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BOSONIC AND FERMIONIC STRING MODELS IN THE SPACE OF LIE GROUPS

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1. Recently, some progress has been made [1,2] in quantization of the two-dimensional chiral field model. As is well known, the quantum inverse scattering method cannot be applied to this model directly because of nonultralocality of the L -operator which appears in the auxiliary spectral problem[3]. On the other hand, there exists a very important, for investigation of fundamental physical processes, example of integrable but also nonultralocal theory, the theory of relativistic string[4]. Thus, we can try to apply the successful methods of the above-mentioned papers[1,2] for the case of the relativistic string too. To do this, however, it is necessary to reveal the connection between the chiral field theory and the relativistic string theory even if on the classical level. The present paper is devoted to the solution of this problem. Briefly, its content consists in the following. A new geometrical model of the chiral field, which possesses a more extended symmetry than the usual one is formulated. The careful investigation of the field dynamics in the new model shows (see section 2) that this dynamics is a "cut" one (because of the constraints on the canonical variables) of the usual chiral field model. On the other hand, one can verify (see section 3) that relativistic string is described by the new model in some special case.

Thus (leaving aside the questions related with conformal anomalies[5,6]), one is under the impression that the quantum theory of the relativistic string should be "put in" the quantum theory of the chiral field. A supersymmetric extension of the examined theory is discussed at the end of the second section.

2. Let us consider the dynamics of a system (in two-dimensional space-time), a physical state of which is described by the matrix field $\mathcal{U}(\xi_1,\xi_2)$. Let the field $\mathcal{U}(\xi_1,\xi_2)=\exp(x_a(\xi_1,\xi_2)T^a)$ be an element of some matrix Lie group G. Here T^a are generators of G

$$[T^a, T^b] = t^{abd}\,T_d \quad , \quad Tr\,(T^a T^b) = -\eta^{ab},$$

t^{abd} are structure constants of G, and $x^a(\xi_1,\xi_2)$ are parameters of this group. The dynamical equations for the field $U(\xi_1,\xi_2)$ are defined by the minimization of the action

$$S = \frac{1}{2\gamma^2}\int_{\partial V} d^2\xi\,\sqrt{-\det\|g_{\alpha\beta}\|} + \frac{N}{24\pi}\int_V d^3\xi\,\varepsilon^{\alpha\beta\gamma}\,Tr(U^{-1}\partial_\alpha U\,U^{-1}\partial_\beta U\,U^{-1}\partial_\gamma U), \qquad (1)$$

where the metric $g_{\alpha\beta} = Tr(\partial_\alpha U^{-1}\partial_\beta U) = -Tr(U^{-1}\partial_\alpha U U^{-1}\partial_\beta U)$,

$\tilde{g} = det\|g_{\alpha\beta}\| = \frac{1}{2}\varepsilon^{\alpha\alpha'}\varepsilon^{\beta\beta'} g_{\alpha\beta}g_{\alpha'\beta'}$, $(\varepsilon^{12} = -\varepsilon^{21} = 1,\ \varepsilon^{11} = \varepsilon^{22} = 0)$.

The quantities γ^2 and N (N is an integer in the quantum case/7/) are parameters of the theory. The action (1) leads to the equation of motion

$$\frac{1}{2\gamma^2}\partial_\alpha(g^{\alpha\beta}\sqrt{-\tilde{g}}\,U^{-1}\partial_\beta U) - \frac{N}{8\pi}\varepsilon^{\alpha\beta}\partial_\alpha(U^{-1}\partial_\beta U) = 0, \qquad (2)$$

where $g^{\alpha\beta} = \frac{1}{\tilde{g}}\varepsilon^{\alpha\alpha'}\varepsilon^{\beta\beta'}g_{\alpha'\beta'}$. Before proceeding to the discussion of the Hamiltonian formulation of the theory (1) and of the Lax representation for the matrix eq. (2) let us emphasize that (1) is invariant under the transformations of reparametrization of the manifold V and its boundary ∂V

$$U(\xi_1,\xi_2,\xi_3) \longrightarrow U(f_1(\xi_1,\xi_2,\xi_3), f_2(\xi_1,\xi_2,\xi_3), f_3(\xi_1,\xi_2,\xi_3)) \qquad (3)$$

(f_1, f_2, f_3 are nonsingular functions) and also under the global transformations

$$U \to g_L U g_R\ , (g_L, g_R \in G) \qquad (4)$$

generating the $G_L \otimes G_R$ group. The symmetry (3) demonstrates that the system (1) is an example of the degenerated Hamiltonian system; thus, we must put constraints on the canonical variables (which will be introduced below).

