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TOWARDS TRANSPORT THEORY OF HADRON GASES

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1. INTRODUCTION

Considering hadron gases, one finds that the standard kinetic theory, see, e.g., [1], developed for the description of atomic gases is far inadequate for studying hadrons. The majority of hadrons is unstable. The lifetime of hadron resonances is so short that the decay width is comparable to the particle mass. The resonances are abundantly produced in hadron-hadron collisions, see, e.g. [2]. Thus, to describe a hadron gas, the resonances have to be taken into account. However, for the inclusion of unstable particles, the Boltzmann equation has to be modified. The aim of this paper is to consider such a minimal modification of the standard classical (non-quantum) theory at the phenomenological level. Namely, besides binary collisions, we take into account two-particle resonance decays and time-reversed processes, i.e., resonance formation. Then, we explicitly include the effect of resonance mass smearing.

There are other characteristic features of hadrons which are still outside our discussion. Of particular importance is, in our opinion, the fact that many hadrons may be produced in hadron collisions. Binary collisions dominate at relatively low incident energies only. The inclusion of such processes is difficult and resembles the problems found in attempts to develop the transport theory of dense atomic gases, see, e.g., [1].

The non-trivial equilibrium properties of a hadron gas have been widely discussed, in numerous papers by R.Hagedorn and collaborators [3,4]. The Gibbs statistical mechanics and the idea of statistical bootstrap model have been used in their considerations. The most important, in our opinion, Hagedorn's result is the prediction of limiting hadron temperature contemporarily interpreted as a temperature of phase transition to quark-gluon plasma [4]. Because the limiting temperature confirmed experimentally is of the order of pion mass, the average incident energy of hadron collision in the gas being in equilibrium does not exceed some hundreds MeV. In this incident energy region binary collisions dominate what makes our considerations (limited to decays, resonance formation and binary collisions) applicable for the description of hadron gas close to equilibrium. Anyhow our discussion is adequate for dilute gases where one can neglect the collisions with more than two particles in an initial state.

Our paper is organized as follows. In Sec. II we defined the classical distribution function of resonance and some macroscopical quantities. The Boltzmann equation is generalized in Sec. III. In Sec. IV the H-theorem is proved and an equilibrium state is considered. The equilibrium characteristics of a hadron gas are discussed in Sec. V. In Sec. VI we conclude our considerations.

II. THE DISTRIBUTION FUNCTION OF A RESONANCE

In our considerations we follow the textbook "Relativistic Kinetic Theory" by de Groot, van Leeuwen and van Weert [5]. Because the energy, E, and momentum, $\bar{p}$, of a resonance are not connected by the mass relation $E^2 - \bar{p}^2 = m^2$ $(c = k = \hbar = 1)$ the four-dimensional, relativistic formalism is a more natural framework for studying hadron gases than the three-dimensional nonrelativistic one.

The Lorentz invariant phase-space element of a stable particle d^3p/E is not adequate for a resonance since the energy and momentum have to be independent (quasi-independent) variables. However,

$$\frac{d^3\bar{p}}{E} = 2d^4p\,\delta(p^2 - m^2)\,\theta(E), \tag{1}$$

where $p^2 \equiv p^\mu p_\mu$, $p \equiv p^\mu = (E, \bar{p})$.

The above expression suggests the form of a resonance phase-space element

$$d^4p\,\Delta(p^2), \tag{2}$$

where the function Δ, later on called the profile function, describes the mass smearing of a resonance. We demand Δ to be a Lorentz scalar.

The profile function is assumed to depend on p^2 while is independent of any gas characteristics. In particular, we assume that a particle lifetime does not depend on the gas density. In general it is not true because the density of final states of a decay process can be significantly different in vacuum and in a dense gas at low temperature. For example, due to the Pauli quenching, the lifetime of the N^* resonance decaying into a pion and a nucleon can be much longer in nuclear matter than in vacuum*. We conclude as follows. Assuming that the profile function depends on p^2 only, we limit our considerations to classical gases. The form of the $\Delta(p^2)$ function will be discussed in the next section, where the connection with experimentally measurable quantities will be established.

