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DUALITIES IN THE d=2 ASYMMETRIC CHIRAL FIELD SIGMA MODELS

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1. Introduction

For the last time a good deal of attention was paid to asymmetric chiral field (ACF) models[1] viz. nonlinear σ models with actions modified by the Wess-Zumino-Witten (WZW) terms[2,3]. Such a generalization can be constructed for any $d=2$ principal chiral field (CF)[1,3] and N=1 supersymmetric extension of the latter[4-6]. It appears in the problem of nonabelian bosonization[3], in string models[7], in higher superextensions of Liouville equation[8,9], etc. In light of this, further understanding the geometric and group-theoretical essentials of ACF seems urgent.

An important ingredient of ordinary symmetric space σ models in $d=2$, including the principal σ models is their on-shell dual symmetry[10-12]. Duality transformations mixe the equations of motion with kinematic Maurer-Cartan equations and play a decisive role in establishing a classical integrability of these models.

The $d=2$ ACF σ models are also integrable, both on classical [13,14] and quantum levels[15,16]. Thus, like their ordinary CF prototypes, these are expected to exhibit a dual invariance. This is certainly the case if the coupling constant of ACF is not subject to the infrared stability condition[3] because within this range the classical ACF equations are reduced to those of standard principal CF simply by a change of variables[13]. However, in order to cover the general situation (at as well as beyond the infrared fixed points) and also having in mind that the equivalence just mentioned may be ruined at the quantum level, it is of interest to see dual invariance and to find an explicit form of duality transformations for ACF directly in original variables. To do it is one of the incentives of the present work.

Dual symmetry of ordinary CF's is manifest with employing the language of Cartan's 1-forms and we begin by translating the ACF equations into this geometric language (Sect. 2). It allows us to get duality transformations for general $d=2$ ACF in a concise simple form and to show that, just as in the conventional case[10-12], they are directly related to the corresponding linear system and to existence of an infinite series of nonlocal conserved currents. Extension to the supersymmetric ACF is given in Sect. 3. In particular, we obtain a manifestly supersymmetric expres-

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sion for generating function of nonlocal vector currents. In Sect. 4, we discuss some equivalences between ACP and other $d=2$ integrable systems (including standard principal CP) within the Cartan's forms approach.

The second main goal of the present paper is to introduce the notion of dual algebra (and dual geometry) of $d=2$ σ-models and to show its usefulness in studying classical and quantum structure of these models. This is done in Sect. 5. We show that the equations of any $d=2$ σ model on symmetric coset space simultaneously describe some σ-model associated with another, dual coset space. For instance, principal CP on a group G is dual to CP on the coset G^c/G, G^c being a complexification of G. As to the ACP models, it turns out that passing to their infrared fixed points can be described on a pure algebraic ground as a sort of contraction of the dual algebra which brings the relevant dual coset space to completely flatten. We discuss also possible applications of dual algebras for the analysis of quantum ACP σ models within the covariant background field method.

Finally, in Sect. 6 we consider along similar lines an asymmetric version of classical CP^1-model suggested in [15]. The techniques of Cartan's forms enables us to readily establish that such a modification yields a trivial dynamics.

2. The Cartan's Forms Description of ACP and Continuous Dual Symmetry

2.1. The equations of $d=2$ ACP on a group G in the standard formulation through currents, are written as[1,3,6,13,14/*)]

$$(1+\eta)\partial_+ \mathcal{J}_- + (1-\eta)\partial_- \mathcal{J}_+ = 0 \tag{2.1}$$

$$\partial_+ \mathcal{J}_- - \partial_- \mathcal{J}_+ + [\mathcal{J}_+, \mathcal{J}_-] = 0 \Leftrightarrow \mathcal{J}_\pm \equiv \mathcal{J}^i_\pm T^i = g(x)\partial_\pm g^{-1}(x),\ g(x)\in G \tag{2.2}$$

$$\eta = \frac{N\lambda^2}{2\pi}\ ,\ (N=0,1,2,\dots). \tag{2.3}$$

where T^i are generators of G, and integer N in eq.(2.3) reflects the quantization of coupling constant in front of the WZW term in the corresponding invariant action[3] (in what follows, we shall never need an explicit form of the latter). When $\eta=0$, we recover the standard equations of principal CP on group G [17-19]. Two other exceptional values of η are ± 1, in quantum case they correspond to the infrared fixed points of the theory, i.e. to zeros of β-function [3,20]. At these values of η eq.(2.1) is drastically simplified:

$$a)\ \eta=1 \Rightarrow \partial_+ \mathcal{J}_- = 0\ ;\qquad b)\ \eta=-1 \Rightarrow \partial_- \mathcal{J}_+ = 0 \tag{2.4}$$

and can be explicitly integrated [3].

In fact, eqs. (2.1), (2.2) are covariant with respect to two independent (but isomorphic) constant parameter groups G_L, G_R whose action on $g(x)$ is defined as follows

$$g'(x) = g_L\, g(x)\, g_R^{-1}. \tag{2.5}$$

The original group G forms a diagonal subgroup in the product $G_L \times G_R$:

$$G:\qquad g_L = g_R$$

and is realized by homogeneous linear transformations. Transformations from the coset $G_L\times G_R/G$ are inhomogeneous and, in any particular parametrization of $g(x)$, are nonlinear. Thus we actually deal with the nonlinear σ model on symmetric space $G_L\times G_R/G$ [12]. Such σ models are treated most naturally in terms of left-invariant Cartan's 1-forms and now we proceed to rewrite eqs. (2.1), (2.2) in this manifestly geometric language.

The standard definition of Cartan's forms can be found, e.g. in [21]. In the present case, both due to the direct product structure of the whole symmetry group and to the existence of the automorphism $G_L \leftrightarrow G_R$ they admit a closed representation through the elements of groups G_L and G_R *):

$$\omega_\pm \equiv \omega^i_\pm T^i = \frac{1}{2}\left[g_L^{-1}(x)\partial_\pm g_L(x) - g_R^{-1}(x)\partial_\pm g_R(x)\right], \tag{2.6}$$

$$\theta_\pm \equiv \theta^i_\pm T^i = \frac{1}{2}\left[g_L^{-1}(x)\partial_\pm g_L(x) + g_R^{-1}(x)\partial_\pm g_R(x)\right]. \tag{2.7}$$

