods bring one out of the framework of the given theory (e.g., out of the framework of equation (I)) at an intermediate stage. We avoid them here, except for Appendix A, where the transparent formula of Bialynicki-Birula/I.8/ is given for decomposition of a Heisenberg field into the N-products of a free one.

The normal form (3.b) is convenient for obtaining the coherent state expectation values

$$\langle \varphi | \hat{\varphi}(x) | \varphi \rangle = \sum_{n=0}^{\infty} \int_{t'} d^4x_1 \dots d^4x_n K(x, x_1 \dots x_n) \varphi(x_1) \dots \varphi(x_n) \ . \qquad (6)$$

Unlike theories I)-3) we cannot identify $\langle \varphi | \hat{\varphi}(x) | \varphi \rangle$ with any classical field $\varphi(x)$. There are two reasons for that: a) $K(x, x_I \dots x_n)$ have no tree form, which is only possible in classical theories, and b) $K(x, x_I \dots x_n)$ contain not only the causal Δ_{ret}-functions, but also the acausal $\Delta^{(I)}$-functions.

It seems however natural to introduce the new operator

$$\hat{\varphi}'(x) = \sum_{n=0}^{\infty} \int_{t'} d^4x_1 \dots d^4x_n K_{ret}(x, x_1 \dots x_n) : \hat{\varphi}(x_1) \dots \hat{\varphi}(x_n): \qquad (7)$$

and to identify the coherent state expectation value of $\hat{\varphi}'(x)$ with a classical field

$$\varphi(x) = \langle \varphi | \hat{\varphi}'(x) | \varphi \rangle = \sum_{n=0}^{\infty} \int_{t'} d^4x_1 \dots d^4x_n K_{ret}(x, x_1 \dots x_n) \varphi(x_1) \dots \varphi(x_n) \ . \qquad (8)$$

We call this prescription 2. It is evident that such a field satisfies the non-linear classical equation

$$\varphi(x) = \varphi(x) + \int_{t'} d^4y \, \Delta_{ret}(x-y) \, j'(\varphi(y)) \ . \qquad (9)$$

Let us recall that $\varphi(x)$ is the following linear functional of $\varphi(\vec{x}, t)$ and $\partial_4 \varphi(\vec{x}, t)$ or of $J(x)$

$$\varphi(x) = i \int d^3x' \, \Delta(x-x') \overleftrightarrow{\partial_4'} \varphi(x') = \qquad (I0.a)$$

$$= i \int d^3x' \, \Delta(x-x') \overleftrightarrow{\partial_4'} \varphi(x') = \qquad (I0.b)$$

$$= -\int_{t''}^{t'} d^4y \, \Delta(x-y) \, J(y) \qquad (I0.c)$$

and according to eq. (9) $\varphi(x)$ is the non-linear functional.

The function j' in eq. (9) is almost the same as j in eq. (I). To avoid the appearance of $\Delta^{(I)}(0)$ in eq. (9) the function j in eq. (I) must be chosen in such a manner, that

$$j(\hat{\varphi}) \to j(\hat{\varphi}) = :j'(\hat{\varphi}): \qquad (II)$$

For example, in the case of φ^3-coupling (see Appendix A)

$$j(\hat{\varphi}) \to j(\hat{\varphi}) = g\left(\hat{\varphi}^2(x) - \frac{1}{2}\Delta^{(1)}(0)\right) = g:\hat{\varphi}^2(x): = :j'(\hat{\varphi}): \ , \quad j'(\varphi) = g\varphi^2(x) \ . \qquad (II')$$

According to the expectation value theorem any operator $\hat{Q}$ can be expressed via its coherent state values[2]

$$\hat{Q} = :\exp\left(\int d^3x' \left(\hat{\dot\varphi}(x')\frac{\delta}{\delta\varphi(x')} + \hat{\varphi}(x')\frac{\delta}{\delta\dot\varphi(x')}\right)\right): \langle\varphi|\hat{Q}|\varphi\rangle_{|\varphi(\vec{y},t')=\dot\varphi(\vec{y},t')=0} = \quad (12.a)$$

$$= :\exp\left(i\int d^3y\, \hat{\varphi}(y)\overleftrightarrow{\partial}_4 \frac{\delta}{\delta J(y)}\right): \langle\varphi|\hat{Q}|\varphi\rangle_{|J=0} = \quad (12.b)$$

$$= \sum_{n=0}^{\infty}\frac{i^n}{n!}\int d^3y_1\ldots d^3y_n :\hat{\varphi}(y_1)\ldots\hat{\varphi}(y_n): \overleftrightarrow{\partial}_{14}\ldots\overleftrightarrow{\partial}_{n4}\frac{\delta}{\delta J(y_1)}\cdots\frac{\delta}{\delta J(y_n)}\langle\varphi|Q|\varphi\rangle_{|J=0} \quad (12.b')$$

The last row gives the exact meaning to the preceding one, the times being non-equal[x]. Using eq. (12), we can, for example, turn back to $\hat{\varphi}'(x)$, eq. (7), from $\varphi(x) = \langle\varphi|\hat{\varphi}'(x)|\varphi\rangle$. The question how, starting from the classical field $\varphi(x) = \langle\varphi|\hat{\varphi}'(x)|\varphi\rangle$, to obtain the expectation values $\langle\varphi|\hat{\varphi}(x)|\varphi\rangle$ and further the operator $\hat{\varphi}(x)$ are answered by the formulas

$$\langle\varphi|\hat{\varphi}(x)|\varphi\rangle = \exp\left(\frac{\hbar}{4}\int d^3\xi\, d^3\xi' \frac{\delta}{\delta J(\xi)}\overleftrightarrow{\partial}_4 \Delta^{(1)}(\xi-\xi')\overleftrightarrow{\partial}'_4 \frac{\delta}{\delta J(\xi')}\right)\langle\varphi|\hat{\varphi}'(x)|\varphi\rangle =$$

$$= \Lambda\langle\varphi|\hat{\varphi}'(x)|\varphi\rangle = \Lambda\varphi(x) \quad (13)$$

$$\hat{\varphi}(x) = :\exp\left(i\int d^3y\, \hat{\varphi}(y)\overleftrightarrow{\partial}_4\frac{\delta}{\delta J(y)}\right): \langle\varphi|\hat{\varphi}(x)|\varphi\rangle_{|J=0} =$$

$$= :\exp\left(i\int d^3y\, \hat{\varphi}(y)\overleftrightarrow{\partial}_4\frac{\delta}{\delta J(y)}\right): \Lambda\varphi(x)_{|J=0}. \quad (14)$$

Note, that the operator Λ keeps us within the framework of this theory unlike the operator $\exp\left(\frac{\hbar}{4}\int\frac{\delta}{\delta\Phi}\Delta^{(1)}\frac{\delta}{\delta\Phi}\right)$ of the Hori's type in /I.8/. Λ operates only on the initial values of the classical field $\varphi(x)$, i.e., on $\varphi(\vec{x},t')$ and $\partial_4\varphi(\vec{x},t')$ or equivalently on $J(x)$.

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x) In what follows eqs. (13), (14), (31.b), (34), (36), (37), (48.b), (49.b), (50) and (51) must be interpreted in the same sense.

3. TRANSITION TO CLASSICAL FIELDS (TO CSR) IN QUANTUM ELECTRODYNAMICS

We start with the Kallen-Yang-Feldman equations

$$\hat{A}_\mu(x) = \hat{A}_\mu(x) + \int_{t'} d^4y\, \Delta_{ret}(x-y)\hat{\mathring{J}}_\mu(y) \quad (15)$$

$$\hat{\psi}(x) = \hat{\psi}(x) - ie\int_{t'} d^4y\, S_{ret}(x-y)\gamma_\mu\hat{A}_\mu(y)\hat{\psi}(y), \quad (16)$$

where

$$\hat{A}_\mu(x) = i\int d^3x'\, \Delta(x-x')\overleftrightarrow{\partial}'_4\hat{A}_\mu(x') \quad (17)$$

$$\hat{\psi}(x) = -i\int d^3x'\, S(x-x')\gamma_4\hat{\psi}(x') \quad . \quad (18)$$

