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# STRONG UNIQUENESS FOR THE TWO-DIMENSIONAL SINE-GORDON MODEL

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## 1. INTRODUCTION. NOTATION AND THE RESULT

Statistical mechanics approach to the two-dimensional super-renormalizable Euclidean scalar (quantum) Euclidean Field Theory can be formulated in terms of the fundamental notions of the Gibbs states [1,2].

The standard measure space $\{S'(R^2), \mathcal{B}\}$, where $S'(R^2)$ stands for the (real part of) Schwartz space of the tempered distributions and $\mathcal{B}$ for the Borel $\sigma$-algebra of subsets in $S'(R^2)$, plays the role of the configuration space in this approach. Let $\mu^{\Gamma}_{0,b}$ be the Gaussian measure with the covariance $(-\Delta^{\Gamma}_b + 1)^{-1}$ and mean equal to zero. Here $\Delta^{\Gamma}_b$ stands for the two-dimensional Laplace operator with some classical boundary condition b imposed on the given picewise $-C^1$ curve $\Gamma$. In particular, we will write $\mu_0$ for the measure with the free boundary condition and $\mu^{\Gamma}_0$ for the Gaussian measure $\mu_0$ with the Dirichlet boundary condition on $\Gamma$. Let us denote by $\Sigma(\Lambda)$ the local $\sigma$-algebras generated by the free Gaussian field $\mu_0$ and by $m_\infty(\Lambda)$, the space of bounded measurable with respect to $\Sigma(\Lambda)$ functionals of the field $\mu_0$.

Let $\{U_\Lambda(\phi)\}$ be an additive functional of the free field such that $\exp(U_\Lambda(\phi)) \in \bigcap_{p\geq 1} L^p(d\mu_0)$. We will say that a probabilistic, Borel cylindric (PBC) measure $\mu$ defines a quantum scalar field with the interaction $\{U_\Lambda\}$ iff

i) $\mu$ is locally absolutely continuous with respect to $\mu_0$, i.e.:

$$\mu|_{\Sigma(\Lambda)} \ll \mu_0|_{\Sigma(\Lambda)} \quad .$$

ii) for any $F \in m_\infty(\Lambda)$ the conditional expectation values of F with respect to the measure $\mu$ and the local $\sigma$-algebra $\Sigma(\Lambda^c)$, $E_\mu\{F|\Sigma(\Lambda^c)\}$, is equal to those computed with respect to the measure $\mu_\Lambda$ and the $\sigma$-algebra $\Sigma(\Lambda^c)$, where $\mu_\Lambda$ is a measure

$$\mu_\Lambda(d\phi) = (Z_\Lambda)^{-1}\exp(U_\Lambda(\phi))\mu_0(d\phi),$$

$$Z_\Lambda = \int \mu_0(d\phi)\exp(U_\Lambda(\phi)); \tag{1.1}$$

iii) the moments of the measure exist and obey standard requirements of the Euclidean Field Theory (such as Glimm-Jaffe axioms [3], etc.).

It is important to note that the family $\{E_{\mu_\Lambda}\{-|\Sigma(\Lambda^c)\}$ defines on the $\{S'(R^2),\mathcal{B}\}$ a local specification in the standard sence [1]. In [4-6] it was proved that

$$E_{\mu_\Lambda}\{-|\Sigma(\Lambda^c)\} = \frac{E_{\mu_0}\{-e^{U_\Lambda(\phi)}|\Sigma(\Lambda^c)\}}{E_{\mu_0}\{e^{U_\Lambda(\phi)}|\Sigma(\Lambda^c)\}} . \tag{1.2}$$

The conditional expectation values $E_{\mu_0}\{-|\Sigma(\Lambda^c)\}(\eta)$ at the randomly chosen point $\eta \in S'(R^2)$ can be written as the solution of the following Dirichlet stochastic problem [4,5,6]:

$$(-\Delta+1)\Psi_\eta^{\partial\Lambda}(x) = 0, \quad x \in \mathrm{Int}\,\Lambda .$$
$$\Psi_\eta^{\partial\Lambda}(x) = \eta(x), \quad x \in \partial\Lambda . \tag{1.3}$$

For a recent, deep discussion of such kinds of the stochastic problems, see the paper by Gallavotti et al. [5]. Here, we recall some basic facts concerning the problem (1.3).

For a given additive functional $\{U_\Lambda\}$ let us denote by $\mathcal{G}^t(U_\Lambda)$ the space of all tempered (i.e., supported on $S'(R^2)$ PBC measures such that

$$\forall_{|\Lambda|<\infty} \quad \mu \circ E_{\mu_\Lambda}\{-|\Sigma(\Lambda^c)\} = \mu \tag{1.4}$$

in the meaning of measures. We shall call elements of the set $\mathcal{G}^t(U_\Lambda)$ the tempered Gibbs measures corresponding to the interaction $\{U_\Lambda\}$.