To investigate the Hamiltonian structure of the system(1), let us first investigate an auxiliary system with the action

$$S = \frac{1}{2\gamma^2}\int_{\partial V} d^2\xi\sqrt{-\tilde{g}} + \frac{N}{24\pi}\int_{\partial V} d^2\xi\,\varepsilon^{\alpha\beta}Tr(\tilde{A}(U)U^{-1}\partial_\alpha U U^{-1}\partial_\beta U) = \int_{\partial V} d^2\xi\,\mathcal{L}\ . \qquad (5)$$

Here $\tilde{A}(U)$ is an "external field". The action (5) can be transformed into the action (1), if we demand that

$$\partial_\alpha\tilde{A}(U)_{ab} = \frac{\partial\tilde{A}_{ab}}{\partial U_{cd}}\partial_\alpha U_{cd} = U^{-1}_{ac}\partial_\alpha U_{cd} \qquad (6)$$

$(a,b,c,d$ are matrix indices of the U field). Thus, we assert that all formulae of the Hamiltonian structure for the system (1) are obtained if we replace the "external field" $\tilde{A}$ in all formulae of the Hamiltonian structure of the system (5) by the formal symbol $\tilde{A}$ from eq. (6). Therefore, let us first consider the Hamiltonian formulation for the system (5). We will regard ξ_1 as "time". Then, for the canonical momentum $\tilde{p}$ we get the expression

$$\tilde{p} = \frac{\partial\mathcal{L}}{\partial\partial_1 U} = \frac{1}{2\gamma^2}\,\frac{-g_{12}U^{-1}\partial_2 U U^{-1} + g_{22}U^{-1}\partial_1 U U^{-1}}{\sqrt{-\tilde{g}}} + \frac{N}{24\pi}(U^{-1}\partial_2 U\tilde{A}U^{-1} - \tilde{A}U^{-1}\partial_2 U U^{-1}). \qquad (7)$$

Let us introduce the new variables p which is not dependent on the "external field" $\tilde{A}$:

$$p = \tilde{p}U - \frac{N}{24\pi}[U^{-1}\partial_2 U, \tilde{A}]\ , \qquad (8)$$

$$p = \frac{1}{2\gamma^2}\,\frac{-g_{12}U^{-1}\partial_2 U + g_{22}U^{-1}\partial_1 U}{\sqrt{-\tilde{g}}} \qquad (9)$$

It is convenient also to introduce variables

$$a = p + \frac{1}{2\gamma^2}U^{-1}\partial_2 U = a^a(\xi_1,\xi_2)T_a\ , \qquad (10)$$

$$b = p - \frac{1}{2\gamma^2}U^{-1}\partial_2 U = b^a(\xi_1,\xi_2)T_a\ .$$

It is easy to check, taking into account (9), that we have the relations

$$Tr(a^2) = Tr(b^2) = 0. \qquad (11)$$

These relations impose constraints on the canonical variables and are the consequence of the local symmetry of the action (5) under the reparametrization (3).

Let us introduce a symplectic structure in the phase space of our auxiliary system according to the canonical Poisson brackets (here U_{ab} are independent functions)

$$\{U_{ab}(\xi_1,\xi_2), \tilde{P}_{cd}(\xi_1,\xi_2')\} = \delta(\xi_2 - \xi_2')\,\delta_{ad}\delta_{cb} \qquad (12)$$

(we will omit the time variable ξ_1 in the formulae below where it will not lead to misunderstanding).

