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SUPERSPACE ACTIONS AND DUALITY TRANSFORMATIONS FOR N=2 TENSOR MULTIPLETS

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1. Introduction

Recently, some attention has been paid to superspace formulation of self-interacting N=2 tensor multiplets. General actions have been given by making use of a certain "subintegration" in N=2 superspace in four[1] and six dimensions[2] as well as of integration over chiral N=2 superspace[3]. Both approaches employ extra bosonic variables which are closely related to the isospinor harmonics $u^{\pm}_i$ of the harmonic superspace[4]. The latter proved to be perfectly adequate to unconstrained formulation of N=2 matter hypermultiplets, Yang-Mills and supergravity theories.

The present paper has two main purposes. The first one is to show that the general self-couplings of N=2 tensor multiplets are equivalent, by duality transformations, to a restricted class of hypermultiplet self-couplings. To this end in sect. 2 we develop a formalism for a manifestly supersymmetric description of N=2 tensor multiplets in the harmonic superspace including the general self-couplings. In sect. 3 we perform for the latter duality transformations resulting in ω - (or q^+_i) hypermultiplets actions with an obligatory symmetry $\omega \to \omega +$ const (or $q^+_i \to q^+_i +$ const$\cdot u^+_i$). A preliminary sketch of sect. 2,3 was given in ref.[5].

It is known[6] that there exists an improved N=2 tensor multiplet which can be coupled to the N=2 conformal supergravity and used as a compensator in constructing Einstein supergravities. Our second aim is to describe this coupling in superspace. It is the harmonic superspace that provides us with the unconstrained superfield prepotentials and the full superconformal gauge group[7]. In sect. 4 we construct the harmonic superspace action for the improved N=2 tensor multiplet and discuss its rigid superconformal invariance. Then by an appropriate duality transformation we demonstrate the equivalence of this action to that for a free q^+_i -hypermultiplet. Further, is sect. 5 we begin with the q^+_i -hypermultiplet action in the conformal supergravity background and then by inverse duality transformation get the corresponding N=2 improved tensor multiplet action.

Sect. 6 contains concluding remarks. Appendix A treats details of the derivation of the superspace action for the improved N=2 tensor multiplet. Descending to the component fields and computation of relevant harmonic integrals is given in Appendix B.

We use conventions of ref.[4] and work in four dimensions. All the considerations are immediately generalizable to six dimensions.

2. N=2 Tensor Multiplet in Harmonic Superspace. General Actions

An N=2 tensor multiplet consists of an SU(2) triplet of physical scalars $L^{(ij)}(x)$, a doublet of Weyl fermions $\psi^i_\alpha(x)$, an antisymmetric gauge tensor $E_{mn}(x)$(notoph[8]) with the field strength $V^m = \frac{1}{2}\varepsilon^{mnkl}\partial_n E_{kl}$ and a complex auxiliary scalar $M(x)$. Its superfield strength is represented in ordinary N=2 superspace $z = \{x^m, \theta^\alpha_i, \bar\theta^{\dot\alpha i}\}$ by a real triplet superfield $L^{(ij)}(z)$ [9] constrained by

$$D^{(i}_\alpha L^{jk)} = \bar D^{(i}_{\dot\alpha} L^{jk)} = 0. \qquad (2.1)$$

Multiplying this equation by SU(2)/U(1) harmonics $u^+_i u^+_j u^+_k$ [4] one obtains

$$D^+_\alpha L^{++}(z,u) = \bar D^+_{\dot\alpha} L^{++}(z,u) = 0, \qquad (2.2)$$

where

$$D^+_\alpha = u^+_i D^i_\alpha \ , \ \bar D^+_{\dot\alpha} = u^+_i \bar D^i_{\dot\alpha} \quad ; \quad L^{++}(z,u) = L^{ij}(z)u^+_i u^+_j . \qquad (2.3)$$

The superfield $L^{++}(z,u)$ obeys the following obvious relations besides eq.(2.2)

$$D^0 L^{++}(z,u) = \left(u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}}\right)L^{++}(z,u) = 2L^{++}(z,u), \qquad (2.4)$$

$$D^{++}L^{++}(z,u) = u^{+i}\frac{\partial}{\partial u^{-i}} L^{++}(z,u) = 0. \qquad (2.5)$$

Here D^0 , D^{++} are (super) covariant harmonic derivatives[4] in (z,u) coordinate frame. Three conditions (2.2), (2.4)and (2.5) are equivalent to the constraint (2.1).

By reasonings of ref.[4] one can immediately recognize that

i) the (z,u) frame is a central basis of the N=2 harmonic superspace;

ii) the constraint (2.2) is an analyticity condition which is solved by passing to the analytic basis $\{x^m_A = x^m - 2i\theta^i\sigma^m\bar\theta^{\dot\imath}\cdot u^+_{(i}u^-_{j)}$, $\theta^\pm_\alpha = \theta^i_\alpha u^\pm_i$, $\bar\theta^\pm_{\dot\alpha} = \bar\theta^i_{\dot\alpha}u^\pm_i$, $u^\pm_i\}$. Here it means simply that L^{++} is independent of $\theta^-_{\alpha(\dot\alpha)}$

$$D^+_{\alpha(\dot\alpha)} L^{++} = 0 \quad \Rightarrow \quad L^{++} = L^{++}(\zeta,u) \quad , \quad (\zeta,u) = (x^m_A, \theta^+_{\alpha(\dot\alpha)}, u^\pm_i). \qquad (2.6)$$

iii) eq.(2.4) implies that L^{++} has U(1)-charge +2 (in general, D^0 counts U(1)-charge of harmonic superfields $D^0 f^{(q)} = q f^{(q)}$ [4]).

iiii) L^{++} is real with respect to the analyticity preserving conjugation[4] $\widetilde{L^{++}} = L^{++}$.