It is not hard to observe that the set $\mathcal{G}^t(U_\Lambda)$ is convex and weakly closed. From the results of Fölmer [7] and the recent, more general of Weizsäcker and Winkler [8,9] it follows that the set $\mathcal{G}^t(U_\Lambda)$ has a structure similar to that of the Choquet simplex: every $\mu \in \mathcal{G}^t(U_\Lambda)$ may be uniquely represented as a resultant of some probabilistic measure $\rho$ supported on the set $\partial\mathcal{G}^t(U_\Lambda)$ of extremal points of $\mathcal{G}^t(U_\Lambda)$ which exist by the Fölmer-Winkler results [7,8].

A Gibbs measure $\mu \in \mathcal{G}^t(U_\Lambda)$ is called the regular Gibbs measure iff its two-point moment can be extended continuously to the Sobolev space $H_{-1}(R^2)$, i.e., there exists a constant $c$ such that

$$\forall_{f \in H_{-1}(R^2)} \quad \int \phi^2(f)\mu(d\phi) \le c\,\|f\|^2_{-1} . \tag{1.5}$$

A Gibbs measure $\mu \in \mathcal{G}^{\pm}(U_\Lambda)$ is called the completely regular Gibbs measure iff there exists a constant $C$ such that

$$\forall_{f \in H_{-1}(R^2)} \quad \int e^{\phi(f)}\mu(d\phi) \le e^{C\|f\|^2_{-1}} \tag{1.6}$$



We denote the set of regular Gibbs measures (resp., completely regular Gibbs measures) by $\mathcal{G}^t_r(U_\Lambda)$ (resp. $\mathcal{G}^t_{cr}(U_\Lambda)$). From the definition it follows that $\mathcal{G}^t_{cr}(U_\Lambda) \subseteq \mathcal{G}^t_r(U_\Lambda) \subseteq \mathcal{G}^t(U_\Lambda)$. From the papers [5,6] we know that for any $\mu \in \mathcal{G}^t_r(U_\Lambda)$ the stochastic Dirichlet problem (1.3) has for almost every $\eta$ with respect to $\mu$ a solution given by the classical Poisson formula:

$$\Psi_\eta^{\partial\Lambda}(x) = \int_{\partial\Lambda} P^{\partial\Lambda}(x,z)\eta(z)dz . \tag{1.7}$$

valid for $x \notin \partial\Lambda$, where $P^{\partial\Lambda}(x,z)$ is the Poisson kernel for the operator $-\Delta+1$. This unique solutions have certain local decay properties as $\Lambda \quad R^2$ (see [6] for greater details).

The most important questions in the general theory of Gibbs states are the questions about the existence and detailed topological structure of the sets like $\mathcal{G}^t(U_\Lambda)$. The existence problem for the field-theoretical Gibbs measures has been treated intensively in the seventies. See [3,10] for references. However, till now there doesn't exist a satisfactory version of the Dobrushin-like theory [33] in the field-theoretical context. Let us remark that it is not known whether every $\mu \in \mathcal{G}^t_r(U_\Lambda)$ for a given interaction $\{U_\Lambda\}$ defines a quantum field theory in the sense of i)-iii). By experience with the lattice spin systems with noncompact state space of the individual spin [11,12], we expect that there may exist some spurious solutions of the DLR equations (1.4) in the space $\mathcal{G}^t(U_\Lambda)$.

Essentially important is the question about the cardinality of the set ${}^{QF}\mathcal{G}^t(U_\Lambda) \cap \mathcal{G}^t(U_\Lambda)$, where we denote by ${}^{QF}\mathcal{G}(U_\Lambda)$ the set of quantum-field theoretical solutions of the D-L-R equations (1.4). Whenever, the above-mentioned set has more than one element, we have to deal with the phenomena of the first order phase transition. Deep results in this direction have been obtained recently by Jmbrie [13] for the case of polynomial interactions. A detailed description of the set $\mathcal{G}^t(U_\Lambda)$ is, however, a very hard mathematical problem (it is very nontrivial already on the level of the two-dimensional Ising model [14,15]). The analysis of the set $\mathcal{G}^t_r(U_\Lambda)$ for a given $\{U_\Lambda\}$ seems to be a much easier problem. In the case of exponential interactions we have proved in [16,17] that the set $\mathcal{G}^t_{cr}(U_\Lambda)$ reduces exactly to one quantum field theory solution of (1.4). This has been proved also in [18] extending the ideas taken from [16].