In fact, the experimentaly measurable lifetime of resonance in nucleus is shorter than that in vacuum because of the resonance collisions with nucleons.

We define the distribution function so that

$$f(p,x)\,d^3\bar{x}\,d^4p\,\Delta(p^2)\,E, \quad x \equiv (t, \bar{x}) \tag{3}$$

gives an average number of resonances being at a moment of time t in the space element $d^3\bar{x}$ with the four-momentum between p and $p + d^4p$. The above definition will be more obvious if we write down the particle four-flow vector

$$N^\mu(x) = \int d^4p\,\Delta(p^2)\,p^\mu f(p,x), \tag{4}$$

which is an analogue of a stable particle four-flow

$$N^\mu_{st}(x) = \int \frac{d^3\bar{p}}{E}\,p^\mu f(p,x).$$

The definition (3), which plays a crucial role in all our considerations, lets us employ the standard scheme of the kinetic theory for studying hadron resonances.

Dealing with decaying particles, we are forced to consider a mixture of many sorts of particles. Thus, we denote by $f_i(x,p)$ the distribution function of an i-th sort of particles. The energy-momentum tensor and the entropy four-flow are the following

$$\left.\begin{aligned} T^{\mu\nu}(x) &= \sum_i \int d^4\tilde{p}_i\,p^\mu p^\nu f_i(p,x), \\ S^\mu(x) &= \sum_i \int d^4\tilde{p}_i\,p^\mu f_i(p,x)\,[\ln f_i(p,x) - 1], \end{aligned}\right. \tag{5}$$

$d^4\tilde{p}$ is the phase-space element of a stable particle (1) or a resonance (2).

III. THE KINETIC EQUATIONS

Let us consider the mixture of N^s and N^u sorts of stable and unstable particles, respectively. Assuming that a resonance decays into two stable particles, one finds the following set of kinetic equations

$$p^\mu\partial_\mu f_i(p,x) = C_i^s + D_i^s, \quad i = 1,2,\dots N^s. \tag{6a}$$

and

$$p^\mu\partial_\mu f_j(p,x) = C_j^u + D_j^u, \quad j = 1,2,\dots N^u, \tag{6b}$$

where C_i is the standard collision term describing the binary interactions, see e.g.,[5]. When a resonance is involved in a collision, the phase-space element (1) has to be replaced by (2).

$$D_i^s \equiv \sum_{j=1}^{N^u} \sum_{k=1}^{N^s} \int d^4 p_j \Delta^j (p_j^2) \frac{d^3 \bar{p}_k}{E_k} \cdot$$

$$\cdot [f_j(p_j, x) W^{j\to ik} (p_j | p, p_k) - f_i(p, x) f_k(p_k, x) W^{ik\to j} (p, p_k | p_j)],$$

$$D_j^u \equiv \sum_{i=1}^{N^s} \sum_{k=1}^{N^s} \int \frac{d^3 \bar{p}_i}{E_i} \frac{d^3 \bar{p}_k}{E_k} \cdot$$

$$\cdot [f_i(p_i, x) f_k(p_k, x) W^{ik\to j} (p_i, p_k | p) - f_j(p, x) W^{j\to ik} (p | p_i, p_k)],$$

where $W^{j\to ik} (p_j | p_i, p_k)$ is the transition rate for the decay of the resonance of an j-th sort having four-momentum p_j into two particles of i-th and k-th sorts with momenta p_i and p_k. $W^{ik\to j} (p_i, p_k | p_j)$ is the transition rate of the inverse process of resonance formation.