As it has been pointed out above, all formulae which we get for the system (5) are the same as for the system (1) if we take into account the formal relation (6). So, for the Poisson bracket of the variables (8), (10) we get (the case of the system (1); now we consider U as an element of the G-group again)

$$\{p(\xi_2)\overset{\otimes}{,}p(\xi_2')\} = \delta(\xi_2-\xi_2')[T^a\otimes T_a, (p - \tfrac{N}{8\pi}U^{-1}\partial_2 U)\otimes 1], \qquad (13a)$$

$$\{a(\xi_2)\overset{\otimes}{,}a(\xi_2')\} = \delta(\xi_2-\xi_2')[T^a\otimes T_a, ((\tfrac{3}{2} - \tfrac{N\gamma^2}{8\pi})a + (-\tfrac{1}{2} + \tfrac{N\gamma^2}{8\pi})b)\otimes 1] - \frac{1}{\gamma^2}T^a\otimes T_a\,\delta'(\xi_2-\xi_2'), \qquad (13b)$$

$$\{b(\xi_2)\overset{\otimes}{,}b(\xi_2')\} = \delta(\xi_2-\xi_2')[T^a\otimes T_a, ((-\tfrac{1}{2} - \tfrac{N\gamma^2}{8\pi})a + (\tfrac{3}{2} + \tfrac{N\gamma^2}{8\pi})b)\otimes 1] + \frac{1}{\gamma^2}T^a\otimes T_a\,\delta'(\xi_2-\xi_2'), \qquad (13c)$$

$$\{a(\xi_2)\overset{\otimes}{,}\, b(\xi_2')\} = \delta(\xi_2-\xi_2')\Big[T^a\otimes T_a,\Big(\frac{1}{2}+\frac{N\gamma^2}{8\pi}\Big)a\otimes 1 + \Big(\frac{1}{2}+\frac{N\gamma^2}{8\pi}\Big)b\otimes 1\Big], \tag{13d}$$

The algebra (13b-d) is the same as that appearing in the theory of the chiral field with the Wess-Zumino term [1,7]. The equations of motion (2), (9) in terms of the variables $a(\xi)$ and $b(\xi)$ have the form

$$\frac{\partial}{\partial\xi_1}a = \frac{1}{2\gamma^2}\frac{\partial}{\partial\xi_2}(fa) + \frac{1}{4}\Big(1-\frac{N\gamma^2}{4\pi}\Big)(f-g)[a,b], \tag{14}$$

$$\frac{\partial}{\partial\xi_1}b = \frac{1}{2\gamma^2}\frac{\partial}{\partial\xi_2}(gb) - \frac{1}{4}\Big(1+\frac{N\gamma^2}{4\pi}\Big)(f-g)[a,b]$$

and can be obtained in a usual manner from the Hamiltonian

$$\mathcal{H}(\xi_1) = \int d\xi_2\Big(\frac{1}{4}f(\xi_1,\xi_2)\,\mathrm{Tr}\,a^2(\xi_1,\xi_2) - \frac{1}{4}g(\xi_1,\xi_2)\,\mathrm{Tr}\,b^2(\xi_1,\xi_2)\Big). \tag{15}$$

Here functions f and g are the Lagrangian multipliers and are connected with the metrics $g_{\alpha\beta}$ through the relations

$$\frac{1}{2\gamma^2}f = \frac{\sqrt{-g}+g_{12}}{g_{22}}, \qquad \frac{1}{2\gamma^2}g = \frac{g_{12}-\sqrt{-g}}{g_{22}}.$$

We emphasize that the case $f=-g=2\gamma^2$ (when eqs. (14) are the same as those in the chiral field theory [1,7]) is equivalent to the choice of the gauge

$$g_{12}=0, \quad g_{11}+g_{22}=0. \tag{16}$$

The system of eqs. (14) can be represented in the Lax form

$$[L,M] = \Big[\frac{\partial}{\partial\xi_2} - \Big(\frac{a}{\mu_1}+\frac{b}{\mu_2}\Big), \frac{\partial}{\partial\xi_1} - \Big(\frac{fa}{\mu_1}+\frac{gb}{\mu_2}\Big)\frac{1}{2\gamma^2}\Big] = 0 \tag{17}$$

if the spectral parameters μ_1 and μ_2 satisfy the condition

$$\mu_2\Big(1-\frac{N\gamma^2}{4\pi}\Big) - \mu_1\Big(1+\frac{N\gamma^2}{4\pi}\Big) = \frac{2}{\gamma^2}. \tag{18}$$

First of all, we see that the L operator is not dependent on arbitrary functions f and g ; therefore, the conservation laws which one can get using the L operator are manifestly gauge invariant.