As a result of infinitely many times of iterations one obtains

$$\hat{A}_\mu(x) = \sum_{m,n=0}^{\infty}\int_{t'} d^4x_1\ldots d^4x_n \int_{t'} d^4y_1\ldots d^4y_m \int_{t'} d^4z_1\ldots d^4z_m\, K^{A}_{ret\,\mu\mu_1\ldots\mu_n}(x,x_1\ldots x_n,y_1\ldots y_m,z_1\ldots z_m)$$

$$\cdot\{\hat{A}_{\mu_1}(x_1)\ldots\hat{A}_{\mu_n}(x_n)\}[\hat{\psi}(y_1)\ldots\hat{\psi}(y_m)\hat{\bar\psi}(z_1)\ldots\hat{\bar\psi}(z_m)] = \quad (19.a)$$

$$= \sum_{m,n=0}^{\infty}\int_{t'} d^4x_1\ldots d^4x_n \int_{t'} d^4y_1\ldots d^4y_m \int_{t'} d^4z_1\ldots d^4z_m\, K^{A}_{\mu\mu_1\ldots\mu_n}(x,x_1\ldots x_n,y_1\ldots y_m,z_1\ldots z_m)$$

$$:\hat{A}_{\mu_1}(x_1)\ldots\hat{A}_{\mu_n}(x_n)\hat{\psi}(y_1)\ldots\hat{\psi}(y_m)\hat{\bar\psi}(z_1)\ldots\hat{\bar\psi}(z_m): \quad (19.b)$$

$$\hat{\psi}(x) = \sum_{m,n=0}^{\infty}\int_{t'} d^4x_1\ldots d^4x_n \int_{t'} d^4y_1\ldots d^4y_{m+1} \int_{t'} d^4z_1\ldots d^4z_m\, K^{\psi}_{ret\,\mu_1\ldots\mu_n}(x,x_1\ldots x_n,y_1\ldots y_{m+1},z_1\ldots z_m)$$

$$\{\hat{A}_{\mu_1}(x_1)\ldots\hat{A}_{\mu_n}(x_n)\}[\hat{\psi}(y_1)\ldots\hat{\psi}(y_{m+1})\hat{\bar\psi}(z_1)\ldots\hat{\bar\psi}(z_m)] = \quad (20.a)$$

$$= \sum_{m,n=0}^{\infty}\int_{t'} d^4x_1\ldots d^4x_n \int_{t'} d^4y_1\ldots d^4y_{m+1} \int_{t'} d^4z_1\ldots d^4z_m\, K^{\psi}_{\mu_1\ldots\mu_n}(x,x_1\ldots x_n,y_1\ldots y_{m+1},z_1\ldots z_m)$$

$$:\hat{A}_{\mu_1}(x_1)\ldots\hat{A}_{\mu_n}(x_n)\hat{\psi}(y_1)\ldots\hat{\psi}(y_{m+1})\hat{\bar\psi}(z_1)\ldots\hat{\bar\psi}(z_m): \quad , \quad (20.b)$$

where the brackets { } and [ ] denote completely symmetrized and antisymmetrized products, respectively. The transitions from (19.a) to (19.b) and from (20.a) to (20.b) are achieved by decompositions of these products into N-products in the same way as in the Wick theorem (like (5)), but with pairings $\frac{1}{i}\delta_{\mu_1\mu_2}\Delta^{(1)}(x_1-x_2)$ and $\frac{1}{i}S^{(1)}(x_1-x_2)$, respectively. The coherent state expectation values for the Heisenberg operators are

$$\langle A\psi|\hat{A}_\mu(x)|A\psi\rangle=\sum_{m,n=0}^{\infty}\int d^4x_1\dots d^4x_n\int d^4y_1\dots d^4y_m\int d^4z_1\dots d^4z_m K^{A}_{\mu\mu_1\dots\mu_n}(x,x_1\dots x_n,y_1\dots y_m,z_1\dots z_m)$$
$$\cdot A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n)\psi(y_1)\dots\psi(y_m)\bar{\psi}(z_1)\dots\bar{\psi}(z_m) \qquad (21)$$

$$\langle A\psi|\hat{\psi}(x)|A\psi\rangle=\sum_{m,n=0}^{\infty}\int d^4x_1\dots d^4x_n\int d^4y_1\dots d^4y_{m+1}\int d^4z_1\dots d^4z_m K^{\psi}_{\mu_1\dots\mu_n}(x,x_1\dots x_n,y_1\dots y_{m+1},z_1\dots z_m)$$
$$A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n)\psi(y_1)\dots\psi(y_{m+1})\bar{\psi}(z_1)\dots\bar{\psi}(z_m) \qquad (22)$$

but they again do not suit to be classical fields, since K^A and K^ψ have no tree structure. For this reason according to prescription 2 the new operators

$$\hat{A}'_\mu(x)=\sum_{m,n=0}^{\infty}\int d^4x_1\dots d^4x_n\int d^4y_1\dots d^4y_m\int d^4z_1\dots d^4z_m K^{A}_{ret\,\mu\mu_1\dots\mu_n}(x,x_1\dots x_n,y_1\dots y_m,z_1\dots z_m)$$
$$:\hat{A}_{\mu_1}(x_1)\dots\hat{A}_{\mu_n}(x_n)\hat{\psi}(y_1)\dots\hat{\psi}(y_m)\hat{\bar{\psi}}(z_1)\dots\hat{\bar{\psi}}(z_m): \qquad (23)$$

$$\hat{\psi}'(x)=\sum_{m,n=0}^{\infty}\int d^4x_1\dots d^4x_n\int d^4y_1\dots d^4y_{m+1}\int d^4z_1\dots d^4z_m K^{\psi}_{ret\,\mu_1\dots\mu_n}(x,x_1\dots x_n,y_1\dots y_{m+1},z_1\dots z_m)$$
$$:\hat{A}_{\mu_1}(x_1)\dots\hat{A}_{\mu_n}(x_n)\hat{\psi}(y_1)\dots\hat{\psi}(y_{m+1})\hat{\bar{\psi}}(z_1)\dots\hat{\bar{\psi}}(z_m): \qquad (24)$$

are untroduced such that now their coherent state expectation values may be identified with classical fields

$$\mathbf{A}_\mu(x)\equiv\langle A\psi|\hat{A}'_\mu(x)|A\psi\rangle=$$
$$=\sum_{m,n=0}^{\infty}\int d^4x_1\dots d^4x_n\int d^4y_1\dots d^4y_m\int d^4z_1\dots d^4z_m K^{A}_{ret\,\mu\mu_1\dots\mu_n}(x,x_1\dots x_n,y_1\dots y_m,z_1\dots z_m)$$
$$A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n)\psi(y_1)\dots\psi(y_m)\bar{\psi}(z_1)\dots\bar{\psi}(z_m) \qquad (25)$$

$$\Psi(x)\equiv\langle A\psi|\hat{\psi}'(x)|A\psi'\rangle=$$
$$=\sum_{m,n=0}^{\infty}\int d^4x_1\dots d^4x_n\int d^4y_1\dots d^4y_{m+1}\int d^4z_1\dots d^4z_m K^{\psi}_{ret\,\mu_1\dots\mu_n}(x,x_1\dots x_n,y_1\dots y_{m+1},z_1\dots z_m)$$
$$A_{\mu_1}(x_1)\dots A_{\mu_n}(x_n)\psi(y_1)\dots\psi(y_{m+1})\bar{\psi}(z_1)\dots\bar{\psi}(z_m) \qquad (26)$$

The coefficient functions K^A_{ret} and K^ψ_{ret} have the tree structure and are constructed from the $\Delta_{ret}(x-y)$-, $S_{ret}(x-y)$- and $\delta^4(x-y)$- functions, and thus satisfy the causality requirement. The classical fields $\mathbf{A}_\mu(x)$ and $\Psi(x)$ obey the classical equations, having the form of the original operator equations

$$\mathbf{A}_\mu(x)=A_\mu(x)+\int_{t'}d^4y\,\Delta_{ret}(x-y)\,\mathring{J}_\mu(y)\quad\left(\mathring{J}_\mu(x)=ie\bar{\Psi}(x)\gamma_\mu\Psi(x)\right) \qquad (27)$$

$$\Psi(x)=\psi(x)-ie\int_{t'}d^4y\,S_{ret}(x-y)\gamma_\mu\mathbf{A}_\mu(y)\Psi(y)\,. \qquad (28)$$

where $A_\mu(x)$ and $\psi(x)$ are the linear functionals of initial values

$$A_\mu(x)=\;i\int d^3x'\,\Delta(x-x')\overleftrightarrow{\partial_4'}\mathbf{A}_\mu(x')= \qquad (29.a)$$
$$=\;i\int d^3x'\,\Delta(x-x')\overleftrightarrow{\partial_4'}A_\mu(x')= \qquad (29.b)$$
$$=-\int_{t'}^{t''}d^4y\,\Delta(x-y)\,J(y) \qquad (29.c)$$

$$\psi(x)=-i\int d^3x'\,S(x-x')\gamma_4\Psi(x')= \qquad (30.a)$$
$$=-i\int d^3x'\,S(x-x')\gamma_4\psi(x')= \qquad (30.b)$$
$$=\int_{t'}^{t''}d^4y\,S(x-y)\,\eta(y)\;. \qquad (30.c)$$

Let us recall that the "classical" fields ψ and Ψ and sources η are anticommuting quantities, ψ and $\bar{\psi}$ being independent (and η and $\bar{\eta}$ too).