So, the field strength of an N=2 tensor multiplet is described in the harmonic N=2 superspace as a real analytic U(1)-charge +2 superfield $L^{++}(\zeta,u)$ constrained by (2.5) that reads in the (ζ,u) frame *)

$$D^{++}L^{++}(\zeta,u) = \left(u^{+i}\frac{\partial}{\partial u^{-i}} - 2i\theta^+\sigma^m\bar\theta^+\partial^A_m\right)L^{++}(\zeta,u) = 0 \qquad (2.5')$$

with the component solution

$$L^{++}(\zeta,u) = L^{ij}(x)u^+_i u^+_j + 2(\theta^{+\alpha}\psi^i_\alpha - \bar\theta^+_{\dot\alpha}\bar\psi^{\dot\alpha i})u^+_i + (\theta^+)^2 M + (\bar\theta^+)^2\bar M + 2i\theta^+\sigma^m\bar\theta^+[V_m + \partial_m L^{ij}u^+_i u^-_j] + 2[(\bar\theta^+)^2\theta^{+\alpha}i\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha i} + (\theta^+)^2\bar\theta^+_{\dot\alpha}i\partial^{\dot\alpha\alpha}\psi^i_\alpha]u^-_i - (\theta^+)^4\Box L^{ij}u^-_i u^-_j , \qquad (2.7)$$

where $\partial_m V^m = 0$, $\overline{(\psi^i_\alpha)} = \bar\psi_{\dot\alpha i}$, $\partial\!\!\!/ = \partial_m\sigma^m$, $\Box = \partial_m\partial^m$.

The free bilinear action for L^{++} is

$$S^{free} = \frac{1}{2}\int d\zeta^{(-4)}du\,[L^{++}(\zeta,u)]^2 \quad , \quad D^{++}L^{++} = 0 , \qquad (2.8)$$

where $d\zeta^{(-4)}du = d^4x_A \cdot d^2\theta^+ \cdot d^2\bar\theta^+ \cdot du$ is an invariant measure in the analytic superspace. In terms of component fields we have

$$S^{free} = \frac{1}{2}\int d^4x\left(\frac{1}{2}\partial^m L^{ij}\cdot\partial_m L_{ij} - V^m V_m - \psi^i_\alpha \cdot i\partial\!\!\!/^{\alpha\dot\alpha}\bar\psi_{\dot\alpha i} + M\bar M\right). \qquad (2.8')$$

Note that after rescaling $L^{++} \to \tilde L^{++} = \varkappa L^{++}$ ($[L^{++}] = m^{-1}$, $[\varkappa] = m^1$, $[\tilde L^{++}] = m^0$) we get

$$S^{free} = \frac{1}{2\varkappa^2}\int d\zeta^{(-4)}du\,(\tilde L^{++})^2 \qquad (2.8'')$$

so the Lagrangian density $(\tilde L^{++})^2$ becomes a <u>dimensionless</u> function of the <u>dimensionless</u> argument $\tilde L^{++}$ (in what follows we suppress tilde over L^{++}).

Now we are prepared to write down the most general supersymmetric actions for the N=2 tensor multiplet. By dimensionality and analyticity arguments they are**)

*) Note a transparent analogy with $N=0$ case ($\partial_m V^m = 0$).

**) This general form was anticipated in ref.[1].

$$S^{general} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)}du\ \mathcal{L}^{(+4)}(L^{++}, u_i^{\pm})\ , \quad D^{++}L^{++} = 0. \tag{2.9}$$

Here $\mathcal{L}^{(+4)}$ is an arbitrary <u>dimensionless</u> function of L^{++} and harmonics $u_i^{\pm}$ provided the total U(1)-charge is +4. Notice, that no dimensionfull parameters could appear in $\mathcal{L}^{(+4)}$. If so, they would entail ∂_m or D_α derivatives and the component action would then contain interactions with higher derivatives. Explicit harmonics in (2.9) break the SU(2) automorphism group of the N=2 supersymmetry still leaving the latter exact.

3. <u>N=2 Tenser Multiplet Versus Hypermultiplet: N=2 Duality Transformation</u>

Variation of the constrained action (2.9) is not straightforward. There are two possibilities to perform it. The first way - it was employed in ref./1,2,3/ - is to solve the constraint (2.5)

$$L^{++} = (D^+)^2(\bar{D}^+)^2\left[D^{\alpha i}D^j_\alpha\ \phi(z) + \bar{D}^i_{\dot\alpha}\bar{D}^{\dot\alpha j}\bar{\phi}(z)\right]u_i^- u_j^- \tag{3.1}$$

to restore the full superspace measure $d\zeta^{(-4)}du(D^+)^2(\bar{D}^+)^2 = d^{12}z\,du$ and then to vary with respect to an unconstrained prepotential $\Phi(z)$. More instructive is second way that uses heavily properties of the harmonic superspace. It is to introduce the constraint (2.5) into the action with the help of a Lagrange multiplier $\omega(\zeta,u)$

$$S_{L^{++},\omega} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)}du\left[\mathcal{L}^{(+4)}(L^{++},u^{\pm}) + \omega\cdot D^{++}L^{++}\right]. \tag{3.2}$$

The real analytic superfield $\omega(\zeta,u)$ is an unconstrained function of its arguments. The ω -variation yields the original action (2.9). Varying instead with respect to the unconstrained analytic superfield L^{++} one gets

$$\frac{\partial\mathcal{L}^{(+4)}}{\partial L^{++}} = D^{++}\omega. \tag{3.3}$$

This is an algebraic equation that can always be solved for L^{++}

$$L^{++} = L^{++}(D^{++}\omega, u^{\pm}) \tag{3.4}$$

provided $\mathcal{L}^{(+4)}$ is nondegenerate. Substituting (3.4) back into eq. (3.2) we arrive at the ω -action

$$S_\omega = \frac{1}{\varkappa^2}\int d\zeta^{(-4)}du\left\{\mathcal{L}^{(+4)}\left[L^{++}(D^{++}\omega,u^{\pm}),u^{\pm}\right] - D^{++}\omega\cdot L^{++}(D^{++}\omega,u^{\pm})\right\} \tag{3.5}$$

(we have also integrated by parts the secnnd term in (3.2)). For example, starting with the free L^{++} -action (2.8) we obtain $L^{++} = D^{++}\omega$ and

$$S^{free}_\omega = -\frac{1}{2\varkappa^2}\int d\zeta^{(-4)}du\,(D^{++}\omega)^2. \tag{3.6}$$

So, we see that the Lagrange multiplier $\omega(\zeta,u)$ becomes simply a matter ω -hypermultiplet. Its free action is given by eq.(3.6) (cf./4/).

Comparing the general L^{++}-action (2.9) and its dual transform (3.5) one observes that <u>the most general self-interactions of the N=2 tensor multiplet are equivalent to a restricted class of ω - hypermultiplet self-interactions</u>*).

Indeed, action (3.4) contains ω only as $D^{++}\omega$ having thus evident invariance under (K is an arbitrary real constant)

$$\omega \to \omega + K \tag{3.7}$$

that in turn implies existence of at least one Killing vector for the σ -model manifolds corresponding to the L^{++} self-couplings/12/. This is of course due to the presence in L^{++} of the notoph's constrained field strength V^m. The latter is dual to a gradient of scalar $\partial^m\varphi(x)$ invariant under $\varphi(x)\to\varphi(x)+K$.