Let us rewrite the equation (6b) in the non-covariant, more familiar, form

$$\frac{\partial}{\partial t} f_j(p, x) + \bar{v}\nabla f_j(p, x) = \frac{C_j^u}{E} + \sum_{i=1}^{N^s} \sum_{k=1}^{N^s} \int d^3 \bar{p}_i \, d^3 \bar{p}_k \cdot$$

$$\cdot [f_i(p_i, x) f_k(p_k, x) \frac{W^{ik\to j} (p_i, p_k | p)}{E E_i E_k} - f_j(p, x) \frac{W^{j\to ik} (p | p_i, p_k)}{E E_i E_k}], \tag{7}$$

where $\bar{v} \equiv \bar{p}/E$. Recalling a physical interpretation of the distribution function, one finds from (7) the following connection of the transition rates with the measurable quantities

$$\frac{(2\pi)^3}{|\bar{v}_i - \bar{v}_k|} \frac{W^{ik\to j} (p_i, p_k | p_j)}{E_i E_j E_k} d^4 p_j \Delta^j (p_j^2) E_j = d\sigma^{ik\to j} \tag{8}$$

and

$$\frac{W^{j\to ik} (p_j | p_i, p_k)}{E_i E_j E_k} d^3 \bar{p}_i \, d^3 \bar{p}_k = d\Gamma^{j\to ik}, \tag{9}$$

where $\sigma^{ik\to j}$ is the cross section of j-th resonance formation; $|\bar{v}_i - \bar{v}_k|$ is the relative velocity of particles with four-momenta

p_i and p_k. $\Gamma^{j\to ik}$ is the partial decay width. The presence of a $(2\pi)^3$ coefficient in the formula (8) is related to the fact that the phase-space elements present in the kinetic equations are not divided by $(2\pi)^3$ while in the units which are used $\hbar = 1$ equals unity. Thus, the $(2\pi)^3$ coefficients are absorbed by the transition rates.

Since a four-momentum is conserved in any reaction, one can write.

$$W^{j\to ik} (p_j | p_i, p_k) = a(p_j | p_i p_k) \delta^{(4)} (p_j - p_i - p_k), \tag{10}$$

$$W^{ik\to j} (p_i, p_k | p_j) = a(p_i p_k | p_j) \delta^{(4)} (p_j - p_i - p_k). \tag{11}$$

Substituting (10) and (11) in (8) and (9), we determine the coefficients a. For the decay process the decay products are assumed to be isotropically distributed in the center-of-mass of decaying particle. In this way we are arrived to the formulae

$$W^{ik\to j} (p_i, p_k | p_j) = \frac{F_{ik} \sigma^{ik\to j}}{(2\pi)^3 \Delta^j (p_j^2)} \delta^{(4)} (p_j - p_i - p_k), \tag{12}$$

$$W^{j\to ik} (p_j | p_i, p_k) = \frac{M_j \Gamma^{j\to ik}}{L_{ik}} \delta^{(4)} (p_j - p_i - p_k), \tag{13}$$

where F_{ik} is the Lorentz invariant flux factor

$$F_{ik} \equiv E_i E_k |\bar{v}_i - \bar{v}_k| = ((p_i p_k)^2 - p_i^2 p_k^2)^{1/2} = \frac{1}{2} ((M_j^2 - m_i^2 - m_k^2)^2 - 4 m_i^2 m_k^2)^{1/2},$$

$$M_j^2 \equiv (p_i + p_k)^2 = p_j^2.$$

L_{ik} is the Lorentz invariant two-particle phase-space

$$L_{ik} \equiv \int \frac{d^3 p_i}{E_i} \frac{d^3 p_k}{E_k} \delta^{(4)} (p_j - p_i - p_k) = \frac{2\pi}{M_j^2} ((M_j^2 - m_i^2 - m_k^2)^2 - 4 m_i^2 m_k^2)^{1/2}.$$

L and F are related by the formula $L_{ik} = \frac{4\pi}{M_j^2} F_{ik}$.