According to the expectation value theorem any operator $\mathbf{Q}$ can be expressed via its coherent state values

$$\mathbf{Q}=:\exp\left(\int d^3x'\left(\hat{A}_\mu(x')\frac{\delta}{\delta A_\mu(x')}+\hat{\dot{A}}_\mu(x')\frac{\delta}{\delta\dot{A}_\mu(x')}+\hat{\psi}(x')\frac{\delta}{\delta\psi(x')}+\bar{\psi}(x')\frac{\delta}{\delta\bar{\psi}(x')}\right)\right):$$
$$\cdot\langle A\psi|\mathbf{Q}|A\psi\rangle|_{A_\mu(\vec{y},t')=\dot{A}_\mu(\vec{y},t')=\psi(\vec{y},t')=\bar{\psi}(\vec{y},t')=0} \qquad (31.a)$$
$$=:\exp\left(i\int d^3y\left(\hat{A}_\mu(y)\overleftrightarrow{\partial_4}\frac{\delta}{\delta J_\mu(y)}-\hat{\psi}(y)\gamma_4\frac{\delta}{\delta\eta(y)}+\hat{\bar{\psi}}(y)\gamma_4\frac{\delta}{\delta\bar{\eta}(y)}\right)\right):\langle A\psi|\mathbf{Q}|A\psi\rangle|_{J=\eta=\bar{\eta}=0} \qquad (31.b)$$

To turn back to the coherent state expectation values of the Heisenberg operators and to the operators themselves not going beyond the framework of this theory the formulas are used

$$\langle A\psi|\hat{A}_\mu(x)|A\psi\rangle = \Lambda_A \Lambda_\psi \langle A\psi|\hat{A}'_\mu(x)|A\psi\rangle \equiv \Lambda_A\Lambda_\psi A_\mu(x) \qquad (32)$$

$$\langle A\psi|\hat{\psi}(x)|A\psi\rangle = \Lambda_A \Lambda_\psi \langle A\psi|\hat{\psi}'(x)|A\psi\rangle \equiv \Lambda_A\Lambda_\psi \psi(x), \qquad (33)$$

where

$$\Lambda_A = \exp\left(\frac{\hbar}{4}\int d^3\xi\, d^3\xi' \frac{\delta}{\delta J_\mu(\xi)} \overleftrightarrow{\partial}_4 \Delta^{(1)}(\xi-\xi') \overleftrightarrow{\partial}'_4 \frac{\delta}{\delta J_\mu(\xi')}\right) \qquad (34)$$

$$\Lambda_\psi = \exp\left(\frac{\hbar}{2}\int d^3\xi\, d^3\xi' \frac{\delta}{\delta \eta(\xi)} \gamma_4 S^{(1)}(\xi-\xi')\gamma_4 \frac{\delta}{\delta \bar{\eta}(\xi')}\right) \qquad (35)$$

$$\hat{A}_\mu(x) = :\exp\left(i\int d^3y\left(\hat{A}_\mu(y)\overleftrightarrow{\partial}_4 \frac{\delta}{\delta J_\mu(y)} - \hat{\psi}(y)\gamma_4\frac{\delta}{\delta\eta(y)} + \hat{\bar{\psi}}(y)\gamma_4\frac{\delta}{\delta\bar{\eta}(y)}\right)\right): \langle A\psi|\hat{A}_\mu(x)|A\psi\rangle\Big|_{J=\eta=\bar{\eta}=0} = \qquad (36.a)$$

$$= :\exp\left(i\int d^3y\left(\hat{A}_\mu(y)\overleftrightarrow{\partial}_4 \frac{\delta}{\delta J_\mu(y)} - \hat{\psi}(y)\gamma_4\frac{\delta}{\delta\eta(y)} + \hat{\bar{\psi}}(y)\gamma_4\frac{\delta}{\delta\bar{\eta}(y)}\right)\right): \Lambda_A\Lambda_\psi A_\mu(x)\Big|_{J=\eta=\bar{\eta}=0} \qquad (36.b)$$

$$\hat{\psi}(x) = :\exp\left(i\int d^3y\left(\hat{A}_\mu(y)\overleftrightarrow{\partial}_4 \frac{\delta}{\delta J_\mu(y)} - \hat{\psi}(y)\gamma_4\frac{\delta}{\delta\eta(y)} + \hat{\bar{\psi}}(y)\gamma_4\frac{\delta}{\delta\bar{\eta}(y)}\right)\right): \langle A\psi|\hat{\psi}(x)|A\psi\rangle\Big|_{J=\eta=\bar{\eta}=0} = \qquad (37.a)$$

$$= :\exp\left(i\int d^3y\left(\hat{A}_\mu(y)\overleftrightarrow{\partial}_4 \frac{\delta}{\delta J_\mu(y)} - \hat{\psi}(y)\gamma_4\frac{\delta}{\delta\eta(y)} + \hat{\bar{\psi}}(y)\gamma_4\frac{\delta}{\delta\bar{\eta}(y)}\right)\right): \Lambda_A\Lambda_\psi \psi(x)\Big|_{J=\eta=\bar{\eta}=0}. \qquad (37.b)$$

4. TRANSITION FROM QUANTUM MECHANICS TO CLASSICAL ONE, AND VICE VERSA

In quantum mechanics with one degree of freedom we take for Heisenberg operator of coordinate $\hat{x}(t)$ the equation/I.8/

$$\hat{x}(t) = x(t) + \int_{t'} du\, G_{ret}(t-u) F(\hat{x}(u)), \qquad (38)$$

where

$$\hat{x}(t) = \frac{\partial}{\partial t'} D(t-t')\cdot\hat{x}(t') - D(t-t')\frac{\partial\hat{x}(t')}{\partial t'} \equiv D(t-t')\frac{\overleftrightarrow{\partial}}{\partial t'}\hat{x}(t'). \qquad (39)$$

The definiton of functions G_{ret}, D, $D^{(I)}$ and coherent states used below see in Appendix B.

Iterating equation (37) infinitely many times, one obtains

$$\hat{x}(t) = \sum_{n=0}^{\infty}\int dt_1\ldots dt_n\, K_{ret}(t,t_1\ldots t_n)\{\hat{x}(t_1)\ldots\hat{x}(t_n)\} = \qquad (40.a)$$

$$= \sum_{n=0}^{\infty}\int dt_1\ldots dt_n\, K(t,t_1\ldots t_n) :\hat{x}(t_1)\ldots\hat{x}(t_n): , \qquad (40.b)$$

the second expression being obtained by decomposition of the symmetrised products into N-products with the pairings

$$\{\underbrace{\hat{x}(t_1)\hat{x}}(t_2)\} = \hbar\, D^{(1)}(t_1-t_2). \qquad (41)$$

The cogerent state expectation value of $\hat{x}(t)$ is equal

$$\langle xp|\hat{x}(t)|xp\rangle = \sum_{n=0}^{\infty}\int dt_1\ldots dt_n\, K(t,t_1\ldots t_n)\, x(t_1)\ldots x(t_n), \qquad (42)$$

where

$$x(t) = \langle xp|\hat{x}(t)|xp\rangle = D(t-t')\frac{\overleftrightarrow{\partial}}{\partial t'}x(t') = -\int_{t'}^{t''} d^4u\, D(t-u) f(u). \qquad (43)$$

The quantity x(t) is the linear function of the two initial values x(t') and $\dot{x}(t')$ and may be equivalently treated as a functional of one function f(t) (the last expression (43)).

The coherent state expectation value (42) cannot be identified with a function, which obeys any classical equation. Again, according to prescription 2 we introduce the new operator

$$\hat{x}'(t) = \sum_{n=0}^{\infty}\int dt_1\ldots dt_n\, K_{ret}(t,t_1\ldots t_n) :\hat{x}(t_1)\ldots\hat{x}(t_n): \qquad (44)$$

and identify its coherent state expectation values with the classical function

$$x(t) = \langle xp|\hat{x}'(t)|xp\rangle = \sum_{n=0}^{\infty}\int dt_1\dots dt_n\, K_{ret}(t,t_1\dots t_n)\,x(t_1)\dots x(t_n) \qquad (45)$$

which satisfy the following non-linear classical equation

$$x(t) = x(t) + \int_{t'} du\, G_{ret}(t-u)\,F'(x(u)) \;. \qquad (46)$$

The functions F and F′ differ from each other like j and j′ in Sec. 2.