In this note we consider the problem of uniqueness (in the space $\mathcal{G}^t_r(U_\Lambda)$) of the solutions of the DLR equations (1.4) for the so-called sine-Gordon interaction. Previously Albeverio and Hoegh-Kröhn using a high-temperature cluster expansion have proved the uniqueness for the weakly coupled sine-Gordon interaction [6]. A similar uniqueness result has been proved in [19]. for the regularized Yukawa $d$-dimensional, neutral gas in the region of couplings where the contraction map principle can be applied to the Kirkwood-Salsburg equations.

Sine-Gordon interactions are defined by:

$$U_\Lambda(\phi) = z \int_\Lambda d^2x : \cos(\epsilon\phi(x) + \Theta): \tag{1.8}$$

Here we consider this model in the region:

$$z \geq 0, \quad \epsilon^2 < \frac{2}{1 - \frac{1}{2}\pi}, \quad \Theta = 0.$$

See the papers [20,21,22] for the construction of the corresponding Gibbs measures with $z \in R^1$, $\Theta \neq 0$ and $\epsilon^2 < 4\pi$.

Our result proved in this note is the following:

Theorem 1. Whenever the infinite volume pressure $p_\infty(z)$ in the model (1.8) is differentiable at $z = z_0$, the set of regular solutions of the corresponding DLR equations at $z = z_0$ which have translationally invariant first moment consists of exactly one element=infinite volume half Dirichlet state.

The uniqueness result of Albeverio and Hoegh-Krohn holds only in the region of the convergence of the Glimm-Jaffe Spencer expansion, i.e., for sufficiently small $|z|$. Taking into account that $p_\infty(z)$ as a concave function of $z$ is almost everywhere differentiable (presumable for all $z$ ), we have that for almost every $z$ there exists a unique pure Gibbs phase corresponding to the interaction (1.8).

The proof we find seems to be very elementary. We adopt to the present case some correlation inequalities found by Frohlich and Pfister in their analysis of the DLR equations for abelian lattice spin systems [23,24,25]. The additional argument we use for the proof is the independence of the infinite volume pressure of boundary condition (generalizing the known result of Guerra-Rosen-Simon [26] and concerning the independence of the so-called classical boundary conditions). This is proven in [47].

The ideas used in this paper can be applied to analyse the class of weakly coupled $\mathcal{P}(\phi)_2$ models or the $\phi^4_2$ -models where the Lee-Yang theorem works. Similar techniques have been applied to the class of charge-symmetric continual systems [27,28].

Finally, let us say few words about organization of this note. In section 2 we review some correlation inequalities reducing the proof of the uniqueness of the solutions of equations (1.4) to the statement about independence of the infinite volume pressure of the boundary conditions. In [47] we prove the claimed independence. Section 3 contains some techniques necessary to complete the proof of Theorem 1.



## 2. REDUCTION OF THE PROOF TO THE STATEMENT ABOUT DIFFERENTIABILITY OF THE PRESSURE

The infinite volume half-Dirichlet sine-Gordon measure corresponding to the interactions (1.8) can be constructed easily by the following correlation inequalities proved in [29]:

$$\langle e^{t\phi(f)}; :\cos\epsilon\phi:(x)\rangle^T_\Lambda \leq 0. \tag{2.1}$$

$$\langle \phi^2(f); :\cos\epsilon\phi:(x)\rangle^T_\Lambda \leq 0. \tag{2.2}$$

where $\langle;\rangle^T$ means the truncated expectation value and $\langle\rangle^0_\Lambda(z)$ means the expectation with respect to the measures:

$$\mu_\Lambda^{\partial\Lambda}(d\phi) = (Z^\circ_\Lambda(z))^{-1} \exp\left(z \int_\Lambda :\cos\epsilon\phi: dx\right) \mu_0^{\partial\Lambda}(d\phi)$$

$$Z^\circ_\Lambda(z) = \int \mu_0^{\partial\Lambda}(d\phi) \exp\left(z \int_\Lambda :\cos\epsilon\phi:(x)dx\right). \tag{2.3}$$