Now, let us consider an auxiliary spectral problem

$$L(\xi_2)\Psi(\xi_2,\mu_1) = \Big(\frac{d}{d\xi_2} - \Big(\frac{a(\xi_2)}{\mu_1}+\frac{b(\xi_2)}{\mu_2}\Big)\Big)\Psi(\xi_2,\mu_1) = 0. \tag{19}$$

We will only discuss the periodic boundary condition $a(\xi_2)=a(\xi_2+2\pi)$, $b(\xi_2)=b(\xi_2+2\pi)$. For the solutions $\Psi(\xi_2;\mu_1)$ of eq. (19) one can construct the monodromy matrix $T_{\xi_2}^{\xi_2+2\pi}(\mu_1)$, such that

$$T_{\xi_2}^{\xi_2+2\pi}(\mu_1)\Psi(\xi_2,\mu_1) = \Psi(\xi_2+2\pi,\mu_1),$$

$$\frac{\partial}{\partial\xi_2}T_{\xi_2}^{\xi_2+2\pi}(\mu_1) = \Big[\Big(\frac{a(\xi_2)}{\mu_1}+\frac{b(\xi_2)}{\mu_2}\Big), T_{\xi_2}^{\xi_2+2\pi}(\mu_1)\Big].$$

It follows from eq. (17) that the function $T(\mu_1) = \mathrm{Tr}\big(T_{\xi_2}^{\xi_2+2\pi}(\mu_1)\big)$ is time-independent

$$\frac{\partial}{\partial\xi_1}T(\mu_1) = \{T(\mu_1), \mathrm{Tr}\,a^2(\xi_2)\} = \{T(\mu_1), \mathrm{Tr}\,b^2(\xi_2)\} = 0. \tag{20}$$

Moreover, the integrals of motion obtained from $T(\mu_1)$ by the expansion around poles $\mu_1=0$ and $\mu_2=0$ generate the involutory sets of conservation laws. This follows from the relation

$$\{T(\mu_1), T(\lambda_1)\} = 0, \quad \forall \mu_1, \lambda_1. \tag{21}$$

The last relation can easily be verified by introducing the new variables

$$A = \Big(1+\frac{N\gamma^2}{4\pi}\Big)a + \Big(1-\frac{N\gamma^2}{4\pi}\Big)b, \tag{22}$$

$$B = \Big(1-\frac{N\gamma^2}{4\pi}\Big)a + \Big(1+\frac{N\gamma^2}{4\pi}\Big)b.$$

These variables generate the extended Kac-Moody algebra

$$\{A(\xi_2)\overset{\otimes}{,}\, A(\xi_2')\} = 2\delta(\xi_2-\xi_2')[T^a\otimes T_a, A(\xi_2)\otimes 1] - \frac{N}{\pi}\delta'(\xi_2-\xi_2')T^a\otimes T_a,$$

$$\{A(\xi_2)\overset{\otimes}{,}\, B(\xi_2')\} = 2\delta(\xi_2-\xi_2')[T^a\otimes T_a, B(\xi_2)\otimes 1], \tag{23}$$

$$\{B(\xi_2)\overset{\otimes}{,}\, B(\xi_2')\} = 2\delta(\xi_2-\xi_2')[T^a\otimes T_a, -A(\xi_2)\otimes 1 + 2B(\xi_2)\otimes 1] + \frac{N}{\pi}\delta'(\xi_2-\xi_2')T^a\otimes T_a.$$

For the L -operator we get the expression

$$L(\xi_2) = \frac{d}{d\xi_2} - \big(\mathcal{F}_1(\mu_1)A(\xi_2) + \mathcal{F}_2(\mu_1)B(\xi_2)\big), \tag{24}$$

where

$$\mathcal{F}_1(\mu_1) = \frac{\frac{2}{\gamma^2}(1+\varkappa) + 4\varkappa\mu_1}{4\varkappa\mu_1\big(\frac{2}{\gamma^2}+\mu_1(1+\varkappa)\big)}, \tag{25}$$

$$\mathcal{F}_2(\mu_1) = \frac{-\frac{2}{\gamma^2}(1-\varkappa)}{4\varkappa\mu_1\big(\frac{2}{\gamma^2}+\mu_1(1+\varkappa)\big)}, \quad \varkappa = \frac{N\gamma^2}{4\pi}.$$