At the same time supersymmetric ω -actions generally depend on ω (containing the scalar $\varphi(x)$ as its lowest component) itself and do not generally have the symmetry (3.7) or any extra symmetries at all.

A careful reader can trace a remarkable analogy with N=0 and N=1 cases. The analytic superfield $\omega(\zeta,u)$ is similar to N=0 scalar $\varphi(x)$ and to N=1 chiral $\phi(x_L,\theta)$ (super) fields while the N=2 tensor multiplet field strength L^{++} ($D^{++}L^{++}=0$) is similar to the N=0 tensor multiplet field strength V^m ($\partial_m V^m = 0$) and to the N=1 one L ($D^\alpha D_\alpha L = \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}L = 0$). The former are unconstrained (super) fields defined on corresponding (super) spaces while the latter are constrained objects. The general self-interactions of the latter are equivalent via duality transformations to restricted ones of the former **).

We conclude this section with two comments.

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*) In a forthcoming paper we extend this statement for other off-shell N=2 matter multiplets with a finite number of auxiliary fields /10,11/. In fact, their self-coupling are equivalent to the L^{++} ones.

**) Contrary to notes of ref./13/.

i) The action (3.2) can be considered as a first order formalism action for the ω-hypermultiplet. As shown in ref./5,14/ one can change field variables

$$q^+_i(\zeta,u) = -u^+_i\,\omega(\zeta,u) + u^-_i L^{++}(\zeta,u)\,, \qquad \widetilde{(q^+_i)} = q^{+i} = \varepsilon^{ij} q^+_j \tag{3.8}$$

$$\omega = u^{-i} q^+_i\,, \quad L^{++} = u^{+i} q^+_i \tag{3.9}$$

thus passing to q^+-hypermultiplet description. After substituting (3.9) into the action (3.2) we get

$$S_{q^+} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)} du \left[\mathcal{L}^{(+4)}(u^{+i}q^+_i, u^\pm) - \frac{1}{2}(u^{+i}q^+_i)^2 - \frac{1}{2} q^{+i} D^{++} q^+_i\right]. \tag{3.10}$$

For example, the free L^{++}-action is equivalent to a free q^+-action

$$S^{free}_{q^+} = -\frac{1}{2\varkappa^2}\int d\zeta^{(-4)} du\, q^{+i} D^{++} q^+_i\,. \tag{3.11}$$

The symmetry (3.7) is replaced by

$$q^+_i \rightarrow q^+_i + K\cdot u^+_i\,. \tag{3.12}$$

In what follows we shall extensively use this q^+-hypermultiplet.

ii) Including several tensor multiplets does not change the above considerations.

4. The Improved Tensor Multiplet

In this section we shall discuss conformal properties of the N=2 tensor multiplet. After constructing a conformally invariant action for it in the harmonic superspace we shall prove equivalence of this theory to that of a free q^+-hypermultiplet by performing a proper duality transformation.

4.1. To start with, the defining constraint (2.5)

$$D^{++} L^{++}(\zeta,u) = 0 \tag{4.1}$$

is invariant under rigid transformations of the N=2 superconformal group SU(2,2/2). A detailed discussion of this group in the harmonic superspace was given elsewhere/7/. So, we confine ourselves here to a few relevant formulae.

First, the field strength L^{++} has to transform infinitesimally as *)

$$\delta L^{++} = L^{++'}(\zeta',u') - L^{++}(\zeta,u) = 2\Lambda\cdot L^{++}(\zeta,u)\,. \tag{4.2}$$

*) Of course this agrees with component field transformations of ref./6

where Λ is related to the Berezinian of the analytic coordinates change/7/

$$\mathrm{Ber}\,\frac{\partial(\zeta',u')}{\partial(\zeta,u)} = 1 - 2\Lambda \;\Rightarrow\; \delta\left(d\zeta^{(-4)}du\right) = -2\Lambda\cdot d\zeta^{(-4)}du\,. \tag{4.3}$$

Second, the harmonic derivative D^{++} transforms

$$\delta D^{++} = -\Lambda^{++}\cdot D^0\,, \tag{4.4}$$

where

$$\Lambda^{++} = D^{++}\Lambda \quad (a), \qquad D^{++}\Lambda^{++} = (D^{++})^2\Lambda = 0\,. \quad (b) \tag{4.5}$$

Now it is easy to check the conformal invariance of (4.1)

$$\delta(D^{++}L^{++}) = -\Lambda^{++}\cdot D^0 L^{++} + D^{++}(2\Lambda L^{++}) = 2\Lambda\cdot D^{++}L^{++} = 0\,.$$

Here we used above equations and eq.(2.4). In fact, the conformal weight of L^{++} in (4.2) is fixed just by requiring (4.1) to be superconformally invariant.

Note that the free bilinear action (2.8) is not conformally invariant *). Having in mind future applications in N=2 supergravity we proceed now to constructing the improved tensor multiplet action with desired conformal invariance.

4.2. To give an idea of the construction it is instructive to recall the N=1 improved tensor multiplet action /15/

$$S^{impr}_{N=1} = \frac{1}{\varkappa^2}\int d^4x\, d^4\theta\; L\ln L \;(a)\,, \qquad D^2 L = \bar{D}^2 L = 0\,. \;(b) \tag{4.6}$$

The action (a) and accompanying constraint (b) are invariant under the N=1 rigid superconformal group SU(2,2/1)

*) At the same time eq.(2.8) is invariant under the $SU(2)_c$ subgroup of SU(2,2/2) having the parameters

$$\Lambda = \Lambda^{(ij)} u^+_i u^-_j = \frac{1}{2} D^{++}(\Lambda^{ij} u^-_i u^-_j)\,, \qquad \Lambda^{ij} = const\,.$$

Indeed

$$\delta S^{free} = \frac{1}{2\varkappa^2}\int d\zeta^{(-4)} du\,(-2\Lambda + 4\Lambda)(L^{++})^2 = -\frac{1}{\varkappa^2}\int d\zeta^{(-4)} du\, \Lambda^{ij} u^-_i u^-_j L^{++}\cdot D^{++} L^{++} = 0\,.$$

This invariance is not accidental. In fact, for the N=2 tensor multiplet this $SU(2)_c$ coincides with $SU(2)_A$ of supersymmetry automorphisms (rotating indices $i, j \ldots$) when applied to the superfield $L^{ij}(x)$ (2.1) or to component fields. These SU(2)'s are realized differently on harmonics /7/ and correspondingly have different appearence when applied to harmonic superfields.