Here, $\omega^i_\pm$ have the meaning of covariant derivatives of parameters of the coset $G_L\times G_R/G$ and $\theta^i_\pm$ are the components of G-connection. These objects are invariant under the left rigid G_L and G_R shifts:

$$g_L(x) \to g_L\cdot g_L(x)\ ,\ g_R(x) \to g_R\, g_R(x). \tag{2.8}$$

Besides, there is a freedom with respect to the right gauge G-transformations:

$$g_L(x) \to g_L(x)h(x),\quad g_R(x) \to g_R(x)h(x), \tag{2.9}$$

$$\omega_\pm \to h^{-1}\omega_\pm h\ ,\ \theta_\pm \to h^{-1}\theta_\pm h + h^{-1}\partial_\pm h\ . \tag{2.10}$$

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*) We use the light cone notation $x^\pm = x^0 \pm x^1$, $\partial_\pm = \frac{\partial}{\partial x^\pm} = \frac{1}{2}\left(\frac{\partial}{\partial x^0} \pm \frac{\partial}{\partial x^1}\right)$

*) A differential 1-form is completely defined by the coefficients in its decomposition in the coordinate differentials. So, when we use the term "Cartan's form" we normally mean the set of corresponding independent coefficients (these are vector ones in ordinary bosonic case and spinor ones in supersymmetric case).

This freedom can be used, e.g. to kill in $g_L(x), g_R(x)$ all the parameters except those coordinatizing the coset $G_L\times G_R/G$ that amounts to the identification: $g_L(x) = g_R^{-1}(x)$.

Now, let us show that the following G-gauge covariant system of equations for $\theta_\pm$, $\omega_\pm$

$$\partial_+\theta_- - \partial_-\theta_+ + [\theta_+,\theta_-] + [\omega_+,\omega_-] = 0 \qquad (2.11a)$$

$$\nabla_+\omega_- - \nabla_-\omega_+ = 0 \qquad (2.11b)$$

$$\nabla_+\omega_- + \nabla_-\omega_+ + 2\eta[\omega_+,\omega_-] = 0, \quad (\nabla_\pm = \partial_\pm - [\theta_\pm, \;\,) \qquad (2.11c)$$

is equivalent to the system (2.1), (2.2). To this end, we identify $g(x)$ with the G-gauge invariant quantity

$$g(x) = g_L(x)\, g_R^{-1}(x) \qquad (2.12)$$

which possesses just the transformation properties (2.5) under the action of groups G_L, G_R. Then, $J_\pm$ is related to $\omega_\pm$ as

$$-\frac{1}{2} J_\pm(x) = g_L(x)\,\omega_\pm(x)\, g_L^{-1}(x) \qquad (2.13)$$

and, up to a numerical factor $-1/2$, can be treated as a special gauge of $\omega_\pm$ which is achieved via a G-gauge transformation with $h(x) = g_L^{-1}(x)$. It follows from eqs.(2.6), (2.7) and the transformation laws (2.9), (2.10) that in this gauge:

$$\theta_\pm = -\omega_\pm = \frac{1}{2} J_\pm$$

Finally, taking into account G-gauge covariance of the system (2.11) we may consider it in this particular gauge and get just eqs. (2.1), (2.2).

In system (2.11), the first two equations are kinematic Maurer-Cartan equations which are identically satisfied with the definition (2.6), (2.7). The third equation is dynamical, the only difference from an analogous equation of principal CF being in an anomalous term $\sim \eta$. In what follows it will be convenient for us, keeping to the philosophy of /22/, to treat (2.11) as a closed system of equations for the G-algebra valued vector fields $\theta_\pm$, $\omega_\pm$. The most important point about this interpretation is the covariance of (2.11) under gauge group (2.10). As was given above, this guarantees that $\theta_\pm$, $\omega_\pm$ essentially depend only on parameters of the coset $G_L\times G_R/G$ which are thus the true independent variables of system (2.11).

2.2. At $\eta=0$, eqs. (2.11) describe the standard principal CF and possess a remarkable invariance under duality transformation /10-12/:

$$\theta'_\pm = \theta_\pm\,, \quad \omega'_\pm = e^{\pm\lambda}\omega_\pm \qquad (2.14)$$

where λ is a real constant parameter. This invariance is essentially on-shell one because (2.14) mixe the dynamical equation of motion (2.11c) (with $\eta=0$) and the kinematic Maurer-Cartan equation (2.11b). Moreover, one of the integrability conditions for the existence of this transformation is just the equation of motion. Implementing (2.14) in terms of group elements $g_L(x)$, $g_R(x)$ according to the representation (2.6), (2.7), we have:

$$g_L'^{-1}(x,\lambda)\,\partial_\pm g_L'(x,\lambda) = \theta_\pm + e^{\pm\lambda}\omega_\pm\,; \quad g_L'(x,0) = g_L(x)\,. \qquad (2.15)$$

$$g_R'^{-1}(x,\lambda)\,\partial_\pm g_R'(x,\lambda) = \theta_\pm - e^{\pm\lambda}\omega_\pm\,; \quad g_R'(x,0) = g_R(x)\,. \qquad (2.16)$$

These equations are solvable with respect to g'_L or g'_R only provided eqs. (2.11) hold (with $\eta=0$). So the latter are nothing else as the integrability conditions for systems (2.15), (2.16) which can thus be treated as variants of linear system for principal CF. More familiar form of linear set /17,18/ comes out when rewriting, e.g., eq. (2.15) through the matrix $\Psi(x,\lambda)$ defined by:

$$\Psi(x,\lambda) = g_L(x)\, g_L'^{-1}(x,\lambda)\,, \quad \Psi(x,0) = I\,. \qquad (2.17)$$

We have

$$\partial_\pm \Psi(x,\lambda) = \frac{1}{2}(e^{\pm\lambda}-1)\, J_\pm\, \Psi(x,\lambda) \qquad (2.18)$$

with e^λ recognized as a spectral parameter.

Thus, the classical integrability of principal CF is directly related to dual symmetry (2.14). While duality transformations are extremely simple when realized on Cartan's forms, in terms of the coset parameters they are nonlinear and nonlocal as follows from eqs. (2.15), (2.16). The same is true for the current $J_\pm(x)$. The dual transformed current is given by

$$J'_\pm(x,\lambda) = g'(x,\lambda)\,\partial_\pm g'^{-1}(x,\lambda) = -2 g_L'(x,\lambda)\,\omega'_\pm\, g_L'^{-1}(x,\lambda) = e^{\pm\lambda}\Psi^{-1} J_\pm \Psi\,. \qquad (2.19)$$

It is conserved as a consequence of the $\eta=0$ eqs.(2.1), (2.2) and eq. (2.18)

$$\partial_+ J'_- + \partial_- J'_+ = 0$$

and so may serve as a generating function for an infinite sequence of nonlocal conserved currents which appear as coefficients of the Taylor expansion of $J'_\pm$ in powers of $\frac{e^\lambda - 1}{e^\lambda + 1}$ /11,12/.