From eq. (46) the Newton equation

$$\ddot{x}(t) = F'(x(t)) \qquad (47)$$

follows/I.8/ (see Appendix B). Starting with this one can turn back to the initial quantum theory. This is achieved by treating the classical coordinate $x(t)$ as a coherent state expectation value and then by using the coherent state expectation value theorem. According to the theorem any Heisenberg operator Q may be expressed through its coherent state expectation values

$$\hat{Q} = :\exp\left(\hat{x}(t')\frac{\partial}{\partial x(t')} + \hat{p}(t')\frac{\partial}{\partial p(t')}\right): \langle xp|Q|xp\rangle\Big|_{x(t')=p(t')=0} = \qquad (48.a)$$

$$= :\exp\left(\hat{x}(\tau)\overleftrightarrow{\frac{\partial}{\partial\tau}}\frac{\delta}{\delta f(\tau)}\right): \langle xp|Q|xp\rangle\Big|_{f=0} \qquad (48.b)$$

In particular,

$$\hat{x}(t) = :\exp\left(\hat{x}(t')\frac{\partial}{\partial x(t')} + \hat{p}(t')\frac{\partial}{\partial p(t')}\right): \langle xp|\hat{x}(t)|xp\rangle\Big|_{x(t')=p(t')=0} = \qquad (49.a)$$

$$= :\exp\left(\hat{x}(\tau)\overleftrightarrow{\frac{\partial}{\partial\tau}}\frac{\delta}{\delta f(\tau)}\right): \langle xp|\hat{x}(t)|xp\rangle\Big|_{f=0} \qquad (49.b)$$

Equation (49.a) is given in/I.8/ x)(eq. (33) there).

Further, the expectation value $\langle xp|\hat{x}(t)|xp\rangle$ can be expressed through the classical coordinate $x(t) = \langle xp|\hat{x}'(t)|xp\rangle$ according to

$$\langle xp|\hat{x}(t)|xp\rangle = \exp\left(\frac{\hbar}{4}\frac{\delta}{\delta f(\tau)}\overleftrightarrow{\frac{\partial}{\partial\tau}}D^{(1)}(\tau-\tau')\overleftrightarrow{\frac{\partial}{\partial\tau'}}\frac{\delta}{\delta f(\tau')}\right)\langle xp|\hat{x}'(t)|xp\rangle =$$

$$= \Lambda\langle xp|\hat{x}'(t)|xp\rangle = \Lambda\, x(t)\;. \qquad (50)$$

Hence, the direct relation between the quantum and classical coordinates $\hat{x}(t)$ and $x(t)$ is

$$\hat{x}(t) = :\exp\left(\hat{x}(\tau)\overleftrightarrow{\frac{\partial}{\partial\tau}}\frac{\delta}{\delta f(\tau)}\right): \Lambda x(t)\Big|_{f=0} \;. \qquad (51)$$

x) Unlike an analogous formulas for field theory, which are absent in/I.8/(possibly because of using an inconvenient form of the coherent states).

5. CLASSICAL EQUATIONS AND TRANSITION AMPLITUDES

After turning back to the quantum equations from the classical ones, we can reconstruct all the quantum theory and, in particular, find transition amplitudes, which are c-number quantities again. As appears, instead of this one can obtain the transition amplitudes directly from the classical equations, that in fact has been done by Feynman long ago/3/.

Let us consider the theories I)-3).

I) Free quantized field. Let us pass from the real classical field (x) to its positive-frequency part, i.e., to the complex amplitude

$$\psi(x) = \varphi^{(-)}(x) \qquad (52)$$

(like one does in classical optics and electronics, see/I/, Appendix B). It obeys the Schrödinger equation

$$\partial_4\psi(x) = -\sqrt{-\Delta+m^2}\,\psi(x) \qquad (53)$$

and usually is used for describing a free quantum.

According to Feynman we can retain within the framework of the Klein-Gordon or Dirac equations

$$(\Box - m^2)\,\varphi(x) = 0 \qquad (54)$$

$$(\gamma\partial + m)\,\psi(x) = 0 \quad , \qquad (55)$$

if we define as the amplitudes

$$\varphi(x) = i\int d^3x''\,\Delta_+(x-x'')\overleftrightarrow{\partial_4''}\varphi(x'') - i\int d^3x'\,\Delta_+(x-x')\overleftrightarrow{\partial_4'}\varphi(x') \qquad (56)$$

$$\psi(x) = -i\int d^3x''\,S_+(x-x'')\gamma_4\psi(x'') + i\int d^3x'\,S_+(x-x')\gamma_4\psi(x') \;, \qquad (57)$$

where t′ and t″ are initial and final times of evolution.

Amplitudes (56) and (57) ar: solutions of the mixed initial-final value problems instead of the initial value problems (see below and Appendix C).

2) Interaction with an external current. The equation

$$\varphi(x) = \varphi(x) + \int_{t'} d^4y\, \Delta_{ret}(x-y)\, j(y) \tag{58}$$

for the classical field corresponds to a Cauchy problem. According to Feynman to describe evolution of the quantum we again must solve the initial-final value problem, which is presented by the equation

$$\varphi(x) = \varphi_+(x) + \int_{t'}^{t''} d^4y\, \Delta_+(x-y)\, j(y)\,, \tag{59}$$

where

$$\varphi_+(x) = i\int d^3x''\, \Delta_+(x-x'')\overleftrightarrow{\partial_4''}\varphi(x'') - i\int d^3x'\, \Delta_+(x-x')\overleftrightarrow{\partial_4'}\varphi(x')\,. \tag{60}$$

Equation (59) follows from eq. (58), if one substitutes

$$\Delta_{ret}(x-y) = \Delta_+(x-y) - \Delta^{(+)}(x-y)\,. \tag{61}$$

Hence, $\varphi_+(x)$ and $\varphi(x)$ are related by

$$\varphi_+(x) = \varphi(x) - \int_{t'}^{t''} d^4y\, \Delta^{(+)}(x-y)\, j(y)\,. \tag{62}$$

3) Interaction with an external field. The integral equation for the classical field

$$\psi(x) = \psi(x) - ie\int_{t'} d^4y\, S_{ret}(x-y)\gamma_\mu A_\mu^{ext}(y)\psi(y) \tag{63}$$

for the Cauchy problem, also by means of the substitution

$$S_{ret}(x-y) = S_+(x-y) - S^{(+)}(x-y)\,, \tag{64}$$

one can transform into the Feynman integral equation

$$\psi(x) = \psi_+(x) - ie\int_{t'}^{t''} d^4y\, S_+(x-y)\gamma_\mu A_\mu^{ext}(y)\psi(y) \tag{65}$$

for the initial-final value problem. In eq. (65)

$$\psi_+(x) = -i\int d^3x''\, S_+(x-x'')\gamma_4\psi(x'') + i\int d^3x'\, S_+(x-x')\gamma_4\psi(x') \tag{66}$$

and $\psi_+(x)$ and $\psi(x)$ are now related by

$$\psi_+(x) = \psi(x) + ie\int_{t'}^{t''} d^4y\, S^{(+)}(x-y)\gamma_\mu A_\mu^{ext}(y)\psi(y) \tag{67}$$

involving the unknown spinor function $\psi(y)$, which, in principle, may be found from eq. (63) or eq. (65) in terms of $\psi(x)$ or $\psi_+(x)$.

Feynman was first treating amplitudes as solutions of the Dirac and Klein-Gordon equations (without or with interaction) or of the corresponding integral equations.