$$\delta L = (\lambda+\bar\lambda)\cdot L \quad , \; D_\alpha \bar\lambda = \bar D_{\dot\alpha}\lambda = 0, \tag{4.7}$$

$$\delta\,(d^4x d^4\theta) = -(\lambda+\bar\lambda)\, d^4x\, d^4\theta . \tag{4.8}$$

The invariant (4.6a) could be derived as follows. Notice that as in the N=2 case neither the naive bilinear action nor any series in powers of L are invariant. So, we are compelled to single a constant part $C \neq 0$ out of L ($[L]=m^0$)

$$L(x,\theta,\bar\theta) = C + \ell(x,\theta,\bar\theta) \tag{4.9}$$

and to try to expand the Lagrange density in powers of the deviation $\ell(x,\theta,\bar\theta)$. The superfield $\ell(x,\theta,\bar\theta)$ obeys the same constraint (4.6b) (because the constant part does) and transforms inhomogeneously under SU(2,2/1)

$$\delta\ell = c\cdot(\lambda+\bar\lambda) + (\lambda+\bar\lambda)\cdot\ell . \tag{4.10}$$

Now in the action supposed

$$S^{trial} = \frac{1}{\varkappa^2}\int d^4x d^4\theta \sum_{n=2}^{\infty} a_n \ell^n \tag{4.11}$$

the bilinear term is invariant under additive part of (4.10) while multiplicative part of (4.10) requires additive variation of the trilinear term and so on. So we get

$$a_{n+1} = -\frac{n-1}{c(n+1)}\cdot a_n \;\Rightarrow\; a_n = \frac{2(-c)^{2-n}}{n(n-1)}\cdot a_2 \tag{4.12}$$

$$\Rightarrow S^{impr}_{N=1} = \frac{2a_2c^2}{\varkappa^2}\int d^4x d^4\theta \cdot \sum_{n=2}^{\infty}\frac{(-1)^n}{n(n-1)}\cdot\left(\frac{\ell}{c}\right)^n =$$
$$= \frac{2a_2c}{\varkappa^2}\int d^4x d^4\theta\left\{(c+\ell)\cdot\ln(c+\ell) - [(c+\ell)\cdot\ln c + \ell]\right\}. \tag{4.13}$$

The terms in square brackets do not contribute due to eq.(4.6b). Note that with $a_2 = (2C)^{-1}$ action (4.13) depends only on L (not C) and coincides with eq.(4.6a).

4.3. Now we return to the N=2 case. The first step is to single out of L^{++} some constant-like part (of U(1)-charge +2) that obeys the constraint (4.1)

$$L^{++}(\zeta,u) = c^{++} + \ell^{++}(\zeta,u), \tag{4.14}$$

$$c^{++} = c^{ij}u^+_i u^+_j\,,\; \overline{(c^{ij})} = \varepsilon_{ik}\varepsilon_{js}c^{ks}\,, \quad c^2 = \tfrac{1}{2}c^{ij}c_{ij} \neq 0 . \tag{4.15}$$

The real analytic superfield ℓ^{++} is constrained

$$D^{++}\ell^{++}(\zeta,u) = 0 \tag{4.16}$$

and transforms as follows

$$\delta\ell^{++} \equiv \ell^{++\prime}(\zeta',u') - \ell^{++}(\zeta,u) = 2\,(\Lambda c^{++} - \Lambda^{++}c^{+-}) + 2\Lambda\cdot\ell^{++} . \tag{4.17}$$

Here we used eq.(4.2) and $u^{\pm}_i$ superconformal transformation law /7/

$$\delta u^+_i = \Lambda^{++}u^-_i \;\Rightarrow\; \delta c^{++} = 2\Lambda^{++}c^{+-} \quad , \; c^{+-} = c^{(ij)}u^+_i u^+_j \; ?$$
$$\delta u^-_i = 0 . \tag{4.18}$$

The power expansion of the desired action

$$S^{trial} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)}du \sum_{n=2}^{\infty} b_n (\ell^{++})^n\cdot(c^{--})^{n-2} \tag{4.19}$$

contains a constant-like quantity of U(1)-charge -2

$$C^{--} = c^{ij}u^-_i u^-_j \;, \quad \delta c^{--} = 0 \tag{4.20}$$

so that each term of the Lagrange density in (4.19) has U(1)-charge (+4). As in the N=1 case the bilinear term is invariant under additive part of (4.17) (taking into account eq.(4.16)) while multiplicative part of (4.17) requires additive variation of the trilinear term, etc. Thus the invariance of the action (4.19) implies a recurrent relation for b_n. Introducing a convenient quantity $f^{++}(\zeta,u)$

$$f^{++} = \frac{2\ell^{++}}{1+(1+\ell^{++}c^{--}/c^2)^{1/2}} \quad , \; \ell^{++} = f^{++}\cdot(1+f^{++}c^{--}/4c^2) \tag{4.21}$$

we finally get the sought action in a rather simple form (for details of derivation see Appendix A).

$$S^{impr} = \frac{1}{2c\varkappa^2}\int d\zeta^{(-4)}du\,(f^{++})^2 \quad (a), \quad D^{++}\left[f^{++} + (f^{++})^2c^{--}/4c^2\right] = 0. \tag{4.22}$$

Now we can check the superconformal invariance of the action (4.22) straightforwardly without using expansion in powers of ℓ^{++}. To this end we use eqs.(4.3), (4.5), identities (A.1, A.2) and the superconformal transformation law for f^{++}

$$\delta f^{++} = (1+f^{++}c^{--}/2c^2)^{-1}\cdot\left[2(\Lambda c^{++} - \Lambda^{++}c^{+-}) + 2\Lambda f^{++}\cdot(1+f^{++}c^{--}/4c^2)\right]. \tag{4.23}$$

As in the N=1 case the Lagrange density acquires a total (harmonic) derivative

$$\delta S^{impr} = \frac{2}{c\varkappa^2}\int d\zeta^{(-4)}du\, D^{++}\left[f^{++}(\Lambda c^{+-} - \Lambda^{++}c^{--})\right] = 0 . \qquad (4.24)$$

The component field action corresponding to the superfield one (4.22) agrees completely with the improved N=2 tensor multiplet action derived several years ago in ref.[6]. The procedure of descending to component fields involves integration over harmonics $u^{\pm}_i$ and is described in Appendix B.

One can also reduce our result to that of ref.[16,3]. To achieve this one has i) to use eq.(3.1) ii) to pass to the central basis of the harmonic superspace and, finally, iii) to integrate over harmonics using integrals of Appendix B. To make a comparison with results of ref.[1,2] one has to choose a different parametrization of the sphere $S^2 \simeq SU(2)/U(1)$; we shall not concern this matter here.