Let us discuss how to determine the profile function. If we assume that the transition rates satisfy the detailed balance condition

$$W^{j\to ik} (p_j | p_i, p_k) = W^{ik\to j} (p_i, p_k | p_j),$$

one gets

$$\Delta^j(p^2) = \frac{1}{(2\pi)^3}\,\frac{L_{ik}\,F_{ik}}{M_j}\,\frac{\sigma^{ik\to j}}{\Gamma^{j\to ik}}\,. \tag{14}$$

Below we will discuss the above formula. But now we show another way leading to Eq. (14). We assume that the transition rates satisfy the bilateral normalization conditions

$$\sum_i \int d^4p\,\Delta^j(p^2)\,W^{ik\to j}(p_i,p_k\,|\,p_j) = \sum_i \int d^4p\,\Delta^j(p^2)\,W^{j\to ik}(p_j\,|\,p_i,p_k) \tag{15}$$

and

$$\sum_{i,k} \int \frac{d^3p_i}{E_i}\,\frac{d^3p_k}{E_k}\,W^{ik\to j}(p_i,p_k\,|\,p_j) = \sum_{i,k} \int \frac{d^3p_i}{E_i}\,\frac{d^3p_k}{E_k}\,W^{j\to ik}(p_j\,|\,p_i,p_k)\,. \tag{16}$$

The above expressions related to unitarity of the S-matrix are briefly discussed in Appendix.

Putting (12) and (13) in (15) and (16), one finds the following equations

$$\sum_j \frac{F_{ik}\,\sigma^{ik\to j}}{(2\pi)^3} = \sum_j \frac{M_j\Gamma^{j\to ik}}{L_{ik}}\,\Delta^j(p^2)$$

and

$$\sum_{i,k} \frac{F_{ik}\,\sigma^{ik\to j}\,L_{ik}}{(2\pi)^3} = \sum_{i,k} M_j\Gamma^{j\to ik}\,.$$

Because the first equation has to be satisfied for any i,k pairs while the second one for any j, we get the relation

$$\frac{F_{ik}\,\sigma^{ik\to j}}{(2\pi)^3} = \frac{M_j\Delta^j(p^2)\,\Gamma^{j\to ik}}{L_{ik}}\,,$$

which is equivalent to Eq. (14). In this approach the formula (14) provides the detailed balance condition. Thus, the detailed balance occured to be a consequence of the bilateral normalization conditions (15) and (16), the four-momentum conservation and the assumption of the isotropic distribution of decay products in the center-of-mass of the decaying particle.

As the profile function characterizes a resonance but not a decay channel, the formula (14) should give the same results for different i,k pairs. We cannot rigorously prove that the profile function described by (14) is unique. However, we present simplified argumentation and then we show that the independence of i,k indexes is realized for the Breit-Wigner form of the cross section.

One expects the following relation

$$\frac{|\mathfrak{M}^{ik\to j}|^2}{|\mathfrak{M}^{\ell n\to j}|^2} = \frac{|\mathfrak{M}^{j\to ik}|^2}{|\mathfrak{M}^{j\to \ell n}|^2}\,, \tag{17}$$

where $\mathfrak{M}$ is the transition matrix of the indicated process. The above relation is strictly correct when the interaction is invariant under time inversion what is the case, at least approximately, for strong interactions. If the decay width can be factorized as follows

$$\Gamma^{j\to ik} = |\mathfrak{M}^{j\to ik}|^2 L_{ik}\,, \tag{18}$$

we get from Eq. (17) the condition

$$\frac{F_{ik}\,\sigma^{ik\to j}}{F_{\ell n}\,\sigma^{\ell n\to j}} = \frac{\Gamma^{j\to ik}/L_{ik}}{\Gamma^{j\to \ell n}/L_{\ell n}}\,,$$

which makes the formula (13) independent of i,k indexes. However, the factorization (18) is totaly justified for narrow resonances only.

