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TOTAL CROSS SECTION FOR RELATIVISTIC POSITRONIUM INTERACTION WITH ATOM

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In the paper [1] it has been suggested that the superpenetrability effect can occur when ultrarelativistic positronia pass through a thin layer of matter. The essence of the effect is in the deviation of probability of such process from the exponential low. Beams of ultrarelativistic positronia are now available and the experimental study of their interactions with matter has started. The quantitative analysis of these processes requires the knowledge of the value of the total cross section for positronium interaction with an atom, since this parameter essentially determines the suppression degree of positronia. A series of papers [2-4] has been devoted to the calculations of the total cross-section of positronium interaction with matter. The critical analysis of these papers is given in Ref. [5]. Avoiding details, we would like to mention one point. In the indicated papers the total cross section has been calculated for such positronium-atom interactions where the atom remains in its ground state, i.e. the so-called coherent part which is proportional to Z^2 (Z is the charge of the atom). The incoherent part of the cross-section which is related to the atom excitation and ionization is proportional to Z. Most probably for this reason the incoherent part of the cross section has been assumed to be small when compared with the coherent part. As a result, the coherent part has been identified with the total cross section. The analysis and numerical calculations performed in this paper show the incorrectness of such identification.

In the one-photon approximation the amplitude of the positronium (A_{2e}) interaction with an atom looks like

$$F_{if,IF}(q)=\frac{2\alpha}{q^2}S_{IF}(q)\cdot\tilde{S}_{if}(q), \qquad (1)$$

where α is the fine structure constant, q is the four-momentum transfer which coincides with the perpendicular momentum transfer at high energy, $i(I)$ is the initial and $f(F)$ is the final positronium (atom) state; $S_{IF}=\langle I|Z-\sum_{k=1}^{Z}e^{i\vec{q}\vec{r}_k}|F\rangle$, $\tilde{S}_{if}=\langle i|e^{i\vec{q}\frac{\vec{r}}{2}}-e^{-i\vec{q}\frac{\vec{r}}{2}}|f\rangle$ are the inelastic form factors of the atom and the positronium respectively, $\bar{z}_k$ is the position vector of an electron in the atom and $\pm\bar{z}/2$ is the position vectors of the electron and positron in A_{2e} .

The total cross section, of course, equals

$$\sigma^{tot}=4\pi\alpha^2\int\frac{dq^2}{q^4}\sum_{f,F}|S_{if}(q)|^2\cdot|\tilde{S}_{IF}(q)|^2. \qquad (2)$$

Performing in (2) summation over the final states f and F and using the completeness conditions

$$\sum_f |f\rangle\langle f|=1,\quad \sum_F |F\rangle\langle F|=1 \qquad (3)$$

we get

$$\sigma^{tot}=8\pi\alpha^2\int\frac{dq^2}{q^4}[1-\tilde{S}(q)]\{Z^2[1-F(q)]^2-Z[1-F^2(q)]\}, \qquad (4)$$

where $\tilde{S}(q)=\int|\psi_{A_{2e}}(z)|^2 e^{i\vec{q}\vec{z}}d\vec{z}$, $F(q)=\int|\varphi(z_1\ldots z_Z)|^2 e^{i\vec{q}\vec{z}_1}d\vec{z}_1\ldots d\vec{z}_{Z-1}$. From (4) it follows that the value of the cross section is determined by the form factors and consequently by the radii of the interacting objects. The ratio of the values of σ^{tot}_{coh} and σ^{tot}_{incoh} is determined as is seen from (4) by Z and, on the other hand by the relation between size parameters of the positronium and the atom. For the qualitative analysis we choose $\tilde{S}(q)$ and $F(q)$ in the pole approximation:

$$\tilde{S}(q)=(1+b^2q^2)^{-1},\quad F(q)=(1+a^2q^2)^{-1}. \qquad (5)$$

Then σ^{tot}_{coh} and σ^{tot}_{incoh} are as follows

$$\sigma^{tot}_{coh}=Z^2\left\{\frac{a^2b^2}{b^2-a^2}+\frac{a^4b^2}{(b^2-a^2)^2}\ln\frac{a^2}{b^2}\right\},$$

$$\sigma^{tot}_{incoh}=Z\left\{\frac{a^2b^2}{b^2-a^2}\left[\ln\frac{b^2}{a^2}-1\right]+\frac{a^2b^4}{(b^2-a^2)^2}\ln\frac{b^2}{a^2}\right\}. \qquad (6)$$

When $b^2\gg a^2$ one gets

$$\sigma^{tot}_{coh}\simeq Z^2a^2,\quad \sigma^{tot}_{incoh}\simeq Za^2\left(2\ln\frac{b^2}{a^2}\right). \qquad (6')$$

Taking into account that $a^2\sim Z^{-2/3}$, one easily finds that the values of σ^{tot}_{coh} and σ^{tot}_{incoh} , are comparable.

When $a^2\gg b^2$

$$\sigma^{tot}_{coh}\simeq Z^2b^2\ln\frac{a^2}{b^2},\quad \sigma^{tot}_{incoh}\simeq Zb^2\ln\frac{a^2}{b^2}. \qquad (6'')$$

In this case the ratio $\sigma^{tot}_{coh}/\sigma^{tot}_{incoh}\sim 1/Z$ and σ^{tot}_{coh} practically coincides with σ^{tot} .

Numerical calculations have been performed as in the papers /2-5/ with the help of the Thomas-Fermi-Molier parametrization of the form factor:

$$F(q)=\frac{0.35}{q^2+\beta_1^2}+\frac{0.55}{q^2+\beta_2^2}+\frac{0.1}{q^2+\beta_3^2}, \qquad (7)$$

$$\beta_1=0.3\cdot(0{,}885\cdot Z^{-1/3}a_0)^{-1},\quad \beta_2=4\beta_1,\quad \beta_3=5\beta_2,\quad a_0=0.529\,\text{Å}.$$

The results of the calculations are presented in the Table:

Table

Z	$\sigma^{tot}_{coh}\cdot 10^{19}$ (cm²)	$\sigma^{tot}_{incoh}\cdot 10^{19}$ (cm²)	$\sigma^{tot}_{incoh}/\sigma^{tot}_{coh}$
6	4.64	4.32	0.93
12	13.08	6.85	0.52
18	18.18	8.89	0.37
24	36.17	10.67	0.29
30	50.00	12.26	0.25
36	65.05	13.71	0.21
42	81.19	15.07	0.19
48	98.32	16.34	0.17
54	116.34	17.53	0.15
60	135.20	18.67	0.14

It is seen that the incoherent part significantly influences the value of the total cross section, in particular, for light elements.

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Пак А.С., Тарасов А.В. E2-85-882

Полное сечение взаимодействия релятивистского позитрония с атомом

Работа посвящена расчету полных сечений взаимодействия релятивистских атомов позитрония с атомами. Расчеты проводятся в рамках потенциальной теории в борновском приближении. Анализируются вклады в полное сечение когерентной (σ_{coh}) и некогерентной (σ_{inc}) частей. Показано, что для легких элементов величина σ_{inc} сравнима с σ_{coh}, а для тяжелых отношение $\sigma_{inc}/\sigma_{coh}$ существенно превышает Z^{-1}/Z -заряд ядра атома/. Приведены результаты численных расчетов. Сделан вывод о важности учета некогерентной части при расчете полных сечений.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Pak A.S., Tarasov A.V. E2-85-882

Total Cross Section for Relativistic Positronium Interaction with Atom

The total cross-sections of relativistic positronium interactions with atoms are calculated. It is shown that the incoherent part which is related to the atom excitation and ionization significantly influences the value of the total cross section.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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