There are conceivable, for example, problems with the conditions;

a) $\psi^{(-)}(\vec{x},t') = f(\vec{x})$, $\psi^{(+)}(\vec{x},t'') = 0$ (electron); b) $\psi^{(-)}(\vec{x},t') = 0$, $\psi^{(+)}(\vec{x},t'') = f(\vec{x})$ (positron)

c) $\psi^{(-)}(\vec{x},t') = f(\vec{x})$, $\psi^{(+)}(\vec{x},t') = 0$ d) $\psi^{(-)}(\vec{x},t') = 0$, $\psi^{(+)}(\vec{x},t') = f(\vec{x})$ (68)

e) $\psi^{(-)}(\vec{x},t'') = f(\vec{x})$, $\psi^{(+)}(\vec{x},t'') = 0$ f) $\psi^{(-)}(\vec{x},t'') = 0$, $\psi^{(+)}(\vec{x},t'') = f(\vec{x})$,

where f(x) is a given spinor function. According to Feynman the electron is described by not merely positive-frequency state, but by a non-local state with initial- final values a)[x], $\psi^{(+)}(\vec{x},t')$ and $\psi^{(-)}(\vec{x},t)$ being unknown quantities, subjected to equations of motion. It is similar for positron. As to problem c) it leads to the the Klein paradox (see Appendix C). The same holds for the problems d)—f). For solving problems a),b); c),d) and e),f) different Green functions must be used (S_+^A, S_{ret}^A, and S_{adv}^A, respectively).

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[x] Apparently, one can consider this as a generalization of the concept "analytical signal" ("pre-envelop") to the case of (non-linear) field theoretical problems.

6. CONCLUDING REMARKS

1. Searching for this form of quantum theory (CSR with both prescriptions 1 and 2) we were guided by the situation in the classical field (wave) theory (see /1/ , Appendix Б). The latter is causal but has both explicitly and implicitly causal objects. Examples of both the kinds of objects are, respectively, $\varphi(x)=\Delta(x-x')$, the real wave with the front, and $\psi(x)=\exp(i\vec{p}\vec{x}-i\omega t)$, the complex plane wave, which is uniformly extended over all the space *.

In quantum field theory we meet, first of all, the objects of the second kind: one-quantum states (actually, the above plane waves or any their superpositions) **, many-quantum ones, and S-matrix elements (the Bogolubov causality condition is non-linear with respect to S /II/).

It appears that the real amplitude $\varphi(x)$, obeying the integral equation for the Cauchy problem, may serve as an object of the second kind.

Note that in the free case the one- and many-quantum complex transition amplitudes

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* $T_{00}(x)=m^2\psi^*(x)\psi(x)+\partial_k\psi^*(x)\partial_k\psi(x)-\partial_4\psi^*(x)\partial_4\psi(x)=2\omega^2$.

** At the early stage of quantum mechanics de Broglie has well understood that the quantum occupies the space entirely/8/ . Of course, information may be transmitted by means of non-strictly localized objects too. Such situations are described by wave groups and by S-matrix elements (possibly, in the framework of the time-energy uncertainty relation/9/). There is a point of view that strict causality is the Ansatz which can be checked only indirectly, through dispersion relations between amplitudes in the p-space./10/ However, for soft photons we can check it directly (in the x-space).

$$\Delta^{(-)}(x-y)$$
$$\sum_{\text{perm. of } y_i}\Delta^{(-)}(x_1-y_1)\ldots\Delta^{(-)}(x_n-y_n) \qquad (69)$$

can be considered as obtained (by means of the Hilbert transforms) from real and causal those

$$\Delta(x-y)$$
$$\sum_{\text{perm. of } y_i}\Delta(x_1-y_1)\ldots\Delta(x_n-y_n)\ . \qquad (70)$$

As to observables we note that in the free case (see eq.(I.65))

$$\frac{\delta^2}{\delta J(x)\,\delta J(y)}\langle\varphi|\hat{P}_\nu|\varphi\rangle=\partial_\nu\Delta(x-y)\ , \qquad (71)$$

i.e., it is expressed in terms of amplitudes (70). The same is true for

$$\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}\langle\varphi|:e^{ix_r\hat{P}_r}:|\varphi\rangle\Big|_{J=0}=\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}\,e^{ix_rP_r}\Big|_{J=0}\ . \qquad (72)$$

2. For the coherent state

$$\langle\varphi|:\hat{\varphi}(x_1)\ldots\hat{\varphi}(x_n):|\varphi\rangle=\varphi(x_1)\ldots\varphi(x_n) \qquad (73)$$

whatever is n *. But, if an upper value n is fixed, one can obtain similar results (up to constant factors)using the truncated states

$$\widetilde{|\varphi\rangle}=\sum_{m=0}^{n}\frac{(-1)^m}{m!}(\varphi,\hat{\varphi})^m|0\rangle \qquad (74)$$

instead of the coherent ones. In the case of validity of the perturbation theory expansion a classical situation, including the inherent to wave theory causality, may be realized with high accuracy in the subspace spanned on the vectors with limited numbers of quanta up to n.

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* Dirac has noted that expectation values have the additive property unlike multiplicative one/12/ But if one defines the product of free field operators to be the N-product then for its coherent state expectation values the multiplicative property is valid too.

3. The complex amplitudes embody the particle aspect* of quantum theory. However, the real amplitudes $\varphi(x)$ are possibly suited to treat wave (classical and quantum) theory as a statistics of paths. In particular, one can represent the Δ-function (and Δ_{ret} too) for m=0 as the spherical mean

$$\Delta(x) = -\frac{t}{4\pi}\int\limits_{\vec{v}^2=1} d^2\Omega_{\vec{v}}\, \delta(\vec{x}-\vec{v}t) , \tag{75}$$

i.e., as the integral over all possible values of velocity. This representation leads immediately to the well-known Poisson formula, which can be written as follows

$$\varphi(x) = \frac{1}{4\pi}\int d^3x'\int d^3v\, \delta(\vec{v}^2-1)\,\delta(\vec{x}-\vec{x}'-\vec{v}(t-t'))\left[1-(t-t')(v_m\partial'_m - i\partial'_4)\right]\varphi(x') . \tag{76}$$

For an arbitrary m we have

$$\Delta(x) = t^3\int d^3v\, \Delta(|t||\vec{v}|,|t|)\,\delta(\vec{x}-\vec{v}t) \tag{77}$$

$$\varphi(x) = \int d^3x'\int d^3v\, \delta(\vec{x}-\vec{x}'-\vec{v}(t-t'))\Big[i\partial'_4\big((t-t')^3\Delta(|t-t'||\vec{v}|,|t-t'|)\big) +$$
$$+ (t-t')^3\Delta(|t-t'||\vec{v}|,|t-t'|)(v_m\partial'_m - i\partial'_4)\Big]\varphi(x') . \tag{78}$$

4. Existence of invariant scalar products $(\varphi,\hat{\varphi})$ and $(\Psi,\hat{\Phi})$ in the cases of external fields and of curved spaces /I5/ permits to construct generalized coherent states similarly to those in /I,2/ and to introduce corresponding CSRs.

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* Thus, multiplying of exponent factors

$$\exp(i\vec{p}_1\vec{x} - i\omega_1 t)\,\exp(i\vec{p}_2\vec{x} - i\omega_2 t) = \exp\left(i(\vec{p}_1+\vec{p}_2)\vec{x} - i(\omega_1+\omega_2)t\right)$$

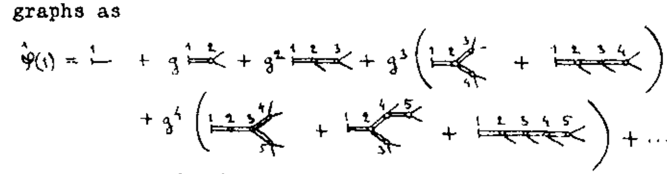
takes into account the Einstein conservation law of 4-momentum in an elementary act /I3/ .

APPENDIX A

As an example of expansion (3.a) let us consider it for the $\hat{\varphi}^3(x)$-coupling. When iterating eq. (I), we obtain, up to the fourth order,

$$\hat{\varphi}(1) = \hat{\varphi}(1) + g\,(12)\,\hat{\varphi}^2(2) + g^2(12)(23)\{\varphi(2),\varphi^2(3)\} +$$
$$+ g^3\Big((12)(23)(24)\tfrac{1}{2}\{\varphi^2(3),\varphi^2(4)\} + (12)(23)(34)\{\varphi(2)\{\varphi(3),\varphi^2(4)\}\}\Big) +$$
$$+ g^4\Big((12)(23)(34)(35)\tfrac{1}{2}\{\varphi(2),\{\varphi^2(4),\varphi^2(5)\}\} + (12)(24)(23)(45)\{\varphi^2(3)\{\varphi(4),\varphi^2(5)\}\} +$$
$$+ (12)(23)(34)(45)\{\varphi(2)\{\varphi(3)\{\varphi(4),\varphi^2(5)\}\}\}\Big) + \dots \tag{A.I}$$

where we set $(I2) \equiv \Delta_{ret}(x_I - x_2)$ and imply the integrations over indices, entering thrice. We may represent this expansion in terms of graphs as



The products in (A.I) are not symmetrized yet, unlike eq. (3.a). However due to the commutation properties of free field

$$\{\varphi(2),\varphi^2(3)\} = \frac{1}{3}\{\varphi(2)\varphi(3)\varphi(3')\}_{3'\to 3} = \frac{1}{3}\int d^4x'_3\,\delta^4(x_3-x'_3)\{\varphi(2)\varphi(3)\varphi(3')\}$$

$$\{\varphi^2(3),\varphi^2(4)\} = \frac{1}{12}\{\varphi(3)\varphi(3')\varphi(4)\varphi(4')\}_{3'\to 3,\,4'\to 4} \tag{A.2}$$

$$\{\varphi(2)\{\varphi(3),\varphi^2(4)\}\} = \frac{1}{6}\{\varphi(2)\varphi(3)\varphi(4)\varphi(4')\}_{4'\to 4}$$

and so on. The numerical coefficients can be found simply counting the terms on left- and right-hand sides.