A resonance formation is usually described by the Breit-Wigner cross section, see, e.g.,[6]

$$\sigma^{ik\to j} = \frac{\pi}{\bar{p}^{*2}}\,\frac{\Gamma^{ik\to j}\,\Gamma_j}{(\sqrt{S}-\bar{M}_j)^2+\Gamma_j^2/4}\,,$$

where $\bar{p}^{*2} = \frac{F_{ik}^2}{S}$ is the CM momentum square, $\sqrt{S}$ is the CM energy and Γ_j, $\Gamma^{j\to ik}$ are the total and partial decay widths, respectively. $\bar{M}_j$ is the average resonance mass. Substituting the above formula in (14), we find

$$\Delta^j(M_j^2) = \frac{1}{2\pi}\,\frac{\Gamma_j}{M_j}\,\frac{1}{(M_j-\bar{M}_j)^2+\Gamma_j^2/4}\,. \tag{19}$$

Thus, uniqueness of the profile function for the Breit-Wigner cross section has been demonstrated.

Considerations similar to those leading to the formula (14) may be repeated for binary collisions. In this case the profile function is expressed through the cross sections of reaction

$a + b \to$ resonance + c and the inverse one. Because the profile function arising from the binary collisions and the three particle reactions (resonance formation and decay) has to be the same, one can get relations between the cross sections of the different processes, where the resonance is involved.

IV. H-THEOREM AND AN EQUILIBRIUM STATE

The entropy production $\mathcal{H}$ is

$$\mathcal{H} = \partial^{\mu} S_{\mu} = \sum_k \int d^4 \bar{p}_k \ln f_k \, p^{\mu} \partial_{\mu} f_k .$$

Assuming that the distribution functions satisfy the kinetic equations (6), $p^{\mu}\partial_{\mu} f_k$ can be replaced by the collision terms of the right side of Eq. (6). If we assume that the transition rates for binary collisions and those of three-particle interactions satisfy the bilateral normalization conditions, we can consider separately the entropy production resulting from the binary collisions and the three-particle reactions. Anyhow it should be stressed that such an assumption is stronger than that arising from unitrarity of the S-matrix, see Appendix.

With the help of the bilateral normalization conditions one finds

$$\mathcal{H} = \mathcal{H}_B + \sum_{i,j,k} \int \frac{d^3\bar{p}_i}{E_i} \frac{d^3\bar{p}_k}{E_k} d^4 p_j \Delta^j (p_j^2) \, .$$

$$\{[\kappa - \ln \kappa - 1] f_j W^{j \to ik}(p_j \,|\, p_i, p_k) + [\kappa^{-1} + \ln \kappa - 1] f_i f_k W^{ik \to j}(p_i, p_k \,|\, p_j)\} ,$$

where $\mathcal{H}_B$ is the entropy production due to the binary reactions, see, e.g., /5/,

$$\kappa \equiv \frac{f_i(p_i) f_k(p_k)}{f_j(p_j)} ,$$

The operations leading to the above formula are quite analogous to those described in Ref.5. It is seen that $\mathcal{H} \geq 0$ and the entropy production vanishes when

$$f_i(p_i) f_k(p_k) = f_j(p_j) \quad \text{for} \quad p_i + p_k = p_j . \tag{20}$$

Equilibrium, defined as a maximum entropy state, is reached when the distribution functions satisfy the functional relation (20). Standard considerations, see, e.g., Ref.5, provide the Jüttner equilibrium function, i.e., a relativistic analogue of the Maxwell-Boltzmann distribution

$$f_j^{eq}(p) = \frac{g}{(2\pi)^3 V} \exp\left(\frac{\mu - u^{\nu} p_{\nu}}{T}\right) , \tag{21}$$

where g is the number of internal degrees of freedom of an j-th sort of particles, V is the volume of the system, u^{ν} is the four-velocity of the system as a whole. Thus, the form of the equilibrium distribution functions of stable and unstable particles is the same.