4.4. There is an interesting peculiarity in action (4.22). It is written down in terms of intermediary quantity f^{++} . Rewriting in terms of the N=2 tensor multiplet field strength L^{++} itself

$$S^{impr} = \frac{1}{2c\varkappa^2}\int d\zeta^{(-4)}du \frac{(2L^{++}-2c^{++})^2}{\{1+[1+(L^{++}-c^{++})c^{--}/c^2]^{1/2}\}^2} , \quad D^{++}L^{++}=0 \qquad (4.25)$$

we find out that unlike the N=1 case (see (4.6a) or (4.13)) the Lagrange density in (4.25) depends explicitly on the constant c^{ij} . A careful reader may wonder what means this "isotriplet coupling constant". The answer is as follows.

i) Surprisingly enough the action (4.25) unlike its Lagrange density is independent of any particular choice of c^{ij} , $c \neq 0$. To see this, we use scale and $SU(2)_c$ symmetries of eq.(4.25) which are subgroups of superconformal group.

Making a finite scale transformation corresponding to infinitesimal one (4.2), (4.3) with parameter $\Lambda =$ const, $\Lambda^{++}=0$ we get

$$S^{impr} = \frac{1}{2c\varkappa^2}\int d\zeta^{(-4)}du \frac{e^{-2\Lambda}[2e^{2\Lambda}L^{++}-2c^{++}]^2}{\{1+[1+(e^{2\Lambda}L^{++}-c^{++})c^{--}/c^2]^{1/2}\}^2} . \qquad (4.26)$$

So, all the Λ-dependence can be absorbed into the constant, $c^{ij} \to e^{-2\Lambda}c^{ij}$

$$S^{impr}(L^{++}, c^{ij}) = S^{impr}(L^{++}, e^{-2\Lambda}c^{ij}) \qquad (4.27)$$

and the action is the same for c^{ij} that differ in norm c . In what follows we put

$$c = 1 . \qquad (4.28)$$

Now it remains to prove that

$$S^{impr}(L^{++}, c^{ij}) = S^{impr}(L^{++}, \Lambda^i{}_k c^{jk}), \qquad (4.29)$$

where $\Lambda^i{}_j$ is a unimodular matrix with constant entries. This proof goes along the above lines with using $SU(2)_c$ symmetry (see footnote on page 7) instead of scaling.

ii) Nevertheless the presence of this unessential constant c^{ij} in the Lagrange density (4.25) is inavoidable. It is due to a topological nontriviality of the improved N=2 tensor multiplet action discovered at the component field level in ref.[6]: when the Lagrange density is written down in terms of the notoph field strength V^m it exhibits a Dirac-like string of singularities that is parametrized by constant c^{ij} . Being rewritten in terms of the notoph potential E_{mn} it becomes independent of c^{ij} .

4.5. It was demonstrated in ref.[6] that the nonlinear σ -model corresponding to the component field action (B.16) has the vanishing Riemann tensor. This is a strong evidence for the absence of interactions in the theory.

The harmonic superfield approach has a privilege to allow an elegant proof that the improved N=2 tensor multiplet theory is equivalent by a duality transformation to a free theory.

The proof goes as follows (compare sect. 3). We implement the constraint (4.22b) in the action (4.22a) by means of a Lagrange multiplier $\omega(\zeta,u)$

$$S = \frac{1}{\varkappa^2}\int d\zeta^{(-4)}du \left\{\frac{1}{2}(f^{++})^2 + \omega \cdot D^{++}\left[f^{++} + \frac{1}{4}(f^{++})^2 c^{--}\right]\right\} . \qquad (4.30)$$

To maintain the superconformal invariance of this action ω would transform as

$$\delta\omega = (1+f^{++}c^{--}/2)^{-1} \cdot (\Lambda c^{+-} - \Lambda^{++}c^{--}) . \qquad (4.31)$$

We are to show the equivalence of this theory to that of a free q^+ -hypermultiplet (3.11)

$$S^{free}_{q^+} = -\frac{1}{2\varkappa^2}\int d\zeta^{(-4)}du\; q^{+i} D^{++} q^+_i \,. \qquad (4.32)$$

As explained in ref. /7/ the action (4.32) is superconformally invariant with q^+ transforming as

$$\delta q^+_i = \Lambda q^+_i \,. \qquad (4.33)$$

Comparison of transformation laws (4.23), (4.31) and (4.33) suggests the following nondegenerate change of superfield variables *)

$$q^+_i = (-2u^+_i + c_{ij}u^{-j}f^{++})\cdot\sin\frac{\omega}{2} + (2c_{ij}u^{+j} + u^-_i f^{++})\cdot\cos\frac{\omega}{2} \,. \qquad (4.34)$$

Now substituting (4.34) into the free action (4.32) we indeed arrive at the action (4.30). This finishes the proof of the equivalence.

5. Coupling to Conformal Supergravity

The main interest to the improved N=2 tensor multiplet stems from a possibility to use it as a compensator in constructing Einstein supergravities on the basis of the conformal one /6/. It is the harmonic superspace that allows to define the gauge group and the unconstrained prepotentials of the N=2 conformal supergravity /7/.

In this section we shall derive the action for the tensor multiplet (and for q^+-hypermultiplet) in a conformal supergravity background. To begin with, the defining constraint reads now

$$(\mathcal{D}^{++} + \Gamma^{++})L^{++}(\zeta,u) = 0 \,. \qquad (5.1)$$

where the supercovariant harmonic derivative $\mathcal{D}^{++}$ contains the analytic prepotentials $H^{++M}(\zeta,u)$ with $M = ++, m, \mu+, \dot\mu+$ (when acting on analytic superfields)

$$\mathcal{D}^{++} = \partial^{++} + H^{++M}\partial_M \;; \qquad \partial^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} ,\; \partial_{++} = \partial^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} \qquad (5.2)$$

and so does the harmonic connection

$$\Gamma^{++}(\zeta,u) = (-1)^{P(M)}\partial_M H^{++M}(\zeta,u) \;; \qquad \begin{array}{l} p(++) = p(m) = 0, \\ p(\mu+) = p(\dot\mu+) = 1 \end{array} \,. \qquad (5.3)$$

---

*) Its Jacobian

$$\det\frac{\partial(q^+_1, q^+_2)}{\partial(\omega, f^{++})} = \frac{\partial q^{+i}}{\partial f^{++}}\cdot\frac{\partial q^+_i}{\partial\omega} = 1 + \frac{1}{2}f^{++}c^{--}$$

is nonzero in some neighbourhood of $\omega = f^{++} = 0$.