From these inequalities it follows easily that the infinite-volume limit $\lim_{\Lambda\uparrow R^2} \langle - \rangle_\Lambda = \langle - \rangle_\infty$ exists (independently how $\Lambda$ $R^2$ ) and fulfills the axioms of [30]. In particular, we have the bound

$$\langle e^{t\phi(f)}\rangle^0_\infty \leq e^{\frac{t^2}{2}\|f\|^2_{-1}} \tag{2.4}$$

which means that $\langle - \rangle^0_\infty \equiv \int - \mu_\infty(d\phi)$ is a complete regular Gibbs measure. Let us introduce the following notation: $c(\phi)(x) = :\cos\epsilon\phi:(x)$; $s(\phi)(x) = :\sin\epsilon\phi(x):$. By $\mathcal{G}^t_r(z)$ we denote the set of regular Gibbs measures corresponding to the interaction (1.8) with fixed $z$ and $\epsilon$. The symbol $\overset{\mu}{\underset{\eta}{\forall}}$ means: for almost every $\eta$ with respect to $\mu$. A conditioned finite-volume Gibbs measure $\mu^\eta_\Lambda(d\phi)$ is given by:

$$\mu^\eta_\Lambda(d\phi) = (Z^\eta_\Lambda(z))^{-1} \exp\left(z \int_\Lambda c(\phi + \Psi^{\partial\Lambda}_\eta)(x)dx\right) \mu_0^{\partial\Lambda}(d\phi) \tag{2.5}$$

where $\Psi^{\partial\Lambda}_\eta$ is the solution of the problem (1.3) and $\eta$ is randomly chosen from $S'(R^2)$.

From the reverse martingale theorem it follows that for a given $\mu \in \mathcal{G}^t(z)$:

$$\mu^\eta_\infty(d\phi) = \lim_{\Lambda\uparrow R^2} \mu^\eta_\Lambda(d\phi)$$

exists for $\mu$ a.e., $\eta \in S'(R^2)$, and defines some PBC measure on $\{S'(R^2), \mathcal{B}\}$. Moreover, the full set $\mathcal{G}^t(z)$ can be obtained as convex superpositions of such limits.

For the conditioned measures $\mu^\eta_\Lambda$ the correlation inequalities (2.1), (2.2) in general fail. However, instead of the correlation inequalities (2.1), (2.2) one can use another set of correlation inequalities in order to analyse the content of the set $\mathcal{G}^t_r(z)$. These correlation inequalities proved below are simple adaptations of the correlation inequalities proved by Fröhlich and Pfister in[23]. They all are simple application of the Ginibre correlation inequalities[31]. For the shorhand let us introduce the notation:

$$c_\Lambda(x_1,\dots,x_n) = \int \mu^{\partial\Lambda}_\Lambda(d\phi)\prod_{i=1}^n c(\phi)(x_i)$$

$$c^\eta_\Lambda(x_1,\dots,x_n) = \int \mu^\eta_\Lambda(d\phi)\prod_{i=1}^n c(\phi)(x_i)$$

$$s^\eta_\Lambda(x_1,\dots,x_n) = \int \mu^\eta_\Lambda(d\phi)\prod_{i=1}^n s(\phi)(x_i) \tag{2.6}$$

$$s_\Lambda(x_1,\dots,x_n) = \int \mu^{\partial\Lambda}_\Lambda(d\phi)\prod_{i=1}^n s(\phi)(x_i)$$

and similarly for the corresponding infinite volume limits. The existence of $\lim_{\Lambda\uparrow R^2} c_\Lambda$ follows from the $\cos\epsilon\phi$: bound[20] and the following correlation inequalities

$$<\prod_{i=1}^n c(\phi)(x_i);\ \prod_{j=1}^n c(\phi)(y_j)>^T_\Lambda(z) \geq 0 \tag{2.7}$$

proved in[29]. The existence of the limits $\lim_{\Lambda\uparrow R^2} c^\eta_\Lambda$, $\lim_{\Lambda\uparrow R^2} s^\eta_\Lambda$ (by subsequences) follows from the correlation inequalities to be proved below (see inequality (2.14)) and the compactness arguments. Moreover, from the results of section 3 it follows that every accumulation point of $c^\eta_\infty$ is equal to $c_\infty$ (at least for regular values of z, see below).

Let us denote by $<-->^{(0,\eta)}_\Lambda$ the expectation on $\{S'(R^2), \}^{\otimes 2}$ with respect to the measure $\mu^{\partial\Lambda}_\Lambda(d\phi)\otimes\mu^\eta_\Lambda(d\phi)$.