How to extend this dual symmetry to the general case with $\eta\neq 0$? To answer this question, we note first that the dual invariance of system (2.11) at $\eta=0$ is mainly related to the fact that the equation of motion (2.11c) and the second Maurer-Cartan equation (2.11b) for $\eta=0$ enter symmetrically. One can restore this symmetry for $\eta\neq 0$ too. To achieve it, let us pass in eqs.(2.11) to a new connection

$$\tilde{\theta}_\pm = \theta_\pm \pm \eta\,\omega_\pm \;; \quad \left(\tilde{\theta}_\pm \big|_{\eta=0} = \theta_\pm \right) \tag{2.20}$$

in terms of which eqs.(2.11) are rewritten as

$$\begin{gathered} \partial_+ \tilde{\theta}_- - \partial_- \tilde{\theta}_+ + [\tilde{\theta}_+, \tilde{\theta}_-] + (1-\eta^2)[\omega_+, \omega_-] = 0. \\ \tilde{\nabla}_+ \omega_- - \tilde{\nabla}_- \omega_+ = 0. \\ \tilde{\nabla}_+ \omega_- + \tilde{\nabla}_- \omega_+ = 0 \end{gathered} \tag{2.21}$$

Now a proper generalization of transformations (2.14) is

$$\tilde{\theta}_\pm' = \tilde{\theta}_\pm \;, \quad \omega_\pm' = e^{\pm\lambda}\omega_\pm \tag{2.22}$$

or, in original variables

$$\theta_\pm' = \theta_\pm \pm \eta(1 - e^{\pm\lambda})\omega_\pm \;, \quad \omega_\pm' = e^{\pm\lambda}\omega_\pm . \tag{2.23}$$

We see that eqs.(2.11) possess a dual symmetry at any η, including exceptional values $\eta = \pm 1$. The duality transformations in the general case are given by eqs.(2.23). For $\eta = 0$ these are reduced to (2.14).

All what was said above about the relation between dual symmetry and integrability of principal CF is true in the case of ACF as well. The $\eta \neq 0$ analogs of eqs. (2.15), (2.18) are

$$g_L^{(\eta)'-1}(x,\lambda)\,\partial_\pm\, g_L^{(\eta)'}(x,\lambda) = \theta_\pm + [(1\mp\eta)e^{\pm\lambda} \pm \eta]\,\omega_\pm \;, \tag{2.24}$$

$$\partial_\pm \Psi^{(\eta)}(x,\lambda) = \frac{1}{2}(1\mp\eta)(e^{\pm\lambda}-1)\, J_\pm \Psi^{(\eta)}(x,\lambda). \tag{2.25}$$

The linear system (2.25) can be cast in the form suggested for ACF earlier (see, e.g. /13/). Needless to say, the infinite series of nonlocal conserved currents is again constructed by the recipe (2.19):

$$J_\pm^{(\eta)'}(x,\lambda) = e^{\pm\lambda}\Psi^{(\eta)-1} J_\pm \Psi^{(\eta)} \;, \quad (1+\eta)\partial_+ J_-^{(\eta)'} + (1-\eta)\partial_- J_+^{(\eta)'} = 0. \tag{2.26}$$

3. Dual Transformations of Supersymmetric ACF

N=1 superextension of ACF is described by the following system of superfield equations /5,6/*):

$$(1+\eta)D_+ I_- - (1-\eta)D_- I_+ = 0 \,. \tag{3.1}$$

$$D_+ I_- + D_- I_+ + \{I_+, I_-\} = 0 \Leftrightarrow I_\pm = g D_\pm g^{-1},\; g = g(x,\theta) \in G \,, \tag{3.2}$$

$$D_\pm = \frac{\partial}{\partial\theta^\pm} + i\,\theta^\pm \partial_\pm \;, \quad D_\pm D_\pm = i\,\partial_\pm \;, \quad \{D_+, D_-\} = 0 \,. \tag{3.3}$$

The basic left-invariant Cartan's forms in this case are defined by their spinor coefficients (cf. eqs.(2.6), (2.7)):

*) It should not lead to confusion that here the $d=2$ spinors have the same indices $\pm$ as vectors $\partial_\pm$, $J_\pm$. These indices merely mark chirality of spinors (under the $d=2$ Lorentz group $SO(1,1)$ the $\pm$ spinors transform independently of each other with the half weights as compared to the $\pm$ components of vector).

$$\Theta_\pm = \frac{1}{2}\left[g_L^{-1}(x,\theta)D_\pm g_L(x,\theta) + g_R^{-1}(x,\theta)D_\pm g_R(x,\theta) \right] , \tag{3.4}$$

$$\Omega_\pm = \frac{1}{2}\left[g_L^{-1}(x,\theta)D_\pm g_L(x,\theta) - g_R^{-1}(x,\theta)D_\pm g_R(x,\theta) \right] . \tag{3.5}$$

One may, of course, consider also vector coefficients $\theta_\pm$, $\omega_\pm$ (by using vector derivatives in (3.4), (3.5) instead of spinor ones) which contain purely bosonic ones (2.6), (2.7) as the lowest components in their θ-expansion. However, due to the anticommutation properties (3.3), these objects turn out to be composed of spinor superfields (3.4), (3.5):

$$\begin{gathered} \theta_\pm = -i\,(D_\pm \Theta_\pm + \Theta_\pm \Theta_\pm + \Omega_\pm \Omega_\pm) \\ \omega_\pm = -i\,(D_\pm \Omega_\pm + \{\Theta_\pm, \Omega_\pm\}) . \end{gathered} \tag{3.6}$$

The N=1 analogs of eqs.(2.11) are as follows

$$\begin{gathered} D_+ \Theta_- + D_- \Theta_+ + \{\Theta_+, \Theta_-\} + \{\Omega_+, \Omega_-\} = 0 \\ \mathcal{D}_+ \Omega_- + \mathcal{D}_- \Omega_+ = 0 \\ \mathcal{D}_+ \Omega_- - \mathcal{D}_- \Omega_+ + 2\eta\{\Omega_+, \Omega_-\} = 0 \,, \quad (\mathcal{D}_\pm = D_\pm + \{\Theta_\pm, \;\;\}). \end{gathered} \tag{3.7}$$