The expansion of the Heisenberg operator in terms of free field can be obtained also by the well-known formula

$$\hat{Q} = S^{-1}(t,t')\,\hat{Q}\,S(t,t') = \tag{A.3}$$
$$= \sum_{n=0}^{\infty}(-i)^n\int_{t'}^{t} d^4x_1\dots d^4x_n\,\theta(t-t_1)\theta(t_1-t_2)\dots\theta(t_{n-1}-t_n)\left[\mathcal{L}_I(x_n)\dots\left[\mathcal{L}_I(x_2)\left[\mathcal{L}(x_1),\hat{Q}\right]\right]\dots\right]$$

Calculating these multiple commutators for $\hat{Q} = \hat{\varphi}(x)$, $\hat{Q} = \hat{\varphi}(x)$ also leads to eq. (3.a) with the tree structure (for the $\hat{\varphi}^3$-coupling to (A.I)).

To obtain the N-ordered form of the Heisenberg operator the Hori approach is used in[I.8] and leads to

$$\hat{\varphi}(x) = :\exp\left(\int \hat{\varphi}\frac{\delta}{\delta\phi}\right): \exp\left(\frac{\hbar}{4}\int\frac{\delta}{\delta\phi}\Delta^{(1)}\frac{\delta}{\delta\phi}\right)\exp\left(i\int\frac{\delta}{\delta\phi}\Delta_{ret}\frac{\delta}{\delta\tilde{\Phi}}\right)$$

$$\exp\left(-\frac{i}{\hbar}\int\left[\mathcal{L}_I(\phi+\tfrac{\hbar}{2}\tilde{\Phi})-\mathcal{L}_I(\phi-\tfrac{\hbar}{2}\tilde{\Phi})\right]\right)\phi(x)\Big|_{\phi=\tilde{\Phi}=0}, \qquad (A.4)$$

where $\phi(x)$ and $\tilde{\Phi}(x)$ are not realistic fields, but arbitrary c-number functions. The coherent state expectation value of $\hat{\varphi}(x)$ is

$$\langle\varphi|\hat{\varphi}(x)|\varphi\rangle = \exp\left(\int\varphi\frac{\delta}{\delta\phi}\right)\exp\left(\frac{\hbar}{4}\int\frac{\delta}{\delta\phi}\Delta^{(1)}\frac{\delta}{\delta\phi}\right)\exp\left(i\int\frac{\delta}{\delta\phi}\Delta_{ret}\frac{\delta}{\delta\tilde{\Phi}}\right)$$

$$\exp\left(-\frac{i}{\hbar}\int\left[\mathcal{L}_I(\phi+\tfrac{\hbar}{2}\tilde{\Phi})-\mathcal{L}_I(\phi-\tfrac{\hbar}{2}\tilde{\Phi})\right]\right)\phi(x)\Big|_{\phi=\tilde{\Phi}=0} = \qquad (A.5.a)$$

$$= \exp\left(\frac{\hbar}{4}\int\frac{\delta}{\delta\phi}\Delta^{(1)}\frac{\delta}{\delta\phi}\right)\exp\left(i\int\frac{\delta}{\delta\phi}\Delta_{ret}\frac{\delta}{\delta\tilde{\Phi}}\right)$$

$$\exp\left(-\frac{i}{\hbar}\int\left[\mathcal{L}_I(\phi+\tfrac{\hbar}{2}\tilde{\Phi})-\mathcal{L}_I(\phi-\tfrac{\hbar}{2}\tilde{\Phi})\right]\right)\phi(x)\Big|_{\substack{\tilde{\Phi}=0\\ \phi(x)=\varphi(x)}}. \qquad (A.5.b)$$

The above operator $\hat{\varphi}'(x)$, eq. (7), and its coherent state expectation value (8) are written in these terms as

$$\hat{\varphi}'(x) = :\exp\left(\int\hat{\varphi}\frac{\delta}{\delta\phi}\right): \exp\left(i\int\frac{\delta}{\delta\phi}\Delta_{ret}\frac{\delta}{\delta\tilde{\Phi}}\right)$$

$$\exp\left(-\frac{i}{\hbar}\int\left[\mathcal{L}_I(\phi+\tfrac{\hbar}{2}\tilde{\Phi})-\mathcal{L}_I(\phi-\tfrac{\hbar}{2}\tilde{\Phi})\right]\right)\phi(x)\Big|_{\phi=\tilde{\Phi}=0} \qquad (A.6)$$

$$\varphi(x) \equiv \langle\varphi|\hat{\varphi}'(x)|\varphi\rangle = \exp\left(\int\varphi\frac{\delta}{\delta\phi}\right)\exp\left(i\int\frac{\delta}{\delta\phi}\Delta_{ret}\frac{\delta}{\delta\tilde{\Phi}}\right)\cdot$$

$$\exp\left(-\frac{i}{\hbar}\int\left[\mathcal{L}_I(\phi+\tfrac{\hbar}{2}\tilde{\Phi})-\mathcal{L}(\phi-\tfrac{\hbar}{2}\tilde{\Phi})\right]\right)\phi(x)\Big|_{\phi=\tilde{\Phi}=0} = \qquad (A.7.a)$$

$$= \exp\left(i\int\frac{\delta}{\delta\phi}\Delta_{ret}\frac{\delta}{\delta\tilde{\Phi}}\right)\exp\left(-\frac{i}{\hbar}\int\left[\mathcal{L}_I(\phi+\tfrac{\hbar}{2}\tilde{\Phi})-\mathcal{L}_I(\phi-\tfrac{\hbar}{2}\tilde{\Phi})\right]\right)\phi(x)\Big|_{\substack{\tilde{\Phi}=0\\ \phi(x)=\varphi(x)}}. \qquad (A.7.b)$$

It has been stressed in[I,2] that the Kallen-Yang-Feldman equations contain exhaustive information concerning causality, and in particular, they are convenient tool to demonstrate commutativity of two local operators for space-like separations. Consider

first the commutator of two field operators. In the free case the demonstration of commutativity is very simple

$$[\hat{\varphi}(x),\hat{\varphi}(z)] = \left[i\int_{t'=z_0} d^3x'\,\Delta(x-x')\overleftrightarrow{\partial'_4}\hat{\varphi}(x'),\hat{\varphi}(z)\right] = i\Delta(x-z). \qquad (A.8)$$

As to $[\hat{\varphi}(x),\hat{\varphi}(z)]$ in the case of an interaction, let us suppose that $x_0 > z_0$ (without loss of generality), costruct free operator (2) with $t' = z_0$ and write equation (I) also with $t' = z_0$. Iterating this equation or using eq. (A.3), we obtain for $\hat{\varphi}(x)$ expansion (3.a) (for example, (A.I)). Commuting term by term, we conclude that each term contains the chain

$$\Delta_{ret}(x-y_1)\Delta_{ret}(y_1-y_2)\ldots\Delta_{ret}(y_n-z) \qquad (A.9)$$

among other factors. This chain is equal to zero for space-like $x-z$.

A commutator of two local quantities of a general form is decomposed into a sum of terms, each containing commutator of two fields. Finally, in a theory with several fields (like electrodynamics) the only distinction is that the above chains may include the retarded functions of the different fields.

The commutativity is held for $[\hat{\varphi}'(x),\hat{\varphi}(z)]$, but, in general, not for $[\hat{\varphi}'(x),\hat{\varphi}(z)]$ and $[\hat{\varphi}'(x),\hat{\varphi}'(z)]$.