At the end of this section let us observe that the decay and formation processes provide an additional contribution to the entropy production. So, the presence of resonances in a system accelerates its equilibration and consequently makes shorter the relaxation time.

V. MACROSCOPIC CHARACTERISTICS OF THE HADRON GAS IN EQUILIBRIUM

In this section we consider macroscopic characteristics like density and internal energy of the gas. We focus our attention on the resonance component of the gas. For simplicity we assume that particles do not carry any conserved charges. Thus, the numbers of particles are unlimited and the chemical potentials of all types of particles are equal to zero.

Using the formulae (4) and (5), one finds the density and the internal energy density of an j-th sort of resonance.

$$n_j = \int d^4 p \, \Delta^j (p^2) \, E f_j^{eq}(p) , \quad U_j = \int d^4 p \, \Delta^j (p^2) \, E^2 f_j^{eq}(p) . \tag{22}$$

The equilibrium distribution function (21) in the rest frame of the system ($u^{\nu} = (1,0,0,0)$) for $\mu = 0$ is

$$f_j^{eq}(p) = \frac{1}{(2\pi)^3 V} e^{-E/T} . \tag{23}$$

In the formulae (22) we change the variables $(E, \bar{p}) \to (M, \bar{p})$, where $M^2 = E^2 - \bar{p}^2$. Putting (23) in (22) and integrating with respect to momenta, we get

$$n_j = \int_0^{\infty} dM M \Delta^j (M^2) \{\frac{1}{2\pi^2 V} T M^2 K_2(M/T)\} ,$$

$$U_j = \int_0^\infty dM M \Delta^j(M^2)\{\frac{1}{2\pi^2 V} T^2 M^2 [\frac{M}{T} K_1(M/T) + 3K_2(M/T)]\}, \qquad (24)$$

where K_1 and K_2 are the so-called MacDonald functions [7]. In the parantheses under the integrals (24) one recognizes the density and internal energy density of stable particles, see, e.g., [5]. The resonance characteristics are those of stable particles averaged over the mass. One may wonder, what is the normalization of the profile function. The explicit calculation shows that for the Breit-Wigner form (19) $\int dM M\Delta(M) = 1$. However, in this case the lower limit of the above integral has to be shifted to minus infinity. This operation is correct for the resonances with $\bar{M} \gg \Gamma$. Indeed the Breit-Wigner formula is of physical meaning for "narrow" resonances only.

Substituting the Breit-Wigner profile function (19) in the formulae (24), we obtain

$$n_j = \frac{1}{4\pi^3}\frac{T\Gamma_j}{V}\int_0^\infty dM \frac{M^2}{(M-\bar{M}_j)^2 - \Gamma_j^2/4} K_2(M/T),$$

$$U_j = \frac{1}{4\pi^3}\frac{T^2\Gamma_j}{V}\int_0^\infty dM \frac{M^2}{(M-\bar{M}_j)^2 + \Gamma_j^2/4}[3K_2(M/T) + \frac{M}{T}K_1(M/T)].$$

Since the above integrals cannot be calculated analytically, let us consider two limits.

For $\bar{M}_j \gg \Gamma_j$ and $T \gg \Gamma_j$ the functions $M^2 K_2$ and $M^2[3K_2 + (M/T)K_1]$, respectively, taken at $M = \bar{M}_j$ can be transferred from the integrals. Elementary integration provides the results

$$n_j = \frac{1}{2\pi^2}\frac{T\bar{M}_j^2}{V}K_2(\bar{M}_j/T),$$

$$U_j = \frac{1}{2\pi^2}\frac{T^2\bar{M}_j^2}{V}[3K_2(\bar{M}_j/T) + \frac{\bar{M}_j}{T}K_1(\bar{M}_j/T)]. \qquad (25)$$

As it would be expected, we have recovered the formulae for stable particles. It should be stressed that this result is not quite trivial as the procedure of determining the profile function is not trivial.