Again, the first two equations are the kinematic Maurer-Cartan equations while the third one is the equation of motion. The equivalence of (3.7) to eqs. (3.1), (3.2) can be proven in the same way as in the bosonic case, by identifying

$$g(x,\theta) = g_L(x,\theta)\, g_R^{-1}(x,\theta).$$

Similarity of systems (3.7) and (2.11) suggests a likeness of their dual invariance properties. Indeed, duality transformations leaving invariant eqs.(3.7) have the same form as in (2.23):

$$\Theta_\pm' = \Theta_\pm \pm \eta(1 - e^{\pm\lambda})\Omega_\pm \;, \quad \Omega_\pm' = e^{\pm\lambda}\Omega_\pm \,. \tag{3.8}$$

It is interesting that the vector Cartan's forms (3.6) reveal now more complicated dual transformation laws:

$$\begin{gathered} \delta\theta_\pm = -\lambda\eta\,\omega_\pm \mp 2i\,\lambda\,\Omega_\pm \Omega_\pm \\ \delta\omega_\pm = \pm\lambda\,\omega_\pm + 2i\,\lambda\,\Omega_\pm \Omega_\pm \,. \end{gathered}$$

This is related to appearance of fermionic source terms in the r.h.s. of the third equation of the bosonic system (2.11) in supersymmetric case.

Dual invariance of the supersymmetric ACF equations (3.7) implies their integrability, in the same fashion as for bosonic ACF. We restrict discussion of this matter to giving explicit expressions for generating functions of the relevant set of nonlocal conserved supercurrents.

The basic spinor current $I_\pm$ is related to $\Omega_\pm$ analogously to the bosonic case (eq.(2.13)):

$$I_\pm = -2\, g_L(x,\theta)\, \Omega_\pm\, g_L^{-1}(x,\theta). \tag{3.9}$$

Performing in eq.(3.9) a duality transformation we get the generating function of nonlocal spinor supercurrents:

$$I_\pm^{(\eta)\prime}(x,\theta,\lambda) = -2e^{\pm\lambda} g_L^{\prime}(x,\theta,\lambda)\,\Omega_\pm(x,\theta)\, g_L^{\prime -1}(x,\theta,\lambda). \tag{3.10}$$

$$D_\pm g_L^{\prime -1}(x,\theta,\lambda) = -\{\Theta_\pm + [(1\mp\eta)e^{\pm\lambda} \pm \eta]\Omega_\pm\}\, g_L^{\prime -1}(x,\theta,\lambda). \tag{3.11}$$

$$D_+ I_-^{(\eta)\prime}(1+\eta) - D_- I_+^{(\eta)\prime}(1-\eta) = 0. \tag{3.12}$$

One may wonder how to construct by $I_\pm^{(\eta)\prime}$ the vector supercurrent $\mathcal{J}_\pm^{(\eta)\prime}$ which would have conventional conservation properties with respect to x -differentiation and start with ordinary conserved current. The answer is

$$\mathcal{J}_\pm^{(\eta)\prime} = i\, D_\pm I_\pm^{(\eta)\prime}. \tag{3.13}$$

The conservation law of $\mathcal{J}_\pm^{(\eta)\prime}$ directly follows from eq.(3.12) and the basic anticommutation relations (3.3):

$$(1+\eta)\,\partial_+ \mathcal{J}_-^{(\eta)\prime} + (1-\eta)\,\partial_- \mathcal{J}_+^{(\eta)\prime} = 0. \tag{3.14}$$

It is easy to obtain for $\mathcal{J}_\pm^{(\eta)\prime}$ a representation in terms of Cartan's forms:

$$\mathcal{J}_\pm^{(\eta)\prime}(x,\theta,\lambda) = -2e^{\pm\lambda} g_L^{\prime}(x,\theta,\lambda)\{\omega_\pm - 2i[(1\mp\eta)e^{\pm\lambda} \pm \eta]\Omega_\pm\Omega_\pm\}\, g_L^{\prime -1}(x,\theta,\lambda). \tag{3.15}$$

Note that the supercurrent (3.13) is a dual transform of the following basic conserved vector supercurrent:

$$\begin{aligned}\mathcal{J}_\pm^{(\eta)} = \mathcal{J}_\pm^{(\eta)\prime}\big|_{\lambda=0} &= -2\, g_L(x,\theta)(\omega_\pm - 2i\,\Omega_\pm\Omega_\pm)\, g_L^{-1}(x,\theta) = \\ &= g(x,\theta)\,\partial_\pm g^{-1}(x,\theta) + i\, I_\pm(x,\theta)\, I_\pm(x,\theta).\end{aligned} \tag{3.16}$$

It is worthwile to note the presence of the term bilinear in fermionic supercurrent which is new compared to the expression for the bosonic current (2.2). In the purely bosonic limit, when all the fermionic components vanish, we recover the standard expression.

4. Classical Equivalencies of ACF to Some Other Integrable $d=2$ Models

In this Section we again return to the bosonic case. Nevertherless, all the conclusions we shall arrive at apply as well to supersymmetric ACF.

We have seen above that the dual invariance properties of ACF and conventional principal CF are strongly similar to each other. In fact, for $\eta \neq \pm 1$ the similarity becomes identity as in this case both models are equivalent (at least, at the classical level). Indeed, in terms of new currents

$$\tilde{\mathcal{J}}_\pm = (1\mp\eta)\,\mathcal{J}_\pm \tag{4.1}$$

equations (2.1), (2.2) take the form[13]:

$$\partial_+\tilde{\mathcal{J}}_- + \partial_-\tilde{\mathcal{J}}_+ = 0,\quad \partial_+\tilde{\mathcal{J}}_- - \partial_-\tilde{\mathcal{J}}_+ + [\tilde{\mathcal{J}}_+, \tilde{\mathcal{J}}_-] = 0. \tag{4.2}$$

Note that the second equation is now satisfied only with taking into account the dynamical equation (2.1). For $\eta \neq \pm 1$ the mapping (4.1) is invertible: eqs.(4.2) imply for $\mathcal{J}_\pm$ just eqs.(2.1), (2.2). This equivalence is seen also when comparing the linear systems (2.18), (2.25).