The constraint (5.1) is invariant under the superconformal gauge transformations. This can be checked by using

$$\delta\mathcal{D}^{++} = -\Lambda^{++}\cdot D^0 \quad (a) \quad , \quad \delta\Gamma^{++} = 2(\Lambda^{++} - \mathcal{D}^{++}\Lambda) \quad (b) , \qquad (5.4)$$

$$\delta L^{++} = 2\Lambda L^{++}. \qquad (5.5)$$

The gauge parameters $\Lambda(\zeta,u)$ and $\Lambda^{++}(\zeta,u)$ have the same origin as in the rigid case *)

$$\mathrm{Ber}\,\frac{\partial(\zeta',u')}{\partial(\zeta,u)} = 1 - 2\Lambda \;\Rightarrow\; \delta(d\zeta^{(-4)}du) = -2\Lambda\, d\zeta^{(-4)}du \qquad (5.6)$$

$$\delta u^+_i = \Lambda^{++}u^-_i \quad , \quad \delta u^-_i = 0 \,. \qquad (5.7)$$

The simplest way to derive an invariant L^{++}-action is to descend from the invariant q^+-hypermultiplet action. The latter is given by

$$S_{q^+} = -\frac{1}{2\varkappa^2}\int d\zeta^{(-4)}du\; q^{+i}\cdot\mathcal{D}^{++}q^+_i \qquad (5.8)$$

and it is obviously invariant under (5.4a), (5.6) and

$$\delta q^+_i = \Lambda q^+_i \,. \qquad (5.9)$$

Note that in eq.(5.8) there is no need in the connection Γ^{++} because $q^{+i}q^+_i = \varepsilon^{ij}q^+_i q^+_j = 0$. Let us now change variables exactly as in (4.34)

$$q^+_i = [-2u^+_i + c_{ij}u^{-j}f^{++}]\cdot\sin\frac{\omega}{2} + [2c_{ij}u^{+j} + u^-_i f^{++}]\cdot\cos\frac{\omega}{2} \qquad (5.10)$$

and substitute this into the action (5.8). After some algebra we obtain

$$S_{\omega,f^{++}} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)}du\,\Big\{\frac{1}{2}(f^{++})^2 - \Gamma^{++}f^{++}c^{+-} - 2H^{(+4)}\cdot(1 + f^{++}c^{--}) + $$
$$+ \omega\cdot(\mathcal{D}^{++} + \Gamma^{++})\cdot[c^{++} + f^{++} + \frac{1}{4}(f^{++})^2c^{--}]\Big\} \,. \qquad (5.11)$$

Variation with respect to ω yields the constraint

$$(\mathcal{D}^{++} + \Gamma^{++})[c^{++} + f^{++} + \frac{1}{4}(f^{++})^2c^{--}] = 0 \,. \qquad (5.12)$$

Comparing eqs. (5.1) and (5.12) we identify (as in the rigid case, see eqs. (4.14), (4.21))

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*) However, in the local case there are no relations like (4.5).

$$L^{++} = c^{++} + f^{++} + \frac{1}{4}(f^{++})^2 c^{--} \,, \qquad (c^2 = 1). \tag{5.13}$$

Finally, we get the sought action of the improved N=2 tensor multiplet in the superconformal background in the form

$$S^{impr} = \frac{1}{\varkappa^2} \int d\zeta^{(-4)} du \left[\frac{1}{2}(f^{++})^2 - \Gamma^{++} f^{++} c^{+-} - 2H^{(+4)} (1 + f^{++} c^{--}) \right] \tag{5.14}$$

with f^{++} related to L^{++} by eq.(5.13), constrained by (5.1) (or (5.12)) and transforming as

$$\delta f^{++} = (1 + f^{++} c^{--}/2)^{-1} \left[2(\Lambda c^{++} - \Lambda^{++} c^{+-}) + 2\Lambda f^{++} (1 + f^{++} c^{--}/4) \right]. \tag{5.15}$$

The local superconformal invariance of the action (5.14) is clear from the derivation procedure. It can be also checked directly *) using eqs. (5.4b), (5.6), (5.7), (5.12), (5.15), identities (A.1), (A.2) and

$$\delta H^{(+4)} = \mathcal{D}^{++} \Lambda^{++} \tag{5.16}$$

which is a consequence of (5.4a).

6. Conclusions

So, we have been convinced that the harmonic superspace approach is an appropriate framework for a manifestly supersymmetric description of N=2 tensor multiplets. In particular, we have recast the results of ref.[1,2,3]. A real advantage of this approach is unconstrained formulation of N=2 matter hypermultiplets ($\omega(\zeta,u)$, $q^+_i(\zeta,u)$). Using the latter we have performed (for the first time) N=2 superfield duality transformations to prove the equivalence of the general N=2 tensor multiplets self-couplings to a restricted class of ω (or q^+) ones. For the improved tensor multiplet action a certain duality transformation has led us to the free q^+-hypermultiplet action. These equivalence proofs have been given at the classical level. Their extension to the quantum level along the lines of ref.[17] seems possible.

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*) The Lagrange density acquires total derivatives. So, the action is invariant

$$\delta S^{impr} = \frac{1}{\varkappa^2} \int d\zeta^{(-4)} du \Big\{ 2\partial^{++} \left[\Lambda f^{++} c^{+-} - \Lambda^{++} (1 + f^{++} c^{--}) \right] +$$
$$+ 2 \cdot (-1)^{p(M)} \partial_M \left[H^{++M} (\Lambda f^{++} c^{+-} - \Lambda^{++} - \Lambda^{++} f^{++} c^{--}) \right] \Big\} = 0 .$$

More important is finding superspace coupling of the q^+-hypermultiplet and of the improved tensor multiplet to the N=2 conformal supergravity. The latter was known for component fields and was used to obtain the third minimal version of Einstein supergravity [6] while the former is essentially new. Both can be used for constructing Einstein supergravities in superspace. To this end one has to add to actions (5.8) or (5.14) the action of the so-called "almost simple" N=2 supergravity[18]. The latter is simply the action of N=2 Maxwell multiplet in the conformal supergravity background. Completing the derivation of N=2 Einstein supergravity actions in superspace will be done elsewhere.