<u>Proposition 2.1.</u> Let $\mu\in\mathcal{G}^t_r(z)$. Then for every $n>0, f_1 \dots, f_n \in S(R^2)$ ; such that $f_i \geq 0$ for $i = 1,2,\dots,n$ ; $g\in S(R^2)$ the following correlation inequalities hold:



$$\underset{\eta}{\overset{\mu}{\forall}}\quad 0 \leq \Big<\Big(\prod_{i=1}^n c(\phi)(f_i) - \prod_{i=1}^n c(\phi')(f_i)\Big) \exp \pm\delta\int d^2x\, g(x)c(\phi)(x)c(\phi')(x)\Big>^{(0,\eta)}_\Lambda(z),\ \delta\in R^1. \tag{2.8}$$

<u>Proof:</u> It is a standard application of the duplicate variable technique. Let $\phi'$ by an identical copy of the field $\phi$. From

$$\exp(z\int_\Lambda c(\phi)(x)dx)\exp(z\int_\Lambda c(\phi'+\Psi^{\partial\Lambda}_\eta)(x)dx)$$

$$=\exp\Big(z\int dx\, c\Big(\frac{\phi+\phi'+\Psi^{\partial\Lambda}_\eta}{2}\Big)(x)c\Big(\frac{\phi-\phi'-\Psi^{\partial\Lambda}_\eta}{2}\Big)(x)\Big) \tag{2.9}$$

and

$$c(\phi)(x)c(\phi')(x) = \frac{1}{2}c\Big(\frac{\phi-\phi'}{2}\Big)(x)c\Big(\frac{\phi+\phi'}{2}\Big)(x) \tag{2.10}$$

we conclude after introducing the orthogonal transformation in the space $\{\phi,\phi'\}$:

$$\Psi_+ = \frac{\phi+\phi'}{\sqrt{2}}\ ;\ \Psi_- = \frac{-\phi+\phi'}{\sqrt{2}} \tag{2.11}$$

that the first exponential and the one coming from the interaction factorize after (convergent for $|\Lambda|<\infty$ ) expansions in z. The terms outside the exponentials factorize using the following trigonometric identities:

$$\prod_{j=1}^n \cos\phi_j = \frac{1}{2}\sum_{\{\epsilon_j\}}\cos\Big(\sum_{j=1}^n \epsilon_j\phi_j\Big),\quad \epsilon_j = \pm 1, \tag{2.12}$$

and

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}. \tag{2.13}$$

Some intermediate UV-regularizations are needed to justify these transformations rigorously but the removal of it is simple so that we omit the details here.

q.e.d.

These correlation inequalities lead to the following inductive statement on the independence of the moments like $c^\eta_\Lambda(x_1,\dots,x_n)$ of the boundary condition.

Remark 2.1. For the eventual future application we define also the following moments of the measure $\mu^{\eta}_{\Lambda}(d\phi)$:

$$\hat{c}^{\eta}_{\Lambda}(x_1,\dots,x_n) = E_{\mu_\Lambda}\{\prod_{i=1}^{n} c(\phi)(x_i) \mid \Sigma(\Lambda^c)\}(\eta) \tag{2.16}$$

$$= \int \mu^{\eta}_{\Lambda}(d\phi) \prod_{i=1}^{n} :\cos(\epsilon\phi(x_i) + \epsilon\Psi^{\partial\Lambda}_{\eta}(x_i)):$$

and

$$\hat{s}^{\eta}_{\Lambda}(x_1,\dots,x_n) = E_{\mu_\Lambda}\{\prod_{i=1}^{n} s(\phi)(x_i) \mid \Sigma(\Lambda^c)\}(\eta) \tag{2.16'}$$

$$= \int \mu^{\eta}_{\Lambda}(d\phi) \prod_{i=1}^{n} :\sin(\epsilon\phi(x_i) + \epsilon\Psi^{\partial\Lambda}_{\eta}(x_i)):$$

Then, it is not hard to observe the correlation inequalities listed above hold also for $\hat{c}^{\eta}_{\Lambda}$.

Proposition 2.6. Let $\mu \in \mathcal{G}^t_r(z)$. Then for every $n \geq 0$, $f_1,\dots,f_n \in S(R^2)$ such that $f_i \geq 0$ for $i = 1,\dots,n$ and $g \in S(R^2)$ the following correlation inequalities hold:

$$\forall_{\mu}^{\eta} : \forall_{\delta\in R} : \quad 0 \leq \langle (\prod_{i=1}^{n} c(\phi)(f_i) - \prod_{i=1}^{n} :\cos(\epsilon\phi' + \epsilon\Psi^{\partial\Lambda}_{\eta}):(f_i)) \tag{2.17}$$

$$\times \exp(\pm\delta \int dx\, g(x) c(\phi)(x) c(\phi + \Psi^{\partial\Lambda}_{\eta})(x))\rangle^{(0,\eta)}_{\Lambda}$$

and

$$|\langle \prod_{i=1}^{n} c(\phi + \Psi^{\partial\Lambda}_{\eta})(f_i)\rangle^{\eta}_{\Lambda}| \leq c^{\circ}_{\Lambda}(f_1,\dots,f_n). \tag{2.18}$$

In particular, this proposition leads to the same bootstrop principle as Corollary 2.2 for the moments

$$\lim_{\Lambda \to R^2} \hat{c}^{\eta}_{\Lambda}(x_1,\dots,x_n) = \hat{c}^{\eta}_{\infty}(x_1,\dots,x_n).$$

The existence of $\hat{c}^{\eta}_{\infty}(x_1,\dots,x_n)$ follows from the application of the reverse martingale theorem and the correlation inequality (2.18). Moreover, from the correlation inequality (2.18) it follows easily that the set of the limits $\lim_{\Lambda} \mu^{\eta}_{\Lambda}$ forms then a weakly

precompact set in the space of all probability measures on the space $\{S'(R^2), \mathcal{B}\}$.