The same phenomenon manifests itself in the Cartan's forms approach too. Rescaling ω_+, ω_- in eqs.(2.21) with $\eta \neq \pm 1$ as

$$\tilde{\omega}_\pm = (1\mp\eta)\,\omega_\pm \tag{4.3}$$

we represent (2.21) as the system of equations of standard principal field:

$$\begin{aligned}&\partial_+\tilde{\theta}_- - \partial_-\tilde{\theta}_+ + [\tilde{\theta}_+, \tilde{\theta}_-] + [\tilde{\omega}_+, \tilde{\omega}_-] = 0\\ &\tilde{\nabla}_+\tilde{\omega}_- + \tilde{\nabla}_-\tilde{\omega}_+ = 0\\ &\tilde{\nabla}_+\tilde{\omega}_- - \tilde{\nabla}_-\tilde{\omega}_+ = 0.\end{aligned} \tag{4.4}$$

The current corresponding to the pair $\tilde{\theta}_\pm$, $\tilde{\omega}_\pm$ is just $\tilde{\mathcal{J}}_\pm$ (4.1). It is worth noting that the system (4.4) can be achieved with a different rescaling of $\omega_\pm$, e.g. with

$$\omega_\pm \to \tilde{\tilde{\omega}}_\pm = \sqrt{1-\eta^2}\,\omega_\pm \quad (\eta^2<1),\quad \tilde{\tilde{\omega}}_\pm = \pm\sqrt{\eta^2-1}\,\omega_\pm \quad (\eta^2>1).$$

However, all these changes of variables are related to (4.3) by duality transformations. In particular, $\tilde{\tilde{\omega}}_\pm$ is a dual transform of $\tilde{\omega}_\pm$ with the parameter $\lambda = \frac{1}{2}\ln\frac{1-\eta}{1+\eta}$. The corresponding currents are related to $\tilde{\mathcal{J}}_\pm$ by nonlocal transformations of the type (2.19).

At exceptional points $\eta = \pm 1$ the above equivalence gets lost. The system (2.21) takes a "triangular" form:

$$\begin{aligned}&\partial_+\tilde{\theta}_- - \partial_-\tilde{\theta}_+ + [\tilde{\theta}_+, \tilde{\theta}_-] = 0\\ &\tilde{\nabla}_+\omega_- - \tilde{\nabla}_-\omega_+ = 0\\ &\tilde{\nabla}_+\omega_- + \tilde{\nabla}_-\omega_+ = 0.\end{aligned} \tag{4.5}$$

It respects as before gauge group (2.10). Since $\tilde{\theta}_\pm$ is now a pure gauge, we may choose

$$\tilde{\theta}_\pm = 0$$

that reduces (4.5) to the following system [9,20]:

$$\left.\begin{aligned}\partial_+\hat{\omega}_- - \partial_-\hat{\omega}_+ = 0\\ \partial_+\hat{\omega}_- + \partial_-\hat{\omega}_+ = 0\end{aligned}\right\} \Rightarrow \hat{\omega}_\pm^i = \partial_\pm\varphi^i,\ \partial_+\partial_-\varphi^i = 0. \tag{4.6}$$

Thus, for $\eta = \pm 1$ the ACF is classically equivalent not to principal CF but to a free massless scalar field [9,20].

We wish to emphasize that the equivalences discussed so far are essentially on-shell phenomena as these exploit an interchange between the kinematic constraints, viz. the Maurer-Cartan equations (which are equally valid on- and off-shell) and the dynamical field equations associated with a definite action. So these equivalences may be broken at the full quantum off-shell level. An example of such an equivalence which holds only in classical theory has been given in /7,23/. It is instructive to reproduce this example in our notation. One may change variables in eqs.(2.21) once more (for definiteness, we set $\eta^2 \leq 1$):

$$\tilde\theta_\pm \to \tilde{\tilde\theta}_\pm = \tilde\theta_\pm + \sqrt{1-\eta^2}\,\omega_\pm = \theta_\pm + (\sqrt{1-\eta^2} \pm \eta)\,\omega_\pm$$
$$\omega_\pm \to \tilde\omega_\pm = \pm\,\omega_\pm$$

to rewrite the system (2.21) as

$$\partial_+ \tilde{\tilde\theta}_- - \partial_- \tilde{\tilde\theta}_+ + [\tilde{\tilde\theta}_+, \tilde{\tilde\theta}_-] = 0$$
$$\tilde{\tilde\nabla}_+ \tilde\omega_- + \tilde{\tilde\nabla}_- \tilde\omega_+ - 2\sqrt{1-\eta^2}\,[\tilde\omega_+, \tilde\omega_-] = 0 \qquad (4.7)$$
$$\tilde{\tilde\nabla}_+ \tilde\omega_- - \tilde{\tilde\nabla}_- \tilde\omega_+ = 0.$$

Fixing G-gauge by the condition $\tilde{\tilde\theta}_\pm = 0$ we put (4.7) in the form:

$$\partial_+ \tilde\omega_- + \partial_- \tilde\omega_+ - 2\sqrt{1-\eta^2}\,[\tilde\omega_+, \tilde\omega_-] = 0$$
$$\partial_+ \tilde\omega_- - \partial_- \tilde\omega_+ = 0$$

which is obviously solved by

$$\tilde\omega_\pm = \partial_\pm \varphi\,, \quad \partial_+\partial_- \varphi - \sqrt{1-\eta^2}\,[\partial_+\varphi, \partial_-\varphi] = 0. \qquad (4.8)$$

The last equation is completely integrable /17,24/ and, as we see, at any η it is classically equivalent to the original system (2.11). However, quantum theories associated with ACF and the field φ satisfying eq.(4.8) radically differ: relevant β-functions do not coincide /23/ (except the points $\eta = \pm 1$), in the second case there are no arguments for η to be quantized, etc. So one must be careful when trying to extend the above equivalences to quantum region.

5. Dual Algebras and Discrete Duality

Here we introduce the concepts of dual algebra and dual σ model and use them to give an algebraic interpretation of interplay between ACF equations in different regimes in parameter η. We discuss also possible advantages of applying these notions in analysis of renormalization structure of quantum ACF models.