APPENDIX B

There are possible different zero approximations. For example, one can choose

$$D(t-t') = \begin{cases} -(t-t') & (a) \\ -\dfrac{\sin\omega(t-t')}{\omega} & (b)\end{cases} \qquad D^{(1)}(t-t') = \begin{cases} \dfrac{1}{m} & (a) \\ \dfrac{\cos\omega(t-t')}{\omega} & (b)\end{cases} \qquad (B.I)$$

The first of them corresponds to free motion[I.8], and the second one to oscillator motion (the latter is closer to field theory, see[I], Appendix B). The Green function $G_{ret}(t-t')$ is given

$$G_{ret}(t-t') = -\theta(t-t')D(t-t') \qquad (B.2)$$

The Newton equations for both cases are

$$\ddot{x}(t) = F'(x(t)) \qquad (B.3.a)$$

$$\ddot{x}(t) + \omega^2 x(t) = F'(x(t)) \qquad (B.3.b)$$

The coherent states can be defined as

$$|xp\rangle = e^{i\left(p(t)\hat{x}(t) - x(t)\hat{p}(t)\right)}|0\rangle \tag{B.4}$$

the quantity $p(t)\hat{x}(t)-x(t)\hat{p}(t)=m(\dot{x}(t)\hat{x}(t)-x(t)\dot{\hat{x}}(t))$ being conserved in both the cases due to the free equations;

$$x(t) = \langle xp|\hat{x}(t)|xp\rangle, \qquad p(t) = \langle xp|\hat{p}(t)|xp\rangle, \tag{B.5}$$

$$\langle xp|\hat{x}(t_1)\ldots\hat{x}(t_n)|xp\rangle = x(t_1)\ldots x(t_n). \tag{B.6}$$

We have noted above that it is sufficiently to use only diagonal elements in coherent states. Others are superfluous (due to overcompleteness of the set of coherent states). Let us give an analogy. In classics a system with n degrees of freedom is characterised by 2n variables such as x_i and p_i $(i=1,\ldots n)$. In quantum mechanics each operator Q for a similar system is also characterized by 2n variables, for example, $\langle x_1''\ldots x_n''|Q|x_1'\ldots x_n'\rangle$ in the x-representation or $\langle p_1''\ldots p_n''|Q|x_1'\ldots x_n'\rangle$ in the mixed x-,p-representation, and so on. The same is valid for the coherent state expectation values $\langle\bar{x}_1\ldots\bar{x}_n\bar{p}_1\ldots p_n|Q|x_1\ldots x_n p_1\ldots p_n\rangle$ too, in contrast to non-diagonal matrix elements, depending on 4n variables.

APPENDIX C

The Green functions $G(x,x')$ for scalar and spinor fields, interacting with an external electromagnetic one, obey the equations

$$\left[\left(\partial'_\mu + ieA_\mu(x')\right)^2 - m^2\right]G(x,x') = -\delta^4(x-x'), \tag{C.1}$$

$$G(x,x')\left[-\gamma_\mu\left(\overleftarrow{\partial}'_\mu + ieA_\mu(x')\right) + m\right] = -\delta^4(x-x'). \tag{C.2}$$

Let us form currents (Wronskians)

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x) Such a representation for the density matrix has been used by D.Blokhintzev [14]. However such matrix elements are complex quantities unlike real those in classics and in CSR.

$$G(x,x')\left(\overleftrightarrow{\partial}'_\mu + 2ieA_\mu(x')\right)\varphi(x') \tag{C.3}$$

$$G(x,x')\gamma_\mu\psi(x') \tag{C.4}$$

which are non-conserved

$$\partial'_\mu\left(G(x,x')\left(\overleftrightarrow{\partial}'_\mu + 2ieA_\mu(x')\right)\varphi(x')\right) = -\delta^4(x-x')\varphi(x') \tag{C.5}$$

$$\partial'_\mu\left(G(x,x')\gamma_\mu\psi(x')\right) = \delta^4(x-x')\psi(x') \tag{C.6}$$

unlike those, containing solutions of equations instead of the Green functions. Integrating over a space-time volume R_4 with boundary S and using Green's theorem, one obtains

$$\int_S d\sigma'_\mu\, G(x,x')\left(\overleftrightarrow{\partial}'_\mu + 2ieA_\mu(x')\right)\varphi(x') = \begin{cases}-\varphi(x) & \text{if } x\in R_4\\ 0 & \text{otherwise}\end{cases} \tag{C.7}$$

$$\int_S d\sigma'_\mu\, G(x,x')\gamma_\mu\psi(x') = \begin{cases}\psi(x) & \text{if } x\in R_4\\ 0 & \text{otherwise}\end{cases} \tag{C.8}$$

For the space-time volume between the boundaries t'=const and t''=const and with constraint $t' < t < t''$ one has

$$\varphi(x) = i\int d^3x''\, G(x,x'')\left(\overleftrightarrow{\partial}''_4 + 2ieA_4(x'')\right)\varphi(x'') - i\int d^3x'\, G(x,x')\left(\overleftrightarrow{\partial}'_4 + 2ieA_4(x')\right)\varphi(x') \tag{C.9}$$

$$\psi(x) = -i\int d^3x''\, G(x,x'')\gamma_4\psi(x'') + i\int d^3x'\, G(x,x')\gamma_4\psi(x') \tag{C.10}$$

For the same equation different problems require different Green functions such as

retarded functions	$\Delta^A_{ret}(x,x')$ and $S^A_{ret}(x,x')$,	
advanced ones	$\Delta^A_{adv}(x,x')$ and $S^A_{adv}(x,x')$,	
symmetrical ones	$\Delta^A_{sym}(x,x')$ and $S^A_{sym}(x,x')$,	
Feynman's ones	$\Delta^A_+(x,x')$ and $S^A_+(x,x')$,	and
antiFeynman's ones	$\Delta^A_-(x,x')$ and $S^A_-(x,x')$.	

They can be defined by the integral equations

$$G(x,x') = G^{(0)}(x-x') - \int_{t'} d^4y\, G^{(0)}(x-y)\left\{2ieA_\mu(y)\partial^y_\mu + e^2A_\mu(y)A_\mu(y)\right\}G(y,x') \tag{C.11}$$

$$G(x,x') = G^{(0)}(x-x') - ie\int_{t'} d^4y\, G^{(0)}(x-y)\gamma_\mu A_\mu(y)G(y,x'), \tag{C.12}$$

where $G^{(0)}(x-x')$ are the corresponding free Green functions:

$$\Delta_{ret}(x) = -\theta(t)\Delta(x) = \Delta_{sym}(x) - \tfrac{1}{2}\Delta(x) \qquad = \frac{1}{(2\pi)^4}\int d^4k\, \frac{\exp(ikx)}{k^2+m^2-i\varepsilon k_0}$$
$$\Delta_{adv}(x) = \theta(-t)\Delta(x) = \Delta_{sym}(x) + \tfrac{1}{2}\Delta(x) \qquad = \frac{1}{(2\pi)^4}\int d^4k\, \frac{\exp(ikx)}{k^2+m^2+i\varepsilon k_0}$$
$$\Delta_{sym}(x) = -\tfrac{1}{2}\varepsilon(t)\Delta(x) = \tfrac{1}{2}\left(\Delta_{ret}(x)+\Delta_{adv}(x)\right) \qquad = \frac{1}{(2\pi)^4}P\int d^4k\, \frac{\exp(ikx)}{k^2+m^2}$$
$$\Delta_+(x) = -\theta(t)\Delta^{(-)}(x) + \theta(-t)\Delta^{(+)}(x) = \Delta_{sym}(x) + \tfrac{i}{2}\Delta^{(1)}(x) = \frac{1}{(2\pi)^4}\int d^4k\, \frac{\exp(ikx)}{k^2+m^2-i\varepsilon} \qquad \text{(C.I3)}$$
$$\Delta_-(x) = \theta(t)\Delta^{(-)}(x) - \theta(-t)\Delta^{(+)}(x) = \Delta_{sym}(x) - \tfrac{i}{2}\Delta^{(1)}(x) = \frac{1}{(2\pi)^4}\int d^4k\, \frac{\exp(ikx)}{k^2+m^2+i\varepsilon}$$
$$S_{(\,)}(x) = (\gamma\partial - m)\Delta_{(\,)}(x) \qquad \text{(C.I4)}$$

Inserting Δ^A_{ret} and S^A_{ret} as $G(x,x')$ to eqs.(C.9) and (C.I0) the latter define solutions of the initial value problems, since the first terms vanish (and dependence on t'' too). Similarly, inserting Δ^A_{adv} and S^A_{adv}, eqs.(C.9) and (C.I0) define solutions of the final value problem, since now the second terms vanish (together with the dependence on t'). In other cases both terms remain in eqs. (C.9) and (C.I0), the latter being solutions of different mixed initial-final value problems.