## 3. COMPLETING OF THE PROOF OF THEOREM 1

The sequence of moments $\{c^{\circ}_{\infty}(x_1,\dots,x_n)\}_{n=1,2,\dots}$ does not describe fully the measure $\mu_\infty$ but rather its restriction to the even part of the $\sigma$-algebra $\Sigma(R^2)$ only. From the independence of the boundary conditions of the moments $\{c^{\eta}_{\infty}(x_1,\dots,x_n)\}_{n=1,\dots}$ it follows that every even function of the field $\phi$ does not depend on the boundary conditions. In particular, taking an arbitrary sequence $f_1,\dots,f_n \in S(R^2)$ we conclude that the moments

$$c^{\eta}_{\infty}(f_1,\dots,f_n \mid (x_1,\dots,x_n)) = \lim_{\Lambda\uparrow R^2} \int \mu^{\eta}_{\Lambda}(d\phi) \prod_{i=1}^{n} :\cos\epsilon\phi_{f_i}:(x_i), \tag{3.1}$$

where

$$:\cos\epsilon\phi_{f_i}:(x_i) = \exp(\frac{\epsilon^2}{2}(S * f_i)(0)) \tag{3.2}$$

$$\cos\epsilon(\phi * f_i)(x_i)$$

does not depend on the given $\eta$ if $c^{\eta}_{\infty}(x_1,\dots,x_n)$ does not depend. By the arguments identical to those used in the proof of Prop.2.4 we have the following correlation inequality:

Lemma 3.1. Let $\mu \in \mathcal{G}^t_r(z)$. Take $f_1,\dots,f_n \in D(R^2)$ and $\alpha_1,\dots,\alpha_n \in [0, 2\pi)$ arbitrary. Then for $\mu$ a.e. $\eta$ the following correlation inequality holds:

$$|\langle \prod_{i=1}^{n} :\cos\epsilon(\phi_{f_i} + \alpha_i):(x_i)\rangle^{\eta}_{\infty}| \leq \langle \prod_{i=1}^{n} :\cos\epsilon\phi_{f_i}:(x_i)\rangle^{0}_{\Lambda}. \tag{3.3}$$

We note the following lemma also:

Lemma 3.2. Take $\mu \in \mathcal{G}^t_r(z)$ and $f_1,\dots,f_n$ as in Lemma 3.1. Then for $\mu$ a.e. $\eta$ we have:

$$\langle \prod_{i=1}^{n} :\cos\epsilon\phi_{f_i}:(x_i) :\sin\epsilon\phi_{f_{n+1}}:(y)\rangle^{\eta}_{\infty} = 0 \tag{3.4}$$

assuming that $\langle c(\phi)(x)\rangle^{\eta}_{\infty} = \langle c(\phi)(0)\rangle^{0}_{\infty}$ holds. From this we easily obtain

Corollary 3.3. Let $\mu \in \mathcal{G}^t_r(z)$ and let $f_1,\dots,f_n \in D(R^2)$. Then assuming $\langle c(\phi)(x)\rangle^{\eta}_{\infty} = \langle c(\phi)(0)\rangle^{0}_{\infty}$ we have:

$$< \prod_{i=1}^{m} s(\phi_{f_i})(x_i)>_{\infty}^{\eta} = < \prod_{i=1}^{m} s(\phi_{f_i})(x_i)>_{\infty}^{\eta} \tag{3.5}$$

and for arbitrary $g_1,\dots,g_m \in D(R^2)$ we have

$$< \prod_{i=1}^{m} s(\phi_{f_i})(x_i) \prod_{j=1}^{n} c(\phi_{g_j})(y_j)>_{\infty}^{\eta} = < \prod_{i=1}^{m} s(\phi_{f_i})(x_i) \prod_{j=1}^{n} c(\phi_{g_j})(y_j)>_{\infty}^{\eta} \tag{3.6}$$

assuming the eqiality $<c(\phi)(x)>_{\infty}^{\eta} = <c(\phi)(0)>_{\infty}^{0}$ . Finally, we are ready to prove Theorem 1.