5.1. Let us begin again with the standard principal CF equations which correspond to setting $\eta = 0$ in eqs.(2.11). By passing to new variables

$$\tilde\omega_\pm = \pm\,\omega_\pm \qquad (5.1)$$

we represent this system as

$$\partial_+\theta_- - \partial_-\theta_+ + [\theta_+, \theta_-] - [\tilde\omega_+, \tilde\omega_-] = 0 \qquad (5.2a)$$
$$\nabla_+ \tilde\omega_- + \nabla_- \tilde\omega_+ = 0 \qquad (5.2b)$$
$$\nabla_+ \tilde\omega_- - \nabla_- \tilde\omega_+ = 0. \qquad (5.2c)$$

Now we wish to treat the second equation as the dynamical one, while eq.(5.2c) as belonging together with eq.(5.2a) to some Maurer-Cartan set. To what algebra such Maurer-Cartan equations may correspond? The answer is rather surprising: this is the algebra of non-compact group G^c, a complexification of G. To see it, we introduce an 1-form $\Sigma_\pm$ with values in this algebra:

$$\Sigma_\pm^{(\eta=0)} = \tilde\omega_\pm^k (iT^k) + \theta_\pm^k T^k \equiv \tilde\omega_\pm^k A^k + \theta_\pm^k T^k \qquad (5.3)$$
$$[A^k, A^l] = -i c^{klm} T^m, \quad [A^k, T^m] = i c^{kmt} A^t, \quad [T^k, T^l] = i c^{klm} T^m. \qquad (5.4)$$

It is simple to check that the Maurer-Cartan equation for $\Sigma_\pm^{(\eta=0)}$:

$$\partial_+ \Sigma_-^{(\eta=0)} - \partial_- \Sigma_+^{(\eta=0)} + [\Sigma_+^{(\eta=0)}, \Sigma_-^{(\eta=0)}] = 0$$

just reproduces eqs.(5.2a), (5.2c) for the coefficients $\tilde\omega_\pm^k$, $\theta_\pm^k$. Keeping in mind that the system (5.2) is covariant as before with respect to G-gauge transformations (2.10) we may identify it as describing the $d=2$ σ model on coset G^c/G *). Thus, in two dimensions there exists a kind of discrete duality among the principal CF on a group G and CF on the coset G^c/G, G^c being a complex extension of G. The crucial ingredient of this correspondence is the exchange of the equation of motion with the second Maurer-Cartan equation. In particular, it follows from the existence of the dual description that the original system (2.11) (with $\eta = 0$) has an another, dual set of independent dynamical variables (besides those parametrizing the coset $G_L \times G_R / G$), which are the parameters of coset G^c/G. Both sets are related by a complicated nonlocal transformation.

We shall call algebras of the type (5.4) dual algebras. A dual algebra (and the corresponding dual σ-model) can be defined for any $d=2$ CF on the group G, not only for principal CF. It always coincides with one of the real forms of G^c-algebra (e.g., dual to the $SU(2)/U(1)$-CF is CF on the coset $SU(1,1)/U(1) \sim SO(1,2)/SO(2)$). It is worthwile to note that the existence of a dual description of CF σ-models seems to indicate the presence of second sequence of

*) Recall that the ghost problem in σ models on noncompact groups does not arise if the stability subgroup (G in the present case) is compact /25/.

nonlocal conserved currents in these models, that one associated with the affine algebra constructed by the dual algebra.

5.2. Now, what about the general situation with $\eta \neq 0$? It turns out that, after passing to $\widetilde{\omega}_{\pm}$, we may again interpret eqs.(2.11a) and (2.11c) as the Maurer-Cartan equations associated with the algebra:

$$[A^i, A^k] = -i\, c^{ikl} T^l + 2i\eta c^{ikl} A^l, \ [A^i, T^l] = i c^{ilk} A^k, \ [T^i, T^l] = i c^{ilk} T^k \qquad (5.5)$$

$$\Sigma_{\pm}^{(\eta)} = \widetilde{\omega}_{\pm}^{k} A^k + \theta_{\pm}^{k} T^k$$

$$\partial_+ \Sigma_-^{(\eta)} - \partial_- \Sigma_+^{(\eta)} + [\Sigma_+^{(\eta)}, \Sigma_-^{(\eta)}] = 0 \Leftrightarrow \text{eqs. } (2.11a), (2.11c)$$

To inspect the structure of the commutation relations (5.5) it is convenient to pass to generators $\widetilde{A}^l = A^l + \eta T^l$:

$$[\widetilde{A}^i, \widetilde{A}^k] = -i(1-\eta^2) c^{ikl} T^l, \ [\widetilde{A}^l, T^k] = i c^{lkm} \widetilde{A}^m, \ [T^l, T^k] = i c^{lkm} T^m \qquad (5.6)$$

(in terms of $\theta_{\pm}$, $\omega_{\pm}$, this rearrangement just corresponds to going over to the system (2.21)). Thus, for $\eta^2 > 1$ the structure relations (5.6) define the algebra of group $G_L \times G_R$, for $\eta^2 < 1$ the algebra of G^c, and for $\eta^2 = 1$ the algebra of inhomogeneous group $P_n \otimes G$ where P_n is an invariant subgroup with n abelian translation generators ($n = \dim G$) belonging to adjoint representation of G :

$$[\widetilde{A}^i, \widetilde{A}^k] = 0, \ [\widetilde{A}^i, T^k] = i c^{ikl} \widetilde{A}^l, \ [T^i, T^k] = i c^{ikl} T^l. \qquad (5.7)$$

Respectively, dual σ-models for these three cases correspond to the cosets $G_L \times G_R / G$, G^c/G and $P_n \otimes G/G$. The first two cases are equivalent to each other by a discrete duality $G_L \times G_R / G \Leftrightarrow G^c/G$.

It should be stressed that the dual σ-models are always ordinary ones, their equations of motion have a uniform standard appearance and follow from the conventional actions with no WZW terms. Also, it is unknown as yet how to understand the quantization of η within the dual description. This constant acquires now a purely algebraic meaning as a parameter of contraction of general dual algebra (5.6) to the algebra (5.7) arising in the limit $\eta = \pm 1$. So the ACP σ-model at $\eta = \pm 1$ is dual to σ-model on the flat coset space $P_n \otimes G/G \sim E_n$, that is a theory of n massless free boson fields, just as stated by eqs.(4.6). The classical equivalence of ACP with $\eta \neq \pm 1$ to principal CP also directly stems from the existence of the dual description and it is another, pure algebraic face of this equivalence.