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Appendix A

Here we give some details of derivation of the improved N=2 tensor multiplet action (4.22). We shall need the following identities

$$D^{++} c^{+-} = c^{++} \,, \quad D^{++} c^{--} = 2 c^{+-} \tag{A.1}$$

$$c^{++} c^{--} - (c^{+-})^2 = \frac{1}{2} c^{ij} c_{ij} = c^2 \tag{A.2}$$

$$c^{++}(c^{--})^{n-2} = [2(n-1)(2n-3)]^{-1} (D^{++})^2 (c^{--})^{n-1} + 2(n-2)(2n-3)^{-1} c^2 (c^{--})^{n-3} \tag{A.3}$$

$$(c^{+-})^2 (c^{--})^{n-3} = [2(n-1)(2n-3)]^{-1} (D^{++})^2 (c^{--})^{n-1} - (2n-3)^{-1} c^2 (c^{--})^{n-3} \,, \quad n = 3,4,5\ldots \tag{A.4}$$

Identity (A.2) can be deduced using the harmonics completeness condition $u^{+i} u^-_j - u^+_j u^{-i} = \delta^i_j$, while (A.3) and (A.4) are consequences of (A.1) and (A.2).

Variation of the trial action (4.19) under superconformal transformations (4.3), (4.17) and (4.20) is

$$\delta S^{trial} = \frac{1}{\varkappa^2} \int d\zeta^{(-4)} du \sum_{n=2}^{\infty} \left\{ 2\Lambda (n-1) b_n (c^{++})^n (c^{--})^{n-2} + 2n b_n (\Lambda c^{++} - D^{++}\Lambda c^{+-}) (c^{++})^{n-1} (c^{--})^{n-2} \right\}. \tag{A.5}$$

In the last term we have used eq.(4.5a). Integrating it by parts with the help of eq.(4.16) and identities (A.1) we obtain

$$\delta S^{trial} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)} du\cdot 2\Lambda\cdot\sum_{n=2}^{\infty}\left\{(n-1)b_n(\ell^{++})^n(c^{--})^{n-2} + 2nb_n(\ell^{++})^{n-1}\cdot\left[c^{++}(c^{--})^{n-2} + (n-2)(c^{+-})^2(c^{--})^{n-3}\right]\right\} \quad (A.6)$$

Next step is to substitute eqs. (A.3), (A.4) into (A.6) and to note that the terms containing $(D^{++})^2$ do not contribute due to eq.(4.5b). So, we arrive at

$$\delta S^{trial} = \frac{1}{\varkappa^2}\int d\zeta^{(-4)} du\cdot 2\Lambda \sum_{n=2}^{\infty}\left\{(\ell^{++})^n(c^{--})^{n-2}\cdot\left[(n-1)b_n + \frac{2c^2(n^2-1)}{(2n-1)}\cdot b_{n+1}\right]\right\}. \quad (A.7)$$

Hence, the invariance of S^{trial} requires

$$b_{n+1} = -\frac{2n-1}{2c^2(n+1)}\cdot b_n \Rightarrow b_n = (-4c^2)^{2-n}\cdot\frac{(2n-2)!}{n!(n-1)!}\cdot b_2 \ . \quad (A.8)$$

Putting $b_2 = (2c)^{-1}$ (as explained in sect. 4.4 the factor c^{-1} is needed to achieve c^2-independence of the action) and summing the series (see, for example ref. /19/)

$$\sum_{n=2}^{\infty}\frac{(2n-2)!\,x^{n+2}}{n!(n-1)!} = \frac{2x^2}{1-2x+\sqrt{1-4x}} = \left(\frac{2x}{1+\sqrt{1-4x}}\right)^2 \quad (A.9)$$

we finally get

$$S^{impr} = \frac{1}{2c\varkappa^2}\int d\zeta^{(-4)} du\left[\frac{2\ell^{++}}{1+(1+\ell^{++}c^{--}/c^2)^{1/2}}\right]^2 . \quad (A.10)$$

Appendix B

Here we shall find the component field action for the improved N=2 tensor multiplet starting with the superspace one (eq.(A.10)). Substituting in the latter θ-expansion (2.7) and putting

$$L^{ij}(x) = c^{ij} + \phi^{ij}(x) \quad , \quad \phi^{++} = \phi^{ij}u^+_i u^+_j \quad (B.1)$$

we obtain after θ-integration $(\hat{c}^{--} = c^{--}/c^2)$

$$S^{impr} = \frac{1}{2c\varkappa^2}\int dx du\left\{(1+\phi^{++}\hat{c}^{--})^{-3/2}\cdot\left(\frac{1}{2}\partial^m L^{ij}\cdot\partial_m L_{ij} - \psi^i_\alpha i\partial^{\dot\alpha\alpha}\bar\psi_{\dot\alpha i} - V^m V_m + \right.\right.$$
$$\left. + M\bar M - 2V^m\cdot\partial_m L^{ij}u^+_i u^-_j\right) + \frac{3}{2}\hat{c}^{--}u^+_i u^+_j\cdot(1+\phi^{++}\hat{c}^{--})^{-5/2}\cdot\left(M\bar\psi^i_{\dot\alpha}\bar\psi^{\dot\alpha j} + \right. \quad (B.2)$$
$$\left.\left. + \bar M\psi^{\alpha i}\psi^j_\alpha + 2iV^{\dot\alpha\alpha}\psi^i_\alpha\bar\psi^j_{\dot\alpha} + i\psi^{(i}_\alpha\bar\psi^{r)}_{\dot\alpha}\cdot\partial^{\dot\alpha\alpha}L^j_r\right) + \frac{15}{4}(\hat{c}^{--})^2(1+\phi^{++}\hat{c}^{--})^{-7/2}\cdot\psi^{\alpha i}\psi^j_\alpha\bar\psi^r_{\dot\alpha}\bar\psi^{\dot\alpha s}u^+_i u^+_j u^+_r u^+_s\right\}.$$

Now we have to compute integrals over harmonics. This could be done straightforwardly using the explicit parametrisation of $SU(2)/U(1) = S^2$ harmonics (see, for example ref. /20/)

$$u^+_1 = i\sin\theta/2\cdot\exp(-i\varphi/2) \quad , \quad u^-_1 = \cos\theta/2\cdot\exp(-i\varphi/2)$$
$$u^+_2 = \cos\theta/2\cdot\exp(i\varphi/2) \quad , \quad u^-_2 = i\sin\theta/2\cdot\exp(i\varphi/2) \quad (B.3)$$

and the corresponding measure on S^2

$$\int du = \frac{1}{4\pi}\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\varphi \ . \quad (B.4)$$