Using the method for the calculation of the Poisson bracket of the monodromy matrix elements in the nonultralocal case [8] and simultaneously taking into account boundary terms we find that the equality (21) can be fulfilled if functions $\mathcal{F}_1(\mu)$ and $\mathcal{F}_2(\mu)$ satisfy the condition

$$\frac{(\mathcal{F}_1(\mu)\mathcal{F}_1(\lambda)-\mathcal{F}_2(\mu)\mathcal{F}_2(\lambda))(2+\frac{N}{2\pi}(\mathcal{F}_1(\mu)+\mathcal{F}_1(\lambda)))}{\mathcal{F}_1(\lambda)-\mathcal{F}_1(\mu)} =$$

$$= \frac{[2(\mathcal{F}_1(\mu)\mathcal{F}_2(\lambda)+\mathcal{F}_2(\mu)\mathcal{F}_1(\lambda)+2\mathcal{F}_2(\mu)\mathcal{F}_2(\lambda))+\frac{N}{2\pi}(\mathcal{F}_1(\mu)\mathcal{F}_1(\lambda)-\mathcal{F}_2(\mu)\mathcal{F}_2(\lambda))(\mathcal{F}_2(\mu)+\mathcal{F}_2(\lambda))]}{\mathcal{F}_2(\lambda)-\mathcal{F}_2(\mu)}.$$

We can prove this condition immediately by noticing (see (25)) that

$$\mathcal{F}_1(\lambda)-\mathcal{F}_1(\mu)=(1+\varkappa)(\mu-\lambda)(\mathcal{F}_1(\lambda)\mathcal{F}_1(\mu)-\mathcal{F}_2(\lambda)\mathcal{F}_2(\mu)),$$

$$\frac{\mathcal{F}_1(\lambda)}{\mathcal{F}_2(\lambda)}-\frac{\mathcal{F}_1(\mu)}{\mathcal{F}_2(\mu)}=\frac{4\varkappa(\mu-\lambda)}{\frac{2}{\gamma^2}(1-\varkappa)}.$$

Thus, we observe that for the considered theory with the action (1) in the Hamiltonian formulation one can construct the auxiliary spectral problem (19) (which absolutely coincides with that in the case of the chiral field theory with the Wess-Zumino term /1/) and then obtain the generating function for two sets of commuting conservation laws. We emphasize here one important distinction of our theory from the chiral field theory. This distinction is related with the existence in our case of the additional relations (11) which represent constraints on the canonical variables. Let us emphasize now that eqs. (14), (17), the auxiliary spectral problem (19) and the monodromy matrix $T^{\xi_2+2\pi}_{\xi_2}(\mu)$ were written in terms of the right-handed quantities (10) (or (22)); i.e. in terms of those quantities which are transformed only by G_R subgroup of $G_R\otimes G_L$. However, one can carry out the whole investigation only in terms of the left-handed quantities. The connection between these two investigations is realized by the following transition from the right-handed L, M operators (17) to the left-handed L_L and M_L:

$$L_L=ULU^{-1}, \quad M_L=UMU^{-1}.$$

In this case the right and left-handed monodromy matrices are connected by the relation $T^{\xi_2+2\pi}_{\xi_2}(\mu)_L=U\,T^{\xi_2+2\pi}_{\xi_2}(\mu)_R\,U^{-1}$. Let us introduce now the left-handed variables $A_L=UAU^{-1}$, $B_L=UBU^{-1}$, where A and B are determined by eqs. (22). The variables A_L and B_L generate the algebra analogous to the algebra (23):

$$\begin{aligned}
&\{B_L(\xi)\overset{\otimes}{,}B_L(\xi')\}=2\delta(\xi-\xi')[B_L(\xi)\otimes 1, T^a\otimes T_a]+\frac{N}{\pi}\delta'(\xi-\xi')T^a\otimes T_a,\\
&\{B_L(\xi)\overset{\otimes}{,}A_L(\xi')\}=2\delta(\xi-\xi')[A_L(\xi)\otimes 1, T^a\otimes T_a], \qquad (26)\\
&\{A_L(\xi)\overset{\otimes}{,}A_L(\xi')\}=2\delta(\xi-\xi')[-B_L(\xi)\otimes 1+2A_L(\xi)\otimes 1, T^a\otimes T_a]-\\
&\qquad -\frac{N}{\pi}\delta'(\xi-\xi')T^a\otimes T_a.
\end{aligned}$$