Knowing the dual algebra and the related dual geometry of $d=2$ σ models may be of use in quantum case, when studying the renormalization structure of these models within the covariant background

field method[26]. Indeed, in actual calculations one may put the background field on-shell where it obeys the system (2.11) (or the similar one in the case of other symmetric space σ models) and hence respects an additional symmetry under the dual group. Therefore, the background field dependence in effective action can be spread over the invariant tensor structures (curvatures, torsions, their covariant derivatives...) in two alternative ways: one may do it covariantly with respect either to the initial geometry or to the geometry associated with dual coset space.*) Consider, e.g. the ACP model near exceptional points $\eta = \pm 1$. An analysis of its renormalization properties in this region on the geometric grounds was performed in [20] (with making use of the world tensor language rather than the Cartan's forms approach corresponding in essence to dealing with the tangent space objects). The authors of[20] have observed as well that the classical ACP equation of motion includes the connection $\widetilde{\theta}_{\pm} = \theta_{\pm} \pm \eta \omega_{\pm}$ (in our notation) whose curvature is proportional to $1 - \eta^2$ and hence vanishes at $\eta = \pm 1$ (cf. eqs.(2.21)). Next they have found that the relevant background field functional (with the background field being on-shell) largely contains just the above generalized curvature and this property has been used to demonstrate ultraviolet finiteness of ACP σ model at $\eta = \pm 1$ ("geometrostasis" [20]) to two-loop order. However, the intrinsic reason why the generalized curvature appears remained unknown. The present study makes it clear that this reason is the invariance of classical ACP on-shell under the dual group with algebra (5.6). The fact that the ACP equations (2.11) at any η admit an equivalent representation as the equations of ordinary σ model on a dual coset space guarantees that the ACP background field expansion may be formulated solely in terms of the corresponding geometric quantities $\widetilde{\theta}_{\pm} = \theta_{\pm} \pm \eta \omega_{\pm}$ and $\widetilde{\omega}_{\pm} = \pm \omega_{\pm}$. As a result, just the curvature of $\widetilde{\theta}_{\pm}$ will be encountered everywhere. In the contraction limit $\eta^2 = 1$, dual coset space of ACP completely flattens (recall that it becomes n-dimensional Euclidean space $E_n \sim P_n \otimes G/G$) and this is likely a general geometric reason for which all ultraviolet counterterms of ACP vanish at these points. Of course, a further labour is needed to prove this conjecture.

In conclusion of this Sect., it is interesting to inquire what is the analog of dual algebras in covariant superstring models[27] which can be treated as $d=2$ σ-models with $d=10$ superspace as the target manifold [28].

6. Asymmetric $\vec{n}$-Field σ Model

The last topic we wish to discuss concerns an asymmetric version

*) This kind of duality by no means suggests the quantum equivalence of the original and dual σ models. Indeed, the form of quantum vertices is fixed by the choice of classical action and is different for both cases.

of CP^1-model /15/. Using the Cartan's forms language we demonstrate that this theory contains no dynamics on the classical level.

We begin by recalling that CP^1-model (or the $\vec{n}$-field model) is associated with symmetric space SU(2)/U(1). The relevant left-invariant Cartan's forms θ, ω are defined as the "vertical" and "horizontal" parts of a special element of SU(2)-algebra:

$$g^{-1}(x)\partial_\pm g(x) = \theta_\pm + \omega_\pm \equiv \theta_\pm^3 T^3 + \omega_\pm^i T^i, \quad i=1,2, \quad g(x)\in SU(2). \tag{6.1}$$

In terms of them, the classical equations of CP^1 model read (see, e.g. /19,22/)

$$\partial_+\theta_- - \partial_-\theta_+ + [\omega_+,\omega_-] = 0 \tag{6.2a}$$

$$\nabla_+\omega_- - \nabla_-\omega_+ = 0 \tag{6.2b}$$

$$\nabla_+\omega_- + \nabla_-\omega_+ = 0, \tag{6.2c}$$

where (6.2a), (6.2b) are kinematic Maurer-Cartan equations while (6.2c) is the equation of motion (as before, we are not interested in corresponding Lagrangians). The system (6.2) is covariant with respect to $U(1)$-gauge transformations:

$$g'(x) = g(x)e^{\varphi(x)}, \quad \theta_\pm' = \theta_\pm + \partial_\pm\varphi, \quad \omega_\pm' = e^{\varphi}\omega_\pm e^{-\varphi}, \quad \varphi(x) = \varphi^3(x)T^3 \tag{6.3}$$

One may define a gauge invariant SU(2) current

$$\mathcal{J}_\pm = g(x)\,\omega_\pm\, g^{-1}(x) \equiv \mathcal{J}_\pm^i T^i + \mathcal{J}_\pm^3 T^3 \tag{6.4}$$

which is conserved in virtue of (6.2):

$$\partial_+\mathcal{J}_- + \partial_-\mathcal{J}_+ = 0. \tag{6.5}$$

An asymmetric variant of CP^1 corresponds to a deformation of this law on the pattern of eq.(2.1) /15/:

$$\partial_+\mathcal{J}_- + c\,\partial_-\mathcal{J}_+ = 0, \tag{6.6}$$

where c is some new parameter.

Using the relation (6.4) it is easy to find how this deformation is displayed in terms of $\theta_\pm$, $\omega_\pm$. The third of eqs.(6.2) is modified as

$$(1+c)\nabla_+\omega_- + (1-c)[\omega_+,\omega_-] = 0 \tag{6.7}$$

(here, eq.(6.2b) has been used). We observe that for $c \neq \pm 1$ this new equation contains both "vertical" and "horizontal" parts which should be zero separately:

$$a)\ \nabla_+\omega_- = 0\ ; \quad b)\ [\omega_+,\omega_-] = i\,\omega_+^k\omega_-^\ell\varepsilon_{k\ell}T^3 = 0. \tag{6.8}$$

Thus, in asymmetric case one is left with the old system (6.2) but supplemented now by the additional nonlinear constraint (6.8b). At $c=-1$, there remain only the Maurer-Cartan equations and the constraint (6.8b), with no further dynamical restrictions.

Let us demonstrate that eq.(6.8b) trivializes the theory. Consider first the case when

$$\omega_+^i = 0 \quad \text{or (and)} \quad \omega_-^i = 0. \tag{6.9}$$

These conditions clearly solve the constraint (6.8b) and the equation (6.8a) (with taking account of the Maurer-Cartan equation (6.2b)). In virtue of eq.(6.2a), $\theta_\pm$ is now a pure U(1)-gauge and we may set it equal to zero (cf. eqs.(4.6)). In this gauge, eqs.(6.2b,c) are

$$\left.\begin{aligned}\partial_+\omega_- - \partial_-\omega_+ = 0\\ \partial_+\omega_- + \partial_-\omega_+ = 0\end{aligned}\right\} \Rightarrow \omega_\pm^i = \partial_\pm\gamma^i(x), \quad \partial_+\partial_-\gamma^i(x) = 0 \tag{6.10}$$

i.e. the model turns out to be gauge-equivalent to a free theory.