The same is true for the free case too, but one can reduce eqs. (C.9) and (C.I0) in all the cases to eqs.

$$\varphi(x) = i\int d^3x'\, \Delta(x-x')\overleftrightarrow{\partial_4'}\varphi(x') \qquad \text{(C.I5)}$$
$$\psi(x) = i\int d^3x'\, S(x-x')\gamma_4\psi(x') \qquad \text{(C.I6)}$$

which are valid without any constraints on t and t$'$, unlike eqs.(C.9) and (C.I0).

All we have said holds for operators $\hat{\varphi}(x)$, $\hat{\Psi}(x)$, $\hat{\varphi}(x)$ and $\hat{\psi}(x)$ too.

Considering evolution of one-positron and one-electron states as the initial value problem, one obtains

$$\hat{\Psi}(x)|0,t'\rangle = i\int d^3x'\, S^A_{ret}(x,x')\gamma_4\hat{\Psi}(x')|0,t'\rangle \qquad \text{(C.I7)}$$
$$\hat{\bar{\Psi}}(x)|0,t'\rangle = -i\int d^3x'\, \hat{\bar{\Psi}}(x')|0,t'\rangle\gamma_4 S^A_{adv}(x'-x) \qquad \text{(C.I8)}$$

and the final states together with the retarded Green functions have frequences of both signs (the Klein paradox).

Actually, in quantum field theory evolution of an electron corresponds to the Feynman initial-final problem. According to eq. (C.I0) one can represent

$$\hat{\Psi}(x) = -i\int d^3x''\, S^A_+(x,x'')\gamma_4\hat{\Psi}(x'') + i\int d^3x'\, S^A_+(x,x')\gamma_4\hat{\Psi}(x') =$$
$$= -i\int d^3x''\, S^A_+(x,x'')\gamma_4\hat{\Psi}_+(x'') + i\int d^3x'\, S^A_+(x,x')\gamma_4\hat{\Psi}_-(x') \qquad \text{(C.I9)}$$

and obtains for the electron-electron transition amplitude

$$\langle 0,t''|\hat{\Psi}_\alpha(x'')\hat{\bar{\Psi}}_\beta(x')|0,t'\rangle = i\langle 0,t''|0,t'\rangle S^A_{+\alpha\beta}(x'',x'). \qquad \text{(C.20)}$$

Similarly, for the positron-positron, vacuum-pair and pair-vacuum transition amplitudes one obtains

$$\langle 0,t''|\hat{\bar{\Psi}}_\beta(x'')\hat{\Psi}_\alpha(x')|0,t'\rangle = -i\langle 0,t''|0,t'\rangle S^A_{+\alpha\beta}(x',x'') \qquad \text{(C.2I)}$$
$$\langle 0,t''|\hat{\bar{\Psi}}_\beta(y'')\hat{\Psi}_\alpha(x'')|0,t'\rangle = -i\langle 0,t''|0,t'\rangle S^A_{+\alpha\beta}(x'',y'') \qquad \text{(C.22)}$$
$$\langle 0,t''|\hat{\Psi}_\alpha(x')\hat{\bar{\Psi}}_\beta(y')|0,t'\rangle = i\langle 0,t''|0,t'\rangle S^A_{+\alpha\beta}(x',y'). \qquad \text{(C.23)}$$

The operators $\hat{\Psi}(x'')$ and $\hat{\Psi}_+(x'')$ are complicated quantities. In terms of $\hat{\Psi}(x)$ eq. (C.I9) has the following non-linear ("many-partiðle") form

$$\hat{\Psi}(x) = -i\int d^3x''\, S^A_+(x,x'')\gamma_4 U^{-1}(t'',t')\hat{\Psi}(x'')U(t'',t') + i\int d^3x'\, S^A_+(x,x')\gamma_4\hat{\Psi}(x') \qquad \text{(C.24)}$$

Apparently, it is more natural to consider the operators

$$U(t'',t')\hat{\Psi}(x) = -i\int d^3x''\, S^A_+(x,x'')\gamma_4\hat{\Psi}(x'')U(t'',t') + iU(t'',t')\int d^3x'\, S^A_+(x,x')\gamma_4\hat{\Psi}(x')$$
$$U(t'',t'):\hat{\Psi}(x)\hat{\bar{\Psi}}(y): = \qquad \text{(C.25)}$$
$$= -\int d^3x''d^3y''\, S^A_+(x,x'')\gamma_4\hat{\Psi}(x'')S^A_+(y,y'')\gamma_4\hat{\Psi}(y'')U(t'',t') -$$
$$- \int d^3x'd^3y''\, S^A_+(y,y'')\gamma_4\hat{\Psi}(y'')U(t'',t')S^A_+(x,x')\gamma_4\hat{\Psi}(x') +$$
$$+ \int d^3x''d^3y'\, S^A_+(x,x'')\gamma_4\hat{\Psi}(x'')U(t'',t')S^A_+(y,y')\gamma_4\hat{\Psi}(y') -$$
$$- U(t'',t')\int d^3x'd^3y'\, S^A_+(x,x')\gamma_4\hat{\Psi}(x')S^A_+(y,y')\gamma_4\hat{\Psi}(y') \qquad \text{(C.26)}$$

and so on. In $:\hat{\varphi}(x)\hat{\varphi}(y):$ it is implied ordering with respect to $\hat{\varphi}(x'')$ and $\hat{\varphi}(x')$. The final expressions are N-ordered with respect to $\hat{\varphi}(x)$ (if it is so for U) and are convenient for taking of coherent state expectation values. The vectors

$$U(t'',t')|0,t'\rangle$$

$$U(t'',t')\hat{\varphi}(x)|0,t'\rangle$$

$$U(t'',t'):\hat{\varphi}(x)\hat{\varphi}(y):|0,t'\rangle$$

. . . .

are result of evolution in terms of the interaction picture. When t_x, t_y,... tend to t' or t'', one obtains the many-quantum interpretation of the negative frequency parts (the solution of the Klein paradox, cf./3, 16/).

As to the initial value problem we note the formulas

$$\frac{\delta}{\delta J(y_1)}\cdots\frac{\delta}{\delta J(y_n)}\varphi(x_1)\cdots\varphi(x_n) = \left(\frac{\delta}{\delta J(y_1)}\cdots\frac{\delta}{\delta J(y_n)}\langle\varphi|:\hat{\varphi}(x_1)\cdots\hat{\varphi}(x_n):|\varphi\rangle\right) =$$

$$= (-i)^n\sum\Delta^A_{ret}(x_1,y_1)\cdots\Delta^A_{ret}(x_n,y_n) \qquad (C.27)$$

$$\frac{\delta}{\delta\eta(y_1)}\cdots\frac{\delta}{\delta\eta(y_n)}\psi(x_1)\cdots\psi(x_n) = \left(\frac{\delta}{\delta\eta(y_1)}\cdots\frac{\delta}{\delta\eta(y_n)}\langle\psi|:\hat{\psi}(x_1)\cdots\hat{\psi}(x_n):|\psi\rangle\right) =$$

$$= i^n\sum(-1)^p S^A_{ret}(x_1,y_1)\cdots S^A_{ret}(x_n,y_n), \qquad (C.28)$$

where the sums are over all n! permutations of $y_1,\ldots,y_n$, p being the parity of permutation.

Note also the identities

$$\tilde{G}(x,y) = i\int d^3x''\, G(x,x'')\left(\overleftrightarrow{\partial}''_4 + 2ieA_4(x'')\right)\tilde{G}(x'',y) - i\int d^3x'\, G(x,x')\left(\overleftrightarrow{\partial}'_4 + 2ieA_4(x')\right)\tilde{G}(x',y) \qquad (C.29)$$

$$\tilde{G}(x,y) = -i\int d^3x''\, G(x,x'')\gamma_4\tilde{G}(x'',y) + i\int d^3x'\, G(x,x')\gamma_4\tilde{G}(x',y), \qquad (C.30)$$

for example,

$$S^A_{ret}(x,y) = i\int d^3x'\, S^A_{ret}(x,x')\gamma_4 S^A_{ret}(x',y) \qquad (C.31)$$

$$S^A_{+}(x,y) = i\int d^3x'\, S^A_{ret}(x,x')\gamma_4 S^A_{+}(x',y) \,. \qquad (C.32)$$

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Received by Publishing Department
on December 31, 1974.