The Poisson brackets of the right-handed quantities A, B (22) with the left-handed quantities A_L and B_L have the form

$$\begin{aligned}
&\{B_L(\xi)\overset{\otimes}{,}A(\xi')\}=0,\\
&\{B_L(\xi)\overset{\otimes}{,}B(\xi')\}=-\frac{N}{\pi}\delta'(\xi-\xi')\Delta^{ab}(\xi')T_a\otimes T_b, \qquad (27)\\
&\{A_L(\xi)\overset{\otimes}{,}A(\xi')\}=-\frac{N}{\pi}\delta'(\xi'-\xi)\Delta^{ab}(\xi)T_a\otimes T_b,\\
&\{A_L(\xi)\overset{\otimes}{,}B(\xi')\}=-\frac{N}{\pi}\delta(\xi'-\xi)\left(\frac{\partial}{\partial\xi}\Delta^{ab}(\xi)\right)T_a\otimes T_b,
\end{aligned}$$

where the variable $\Delta^{ab}(\xi)=\mathrm{Tr}(T^aUT^bU^{-1})$ obeys the following commutation relations

$$\begin{aligned}
&\{\Delta^{ab}(\xi), A^c(\xi')\}=\{\Delta^{ab}(\xi), B^c(\xi')\}=-2\delta(\xi-\xi')t^{cb}{}_d\,\Delta^{ad}(\xi),\\
&\{\Delta^{ab}(\xi), A^c_L(\xi')\}=\{\Delta^{ab}(\xi), B^c_L(\xi')\}=-2\delta(\xi-\xi')t^{ac}{}_d\,\Delta^{db}(\xi), \qquad (28)\\
&\{\Delta^{ab}(\xi), \Delta^{cd}(\xi')\}=0.
\end{aligned}$$

The Poisson brackets (23), (27) and (28) show that the variables A, B, A_L, B_L and Δ^{ab} generate the closed Lie algebra which, apparently, contains the whole information about a dynamical system. Let us emphasize that this algebra has a subalgebra with generators A and B_L, which is a direct product of the two Kac-Moody algebras.

It is interesting to consider now the case when constants γ^2 and N are connected by the relation /7/

$$\frac{N\gamma^2}{4\pi}=1. \qquad (29)$$

We get from eqs. (22) that $a=\frac{1}{2}A$ and $b=\frac{1}{2}B=\frac{1}{2}U^{-1}B_LU$. So, when the relation (29) is fulfilled, the constraints (11) (stress-energy tensor components) will be expressed only in terms of generators A and B_L. In this case the quantities A and $\mathrm{Tr}(A^2)$ (and also the quantities B_L and $\mathrm{Tr}(B_L^2)$) represent the generators of a new closed algebra with respect to the Poisson brackets. A quantum analogy of these algebras has been investigated recently in the paper /10/. It is interesing, that the variables Δ^{ab} entering into eqs. (27) were treated as the "primary fields" /11/ in the paper /10/.

Now a few words about the construction of the superextension of the examined theory. I know a manifest contruction of the supersymmetric conformal-invariant theory only when the relation (29) is fulfilled /12/. I don't write here the super-invariant action which generalizes the Wess-Zumino action and pass immediately to the Hamiltonian formulation of the theory. The phase space M of this theory is defined by the free Majorana field (ψ^a_L, ψ^a_R) (odd elements of the Grassman algebra, which are transformed under the group $G_L \otimes G_R$ in the adjoint representation) and by the two generators A and B_L of the Kac-Moody algebras. Let us consider the periodic case when $\psi^a_{L(R)}(\xi) = \psi^a_{L(R)}(\xi + 2\pi)$. For the relativistic string this case is equivalent to the Ramond model[13]. The case when $\psi^a_{L(R)}(\xi) = -\psi^a_{L(R)}(\xi + 2\pi)$ (the Neveu-Schwarz model of the open string[13]) is considered in full analogy.