But the simplest way is to employ $SU(2)_c$ symmetry of (B.2). This subgroup of SU(2,2/2) is realized as follows (see also footnote on p.7)

$$\delta u^+_i = \Lambda^{++}\cdot u^-_i \quad , \quad \delta u^-_i = 0 \quad , \quad \delta(du) = 2\Lambda du$$
$$\delta c^{--} = 0 \quad , \quad \delta\phi^{++} = 2(\Lambda c^{++} - \Lambda^{++}c^{+-} + \Lambda\phi^{++}) \quad (B.5)$$
$$\Lambda = \Lambda^{(ij)}u^+_i u^-_j \quad , \quad \Lambda^{++} = \Lambda^{ij}u^+_i u^+_j = \partial^{++}\Lambda \ .$$

We observe that the first type of harmonic integrals encountered in (B.2)

$$I_1 = \int du\cdot(1+\phi^{++}c^{--}/c^2)^{-3/2} \quad (B.6)$$

is $SU(2)_c$ invariant. Now we use the automorphism $SU(2)_A$ group to cast c^{ij} in the form

$$c^{11} = c^{22} = 0 \quad , \quad c^{12} = ic \quad (B.7)$$

and then $SU(2)_c$ (which does not change c^{ij} (B.5)) to rotate the tensor $c^{ij} + \phi^{ij} = L^{ij}$ into

$$c^{11} + \phi^{11} = c^{22} + \phi^{22} = 0 \ . \quad (B.8)$$

Hence

$$\phi^{11} = \phi^{22} = 0 \ , \quad \phi^{12} = i(L-c) \ ; \quad L \equiv \left(\frac{1}{2}L^{ij}L_{ij}\right)^{1/2} . \quad (B.9)$$

In this frame

$$I_1 = \int du\cdot(1+4u^+_1u^+_2u^-_1u^-_2\,\phi^{12}\cdot c^{12}/c^2)^{-3/2} =$$
$$= \frac{1}{4\pi}\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\varphi\,(1 - i\sin^2\theta\,\phi^{12}/c)^{-3/2} = c/L \ . \quad (B.10)$$

The second type of harmonic integrals in eq. (B.2)

$$I_2 = -2\int dx du\cdot(1+\phi^{++}c^{--}/c^2)^{-3/2}\cdot V^m\cdot\partial_m L^{ij}\cdot u^+_i u^-_j \quad (B.11)$$

is $SU(2)_c$ invariant only with the account of x-integration and the constraint $\partial_m V^m = 0$. To get an $SU(2)_c$ covariant u-integral we recall that $V^m = \frac{1}{2}\varepsilon^{mnkl}\partial_n E_{kl}$ and integrate by parts in (B.11)

$$I_2 = -\frac{3}{4}\int dx du\,\varepsilon^{mnkl}E_{kl}\cdot\partial_n L^{(ir}\cdot\partial_m L^{j)}_r\cdot u^+_i u^+_j\cdot(1+\phi^{++}c^{--}/c^2)^{-5/2}\cdot c^{--}/c^2 \ . \quad (B.12)$$

Here we used also the identity

$$u^+_i u^+_j u^+_{(r}u^-_{s)} = u^+_{(i}u^+_j u^+_r u^-_{s)} + \frac{1}{4}\left[\varepsilon_{i(r}u^+_{s)}u^+_j + \varepsilon_{j(r}u^+_{s)}u^+_i\right] . \quad (B.13)$$

The integral over harmonics in (B.12) (appearing also in the second line of (B.2)) can be derived from (B.6), (B.10)

$$\int du\cdot(1+\phi^{++}c^{--}/c^2)^{-5/2}\cdot u^+_i u^+_j c^{--}/c^2 = -\frac{2}{3}\frac{\partial}{\partial\phi^{ij}} I_1\Big|_c = \frac{1}{3}\frac{L_{ij}c}{L^3}\ . \tag{B.14}$$

The last integral in (B.2) is calculated in a similar way

$$\int du\,(1+\phi^{++}c^{--}/c^2)^{-7/2}\, u^+_i u^+_j u^+_r u^+_s\,(c^{--})^2/c^4 = \frac{1}{5}\frac{L_{(ij}L_{rs)}\cdot c}{L^5}\ . \tag{B.15}$$

Finally, we collect equations above to reproduce the component field action for the improved N=2 tensor multiplet

$$\begin{aligned} S^{impr} = \frac{1}{2\varkappa^2}\int dx \Big\{ &\Big(\frac{1}{2}\partial^m L^{ij}\cdot\partial_m L_{ij} - \psi^i_\alpha i\partial^{\alpha\dot\alpha}\bar\psi_{\dot\alpha i} - V^m V_m + M\bar M\Big)\cdot L^{-1} + \\ &+ \frac{L_{ij}}{2L^3}\Big(M\bar\psi^i_{\dot\alpha}\bar\psi^{\dot\alpha j} + \bar M\psi^{\alpha i}\psi^j_\alpha + 2iV^{\alpha\dot\alpha}\psi^i_\alpha\bar\psi^j_{\dot\alpha} + i\psi^i_\alpha\bar\psi^r_{\dot\alpha}\cdot\partial^{\alpha\dot\alpha}L^j_r - \\ &- \frac{1}{2}\varepsilon^{mnkl}\cdot E_{kl}\,\partial_n L^{ir}\cdot\partial_m L^j_r\Big) + \frac{3\,L_{(ij}L_{rs)}}{4\,L^5}\,\bar\psi^i_{\dot\alpha}\bar\psi^{\dot\alpha j}\psi^{\alpha r}\psi^s_\alpha \Big\}\ . \end{aligned} \tag{B.16}$$

Note, that here the constant c^{ij} disappears and this action containing notoph potential E_{kl} is manifestly SU(2) invariant (cf. sect. 4.4).

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Гальперин А., Иванов Е., Огиевецкий В. E2-85-897
Взаимодействия и преобразования дуальности тензорных N=2 мультиплетов

В рамках гармонического суперпространства рассмотрен общий функционал действия для тензорных N=2 мультиплетов. С помощью суперполевого преобразования дуальности доказано, что всегда имеет место эквивалентность ограниченному классу действий гипермультиплетов. В частности, улучшенный тензорный мультиплет дуально эквивалентен свободному гипермультиплету. Построены также суперполевые действия улучшенного тензорного мультиплета и гипермультиплета на фоне конформной супергравитации.

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Superspace Actions and Duality Transformations for N=2 Tensor Multiplets

General actions for self-interacting N=2 tensor multiplets are considered in the harmonic superspace approach. All of them are shown to be equivalent, by superfield duality transformations, to some restricted class of the hypermultiplets actions. In particular, the improved tensor multiplet theory is dual to a free hypermultiplet one. We present also superspace couplings of these matter multiplets to conformal supergravity.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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