Unstability should strongly manifest itself at $\Gamma_j \gg T$. Because, we are interested in the qualitative effects of the mass smearing we use the Breit-Wigner formula for the "wide" resonance what is not quite correct. See the comment at the end of this section. We assume that Γ_j is of the order of $\bar{M}_j$ what additionally provides $\bar{M}_j \gg T$. Under such conditions we can put the function $[(M-\bar{M}_j)^2 + \Gamma_j^2/4]^{-1}$ taken at $M=0$ in front of the integrals. Then

$$n_j \cong \frac{3}{8\pi^2 V}\frac{T^4\Gamma_j}{\bar{M}_j^2 + \Gamma_j^2/4}, \qquad U_j \cong \frac{3}{2\pi^2 V}\frac{T^5\Gamma_j}{\bar{M}_j^2 + \Gamma_j^2/4}. \qquad (26)$$

We have used the equality [7]

$$\int_0^\infty x^{\alpha-1}K_\nu(x)\,dx = 2^{\alpha-2}\Gamma(\frac{\alpha+\nu}{2})\Gamma(\frac{\alpha-\nu}{2}),$$

where $\Gamma(z)$ is the Euler gamma function and $\mathrm{Re}\,\alpha > |\mathrm{Re}\,\nu|$. Let us compare the formulae (26) with analogous expressions for stable particles (formulae (25) for $\bar{M}_j \gg T$).

$$n_j^{st} \cong \frac{1}{V}(\frac{T\bar{M}_j}{2\pi})^{3/2} e^{-\bar{M}_j/T}[1 + \frac{15}{8}\frac{T}{\bar{M}_j}],$$

$$U_j^{st} \cong \frac{1}{V}(\frac{T\bar{M}_j}{2\pi})^{3/2}\bar{M}_j e^{-\bar{M}_j/T}[1 + \frac{27}{8}\frac{T}{\bar{M}_j}]. \qquad (27)$$

We see that the concentration of the resonances of average mass $\bar{M}$ highly exceeds the concentration of stable particles with mass $\bar{M}$.

It is the well-known experimental fact [2] that in hadron-hadron collisions at high energy there is an abundant resonance production as compared to pion yield. This abundance seems to decrease with incident energy. For many authors a big yield of relatively massive resonances was a crucial argument against thermodynamical approaches to particle production in hadron collisions since it was asserted that the generation of massive particles was exponentially suppressed according to the formula (27). As shown, the formula (27) can highly understimate the resonance yield what seems to invalidate the above argumentation.

From the formulae (26) one can find the energy per particle for a "wide" resonance at low temperature $\mathcal{E} = 4T$. The above expression resembles the one for massless particles. It shows how important the effect of mass smearing can be.

At the end of this section the comment is in order. Our results concerning "wide" resonances are based on the Brei-Wigner profile function (19). There is a common concensus, that the energy distribution of the resonance should be of the Breit-Wigner form near a maximum of the mass distribution. The problem of distribution "tails", which are important for the validity of the formulae (26), is cumbersome. There are rigorous arguments that the "tails" should deviate from the Breit-Wigner form while it is not clear how to modify them. For extensive discussion of the problem which, is, on the other hand, related to a non-exponential character of the decay law, see the review [8]. In the context of hadron resonances the problem of mass distribution has been discussed in Ref.9.

We conclude this section as follows. While the formulae (26) may be invalid due to uncertainties of the Breit-Wigner distribution "tails", the qualitative results of this section seem to be correct.

VI. CONCLUDING REMARKS

Let us discuss the assumptions leading to our kinetic theory model of hadron gas. The first important assumption occurs in the distribution function definition (3). Namely, we assume that the profile function is position-independent. As it has been argued in this way, quantum effects have been neglected. In the other case it would not be possible to determine the profile function with the help of the formula (14). Since the profile function present in (3) is not specified, no other assumptions are made at this step of model formulation. Then, the kinetic equations have been considered and the collision terms have been defined. We have assumed that the profile function can be extracted from the transition rates in the way analogous to the extraction of the delta functions $\delta(p^2 - m^2)$ for stable particles with mass m. The precise meaning of this operation is stated in the formulae (8) and (9), where the transition rates are connected with the experimentally measurable quantities. Later on, no assumptions characteristics for our model are made.