The constraints on the Hamiltonian variables ψ^a_L, ψ^a_R, A and B_L which define the whole dynamics of the system have the form

$$F_L = \frac{1}{2}\psi^a_L B_{La} + \frac{i}{6} t_{abc}\psi^a_L \psi^b_L \psi^c_L = 0, \tag{30a}$$

$$T_L = B^a_L B_{La} + \frac{i}{\pi} k \psi^a_L \psi'_{La} = 0, \tag{30b}$$

$$F_R = \frac{1}{2}\psi^a_R A_a - \frac{i}{6} t_{abc}\psi^a_R \psi^b_R \psi^c_R = 0, \tag{30c}$$

$$T_R = A^a A_a - \frac{i}{\pi} k \psi^a_R \psi'_{Ra} = 0. \tag{30d}$$

Here $k = N$. As one can see below, K will be renormalized by the finite value in the quantum case.

The symplectic structure in the phase space M is defined by the relations (23), (26), (27) and

$$\begin{gathered} \{\psi^a_L(\xi), \psi^b_L(\xi')\} = \{\psi^a_R(\xi), \psi^b_R(\xi')\} = -i\eta^{ab}\delta(\xi - \xi'), \\ \{\psi^a_L(\xi), \psi^b_R(\xi')\} = \{\psi^a_{L(R)}, A\} = \{\psi^a_{L(R)}, B_L\} = 0. \end{gathered} \tag{31}$$

Taking into account these relatinns it is easy to check that (30) is a classic analogue of the direct sum of the two Neveu-Schwarz-Ramond algebras[13]. One of these algebras is left-handed: $\{F_L, T_L\}$ and the other is right-handed $\{F_R, T_R\}$. Since these algebras are identical, let us concentrate only on one of them, for example, on the left-handed algebra. A quantum analogue of the algebra which is a semi-direct sum of the algebras $\{F_L, T_L\}$ and $\{B^a_L, \psi^a_L\}$ has the following construction. For describing this construction let us pass to the Fourier components of the variables $F_L(\xi), T_L(\xi), B^a_L(\xi)$ and $\psi^a_L(\xi)$:

$$\begin{gathered} B^a_L(\xi) = -\frac{2i}{2\pi}\sum_{n=-\infty}^{\infty} e^{in\xi} J^a_n, \quad \psi^a_L(\xi) = \frac{1}{(2\pi)^{1/2}}\sum_{n=-\infty}^{\infty} e^{in\xi}\psi^a_n, \\ F_L(\xi) = \frac{1}{i(2\pi)^{1/2}}\sum_{n=-\infty}^{\infty} e^{in\xi} F^n_L, \quad T_L(\xi) = \frac{N - c_V}{(-\pi^2)}\sum_{n=-\infty}^{\infty} e^{in\xi} T^n_L. \end{gathered} \tag{32}$$

Here $c_V = \dfrac{t_{abc}t^{abc}}{D}$, $D = \eta^a{}_a$. The commutation relations (26), (31) give in the quantum case the following relations

$$\begin{gathered} [\psi^a_n, \psi^b_m]_+ = \eta^{ab}\delta_{n+m,0}, \quad [\psi^a_n, J^b_m]_- = 0, \\ [J^a_n, J^b_m]_- = t^{ab}{}_d J^d_{n+m} + \frac{N}{2} n \eta^{ab}\delta_{n+m,0}. \end{gathered} \tag{33}$$

It is clear from these relations that one can define (without contradictions) vacuum vector $|0\rangle$ which satisfies the equations $\psi^a_n|0\rangle = J^a_n|0\rangle = 0$ (when $n > 0$).