Proof of Theorem 1. It is well known that $p_{\infty}^{0}(z)$ is a concave function of the coupling constant $z$. From this it follows (see[41,42]) that $p_{\infty}^{0}(z)$ is almost everywhere differentiable function (except at most a countable set of values) and that we have the equality:

$$\frac{d}{dz} \lim_{\Lambda\uparrow R^2} p_{\Lambda}^{0}(z) = \lim_{\Lambda\uparrow R^2} \frac{d}{dz} p_{\Lambda}^{0}(z).$$

at the points of differentiability. The arguments of [47] can easily be extended to treat the following perturbed pressure:

$$p_{\Lambda}^{\eta}(z,\lambda) = -\frac{1}{|\Lambda|} \ln \int e^{\lambda \int_{\Lambda} c(\phi)(x)dx} e^{z\int c(\phi+\Psi_{\eta}^{\partial\Lambda})(x)dx} \mu_0^{\partial\Lambda}(d\phi). \tag{3.7}$$

In particular, we obtain that the unique thermodynamic limit

$$p_{\infty}^{\eta}(z,\lambda) = \lim_{\Lambda\uparrow R^2} p_{\Lambda}^{\eta}(z,\lambda) \tag{3.8}$$

exists (whenever $\Lambda\uparrow R^2$ as in[47]) and is independent of the typical boundary condition $\eta$. Moreover, the limit is differentiable at $\lambda$ and $z$ almost everywhere. Assuming that $p_{\infty}^{0}(z)$ is differentiable at the point $z=z_0$ we obtain:

$$\frac{d}{d\lambda} p_{\infty}^{\eta}(z_0,\lambda)\Big|_{\lambda=0} = \lim_{\Lambda\uparrow R^2} \frac{1}{|\Lambda|} \int_{\Lambda} dx < c(\phi)(x)>_{\Lambda}^{\eta}(z_0) = <c(\phi)(0)>_{\infty}^{0}(z_0) \tag{3.9}$$

which shows that $p_{\infty}^{\eta}(z_0,\lambda)$ is then differentiable at the point $\lambda=0$ and that (assuming the limiting measure $< >_{\infty}^{\eta}(z_0)$ has translationally invariant first moment) the following equality holds:

$$<c(\phi)(0)>_{\infty}^{0}(z_0) = <c(\phi)(0)>_{\infty}^{\eta}(z_0). \tag{3.10}$$

Thus, the bootstroop principle of Corollary 2.3 then is applicable with the result that if $\mu$ is any regular measure then for $\mu$ a.e. $\eta \in S'(R^2)$ and $n \geq 1$ we have:



$$< \prod_{i=1}^{n} c(\phi)(f_i)>_{\infty}^{\eta}(z_0) = < \prod_{i=1}^{n} c(\phi)(f_i)>_{\infty}(z_0). \tag{3.11}$$

For the regularized moments of the form $< \prod_1^n c(\phi_{f_i}) \prod_1^m s(\phi_{g_j})>$ we then apply Corollary 3.3. The limits $f_i \downarrow \delta, g_j \downarrow \delta$ of both the sides of equality (3.6) can easily be controlled by the application of the $\cos \epsilon\phi$-type of bounds for the measure $\mu_{\infty}^{0}(d\phi)$.

q.e.d.

Remark 3.1. Taking into account Remark 2.1 we can eliminate the $\lambda$-perturbation argument used above.

## 4. CONCLUDING REMARKS

The main motive for writing this paper is the question about Global Markov Property for the two-dimensional scalar fields. In the case of lattice systems some results concerning this problem have been obtained in[37,38]. The main strategy coming back to Preston[1] and Fölmer[39] is to introduce certain order (the FKG order) into the set of Gibbs measures. Some simplifications have been made in the paper by Goldstein[40]. The method of this paper combined with the superstability estimates has been applied by the author to show the Global Markov Property also for some nonferromagnetic continuous spin systems in[43].

One of the main obstacles to apply immediately the techniques of FKG order to the continual case is that we do not know whether such an order can be defined in space of the Gibbs measures describing the continual fields. The intriguing question is to find a suitable notion of the lattice regularization which discretizes the Dirichlet problem (1.3) in a proper sense by which we mean, firstly, that the discrete versions of the corresponding local specifications are convergent surely to the continual one. Secondly, the shift transformation exists which transform the discrete versions of local specifications to the forms considered in[37,38]. Then, the FKG order may be induced into the set of the continual Gibbs measures on the account of the assured convergence. But we have not checked any details of this intriguing programme.