The case with $\omega_+^i \neq 0$, $\omega_-^i \neq 0$ is most convenient to treat in terms of the familiar correspondence between CP^1model and sine-Gordon equation /10,17,22/. In this case, there exists an x-frame where eqs.(6.2) are reduced to

$$\partial_+\partial_-\sigma(x) = \sin\sigma(x)\ ; \sin\sigma(x) \sim \omega_+^i\omega_-^j\varepsilon_{ij}. \tag{6.11}$$

Then, the constraint (6.8b) singles out a subclass of trivial constant solutions in the phase space of sine-Gordon equation:

$$\sin\sigma(x) = 0 \Rightarrow \sigma(x) = \pi\cdot N \quad (N=0,1,2,\dots).$$

As to the dual symmetry, eqs.(6.2a,b) and (6.7) for arbitrary c are invariant under the standard duality transformation /10-14/:

$$\delta\theta_\pm = 0, \quad \delta\omega_\pm = \pm\lambda\,\omega_\pm.$$

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References

1. Novikov S.P., Dokl. Acad.Nauk SSSR, 1981, 260, N1, 31-35; Uspekhi Mat.Nauk, 1982, 37, N5, 3-49.
2. Wess J., Zumino B., Phys.Lett., 1971, 37B, 95-97.
3. Witten E., Commun.Math.Phys., 1984, 92, 455-472.
4. Curtright T.L., Zachos C.K., Phys.Rev.Lett., 1984, 53, 1799-1801.
5. Abdalla E., Abdalla M.C.B., Phys.Lett., 1985, 152B, 59-62. di Vecchia P., Knizhnik V.G., Petersen J.L., Rossi P. Nucl. Phys., 1985, B253, 701.
6. Volovich I.V. Teor.Mat.Fiz., 1985, 63, N2, 312-314.
7. Nemeschansky D., Yankielowicz Sh., Phys.Rev.Lett., 1985, 54, 620-623.

8. Ivanov E.A., Krivonos S.O. In: Proc. VII Int. Conference on the Problems of Quantum Field Theory, Alushta, 1984, JINR D2-84-366, 257-278, Dubna, 1984; J.Phys.A: Math.Gen., 1984, 17, L671-L676.
9. Ivanov E.A., Krivonos S.O., Teor.Mat.Fiz., 1985, 63, N2, 230-243.
10. Pohlmeyer K., Commun. Math.Phys., 1976, 46, 207-221.
11. Luscher M., Pohlmeyer K., Nucl.Phys., 1978, B137, 46-54.
12. Eichenherr H., Forger M., Nucl.Phys., 1979, B155, 381-393; Commun.Math.Phys., 1981, 82, 227-255.
13. de Vega H.J. Phys.Lett., 1979, 87B, 233-236; Veselov A.P., TakhtadjanL.A., Dokl.Acad.Nauk SSSR, 1984, 279, N5, 1097-1100.
14. Abdalla M.C.B., Phys.Lett., 1985, 152B, 215-217.
15. Polyakov A.M., Wiegmann P.B., Phys.Lett., 1984, 141B, 223-226.
16. Faddeev L.D., Reshetikhin N.Yu. In: Proc. VII Int. Conference on the Problems of Quantum Field Theory, Alushta, 1984, JINR D2-84-366, 37-55, Dubna, 1984.
17. Zacharov V.E., Mikhailov A.V., ZhETF, 1978, 74, 1953-1973.
18. Zacharov V.E., Manakov S.V., Pitaevski L.P. Theory of Solitons. Moscow, Nauka, 1980.
19. Perelomov A.M., Uspekhi Fiz. Nauk, 1981, 134, 577-609.
20. Braaten E., Curtright T., Zachos C. Torsion and Geometrostasis in Nonlinear Sigma Models. Preprint EFTP-85-01 and ANL-HEP-PR-85-03, Florida, 1985.
21. Volkov D.V. E.Ch.A.Ya., 1973, 4, N1, 3-41.
22. Semenov-Tyan-Shansky M.A., Faddeev L.D. - Vestnik Leningr. Univer. 1977, N13, 81-88.
23. Nappi C.R. Phys.Rev., 1980, D21, 418-420.
24. Zacharov V.E. In: "Solitons", ed. R.Bullogh., Springer 1976. Kulish P.P., Teor.Mat.Fiz., 1977, 33, N2, 272-275.
25. Julia B., Luciani J.F., Phys.Lett., 1980, 90B, 270-274.
26. Honerkamp J., Nucl.Phys., 1972, B36, 130-140. Kazakov D.I., Pervushin V.N., Pushkin S.V., J.Phys.A: Math.Gen. 1978, 11, 2093-2105. Alvarez-Gaume L., Freedman D.Z., Mukhi S., Ann.Phys. (NY), 1981, 134, 85-109.
27. Green M., Schwarz J., Phys.Lett., 1984, 136B, 367-370; Nucl.Phys., 1984, B243, 285-306.
28. Henneaux M., Mezincescu L., Phys.Lett., 1985, 152B, 340-342.

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Дуальность в d = 2 моделях асимметричного кирального поля

Выявлена непрерывная дуальная симметрия уравнений асимметричного кирального поля /АКП/ в d = 2 /уравнений нелинейных σ - моделей с многозначным действием/ и найдена реализация преобразований дуальности на явно геометрическом языке форм Картана. Выяснена связь этой симметрии с интегрируемостью АКП. Рассмотрены как простой, так и суперсимметричный случаи. Введены понятия дуальной алгебры и дуальной σ-модели и показана их важная роль для понимания классической и квантовой структуры d = 2 моделей АКП. В частности, показано, что переход к точкам инфракрасной стабильности АКП можно описать чисто алгебраически как контракцию дуальной алгебры, приводящую к тому, что фактор-пространство соответствующей дуальной σ-модели становится плоским. С аналогичной точки зрения анализируются также уравнения асимметричной модели $\vec{n}$-поля. Метод форм Картана позволяет установить, что классическая динамика этой модели тривиальна.

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Ivanov E. E2-85-865

Dualities in the d=2 Asymmetric Chiral Field Sigma Models

Using a manifestly geometric language of Cartan's forms, we expose continuous dual symmetry of general d=2 asymmetric chiral field (ACF) models (another name of d=2 σ-models with multivalued action) and find corresponding duality transformations. Both ordinary and supersymmetric ACF are treated. Like in the case of standard chiral field, duality transformations of ACF prove to be intimately related to its integrability. We introduce also the notions of dual algebra and dual σ-model and demonstrate their important role for understanding the classical and quantum structure of ACF. It is shown, in particular, that passing to infrared fixed points of ACF models can be described in a pure algebraic way as a contraction of dual algebra, with which the coset space of dual σ-model becomes flat. Finally, we analyze, in an analogous context, the classical equations of asymmetric version of $\vec{n}$-field σ model. These are found to yield a trivial dynamics.

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