The results of Sec.V are more or less obvious. Macroscopic characteristics of resonances are those of stable particles averaged over mass. Anyhow there are two important ingredients of the formulae (24). It has been shown that the equilibrium functions of resonances coicide with those of stable particles. On the other hand, the profile function, i.e., the weight function in (24), has been uniquely determined.

We conclude as follows. The approach based on the distribution function definition (3) and the notion of profile function provides the self-consistent formalism very similar to the standard one and compatible with physical intuition concerning unstable particles.

I am grateful to Prof. G.M.Zinovjev and his collaborators for fruitful discussions.

APPENDIX

Unitarity of the S operator provides two equalities

$$\sum_{\beta} |<\beta|S|\alpha>|^2 = \sum_{\alpha} |<\beta|S|\alpha>|^2 = 1 \,. \tag{A1}$$

From Eq.(A1) we get the bilateral normalization condition

$$\sum_{\alpha} (|<\beta|S|\alpha>|^2 - |<\alpha|S|\beta>|^2) = 0 \,. \tag{A2}$$

Let us decompose the complete set of states α into states with definite number, N, of particles $\{\alpha\} = \sum_N \{\alpha_N\}$. We rewrite Eq. A2 in the form

$$\sum_N \Big(\sum_{\alpha_N} (|<\beta|S|\alpha_N>|^2 - |<\alpha_N|S|\beta>|^2) \Big) = 0 \,.$$

For determining the profile function and proving the H-theorem we have used the assumption that

$$\sum_{\alpha_N} (|<\beta|S|\alpha_N|^2 - |<\alpha_N|S|\beta>|^2) = 0 \,, \tag{A3}$$

which is stronger than Eq.(A2) arising from unitarity of the S-matrix. It is seen, however, that Eq.(A3) is strictly correct for interactions invariant under time inversien what is the case (at least approximately) for strong interactions.

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Мрувчински С. E2-85-85

К теории транспорта адронных газов

Аргументируется существенная роль адронных резонансов в определении характеристик адронного газа. В работе развивается кинетическая модель адронов. С помощью функции профиля, которая является аналогом дельта-функции массовой поверхности для стабильных частиц, определяется классическая неквантовая функция распределения резонансов. Для учета процессов формирования и распадов резонансов обобщается уравнение Больцмана. Чтобы определить неизвестную функцию профиля, предполагается, что скорости перехода удовлетворяют условию двусторонней нормировки или принципу детального равновесия. Функция профиля выражается через сечение формирования резонанса и ширину его распада. Доказана Н-теорема и показано, что форма равновесной функции распределения резонансов совпадает с функцией распределения стабильной частицы. Изучаются равновесные характеристики адронного газа и демонстрируется важность эффекта неопределенности массы резонанса.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Mrówczyński S. E2-85-85

Towards Transport Theory of Hadron Gases

An important role of hadron resonances for determining the characteristics of hadron gases is argued. A kinetic theory model of hadron gas is developed. A classical, non-quantum, distribution function of a resonance is defined with the help of the profile function being an analogue of the mass shell delta function of stable particles. The Boltzmann equation is generalized to include the resonance decay and resonance formation processes. To determine the unknown profile function, the transition rates are assumed to satisfy the bilateral normalization or the detailed balance condition. The profile function is expressed through the resonance formation cross section and the decay width. The H-theorem is proved, and it is shown that the form of the equilibrium distribution function of a resonance coincides with the one of a stable particle. Macroscopic equilibrium characteristics are studied. Significance of the resonance mass smearing effect is demonstrated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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