On the other hand, the methods of the present paper do not use any kind of ferromagnetic properties of the continual fields. Therefore, they seem to be very useful in the study of the DLR equations for continual fields which are defined by the trigonometric perturbations of a Gaussian, generalized fields. Such an analysis has been performed by the author in papers[44,45,46].

APPENDIX

In this Appendix we review some results obtained by Fölmer [7], Weizsacker and Winkler [8,9] which are relevant for us. Related results can be found also in the second chapter of Preston [1].

Let $\{J, \alpha\}$ be an increasing net which is countably generated. Assume that $\{\Omega,\Sigma\}$ is a standard Borel space and that for every $i \in J$ there is a sub $\sigma$-algebra $\Sigma_i$ of $\Sigma$ such that $i\alpha j \Rightarrow \Sigma_j \subset \Sigma_i$. A collection $\mathfrak{C}$ of stochastic kernels $\{P_i, i \in J\}$ from $\{\Omega,\Sigma\}$ to $\{\Omega,\Sigma\}$ is called specification iff

s1) $\forall_{i\in J} \; \forall_{F\in\Sigma} \; P_i(\cdot, F)$ is $\Sigma_i$ measurable

s2) $\forall_{i\in J} \; \forall_{F\in\Sigma_i} \; P_i(\cdot, F) = 1_F$

s3) $i\alpha j \Rightarrow P_j P_i = P_j$

A probability measure $\mu$ on $\{\Omega,\Sigma\}$ is called the Gibbs state corresponding to the given specification $\mathfrak{C}$ iff it satisfies the DLR-equations

DLR) $\forall_{i\in J} \; \mu \circ P_i = \mu$.

We collect the fundamental results obtained in [7,8,9,36] in the following theorem:

Theorem A.1. There exist a standard Borel space $\{\Omega_\infty, \Sigma_\infty\}$ and a stochastic kernel $P_\infty$ from $\Omega_\infty$ to $\Omega$ such that the mapping $\mu \to \mu P_\infty$ is an affine bijection from the set of probabilistic measures on $\{\Omega_\infty, \Sigma_\infty\}$ onto $\mathcal{G}(\mathfrak{C})$, in particular,

$$\mathcal{G}(\mathfrak{C}) = \{\mu P_\infty \mid \mu \text{ runs over probabilistic measures on } \{\Omega_\infty, \Sigma_\infty\}\}.$$

and the extremal points of $\mathcal{G}(\mathfrak{C})$ (the so-called Martin-Dynkin boundary)

$$\partial\mathcal{G}(\mathfrak{C}) = \{P_\infty(\omega_\infty; -) \mid \omega_\infty \in \Omega_\infty\}.$$

The set $\partial\mathcal{G}(\mathfrak{C})$ of extremal points of $\mathcal{G}(\mathfrak{C})$ is measurable with respect to the evaluation $\sigma$-algebra $\Sigma$ and for each Gibbs state $\mu$ there is a unique probabilistic measure $\rho$ on $\{\partial\mathcal{G}(\mathfrak{C}), \partial\mathcal{G} \cap \Sigma\}$ such that

$$\forall_{B\in\Sigma} \quad \mu(B) = \int_{\partial\mathcal{G}(\mathfrak{C})} \nu(B)\, d\rho(\nu).$$



For the application to field theory we put $\{\Omega,\Sigma\} = \{S'(R^2), \mathcal{B}\}$, $\{J = \{\Lambda_n\}, c\}$ any monotone sequence of bounded regular subsets of $R^2$ tending to $R^2$ monotonously and by inclusion. Then $\Sigma_n = \Sigma(\Lambda_n^c)$.

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Гелерак Р. E2-85-898
Строгая единственность для двумерной модели сина-Гордона

Исследуются равновесные уравнения типа Добрушина - Ланфорда - Руеля в случае двумерной эвклидовой модели типа сина-Гордона. Доказывается теорема о единственности предельного, трансляционно-инвариантного, равновесного состояния Гиббса, которая значительно усиливает результат, полученный ранее Альбевере и Хаг-Кроном. Использование некоторых корреляционных неравенств типа Жинибра сводит задачу единственности трансляционно-инвариантных, регулярных мер Гиббса к задаче независимости свободной энергии от граничных условий.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Gielerak R. E2-85-898
Strong Uniqueness for the Two-Dimensional Sine-Gordon Model

Dobrushin - Lanford - Ruelle equation is studied in a certain space of measures in the case of two-dimensional trigonometric interactions. Uniqueness theorem extending the results of Albeverio and Hoegh-Krohn is proved. The extension is obtained by the application of some correlation inequalities of the Ginibre-type, that reduce the proof of the uniqueness of the translationally invariant, regular, tempered Gibbs states to the question on the independence of the infinite volume free energy of the boundary conditions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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