The operators T^n_L and F^n_L (32) have the form

$$F^n_L = \sum_m J^a_{n-m}\psi_{ma} - \frac{1}{6} t_{abc}\sum_{m,e} :\psi^a_{n-m-e}\psi^b_m\psi^c_n:, \tag{34}$$

$$(N - c_V)T^n_L = \sum_m :J^a_{n-m}J_{ma}: + \frac{(N - c_V)}{2}\sum_m m :\psi^a_{n-m}\psi_{ma}: \tag{35}$$

Here $: :$ denotes the normal ordering. Comparing eqs. (35) and (30b) one may conclude that the coefficient K (see eq. (30b)) is equal to $N - c_V$ in the quantum case. This change of K is determined uniquely by the requirement that F^n_L and T^n_L generate the super-algebra*)

$$\begin{gathered} \frac{1}{N - c_V}[F^n_L, F^m_L]_+ = T^{n+m}_L + \left(\frac{c}{4}n^2 + \frac{c_0}{24}\right)\delta_{n+m,0}, \\ [T^m_L, F^n_L]_- = \left(\frac{m}{2} - n\right)F^{m+n}_L, \\ [T^n_L, T^m_L]_- = (n - m)T^{n+m}_L + \left(\frac{c}{8}n^3 + \frac{c_0}{12}n\right)\delta_{n+m,0}, \end{gathered} \tag{36}$$

where constants c and c_0 are equal

$$c = \frac{(N - c_V/3)D}{N - c_V}, \quad c_0 = \frac{c_V D}{c_V - N}.$$

There are the conserved charges in this theory, which generate the $G_L \otimes G_R$ transformations:

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*) The same explicit construction of the Neveu-Schwarz-Ramond superalgebra has been obtained recently by P. Goddard and D.Olive /14/ in another context (see also /12/).

$$q^a_L = -\frac{1}{2}\int_0^{2\pi} d\xi \left(B^a_L(\xi) + i t^{abc} :\Psi_{Lb}(\xi)\Psi_{Ld}(\xi):\right),$$

$$q^a_R = \frac{1}{2}\int_0^{2\pi} d\xi \left(A^a(\xi) - i t^{abc} :\Psi_{Rb}(\xi)\Psi_{Rd}(\xi):\right).$$

I think that there are also an infinite number of the conserved quantities in this theory (at least, on the classical level) in full analogy with the case of the supersymmetric string [15].

3. Let us demonstrate now how the model with the action (1) can be transformed into the relativistic string model in d -dimensions. For this let us notice that if some integrable system has a small parameter "m", then a system obtained from the first one at small "m" will also be integrable. To make use of this, let us introduce the small parameter "m" in our theory as follows:

$$U \to \exp\left(m\, x^a(\xi_1,\xi_2) T_a\right)$$
$$\gamma^2 \to \gamma^2 m^2 \quad , \quad N \to \frac{N}{m^3} . \tag{37}$$

The number of generators T_a is equal to d. Substituting formulae (37) into the action (1) and taking the limit $m \to 0$ we get the system with dynamical equations which can be represented in the form (17). The action of this system is

$$S = \int d^2\xi \left(\frac{1}{2\gamma^2}\sqrt{-\det\|\partial_\alpha x^a \partial_\beta x_a\|} + \frac{N}{24\pi} t^{abd}\varepsilon^{\alpha\beta} x_a \partial_\alpha x_b \partial_\beta x_d\right). \tag{38}$$

This is the well-known model [9] of the relativistic string moving in the external field $F^{bd} = t^{abd} x_a$. We emphasize here that there are no quasiclassical arguments indicating that N is integer in the action (38).

In conclusion let us say some words about the quantization of the model with the action (1). As it has been pointed out in the introduction, the papers [1,2] have appeared, in which projects of construction of the quantum version for the chiral field theory have been discussed. As we can see, the dynamics in the model with the action (1) differs from that in the chiral field theory due to constraints (11) on the canonical variables. Hence, there appears a possibility of using the constructions of the papers [1,2] in our case. Especially, this concerns the paper [1], because its approach originates from the diagonalization of the operators $Tr(a^2)$ and $Tr(b^2)$ in the space of states created by applying the scattering data operators to the pseudovacuum. At present this possibility is substantially verified.

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Исаев А.П. E2-85-82
Модели бозонной и фермионной струны в пространствах групп Ли

Рассматривается двумерная теория, сочетающая в себе черты двух моделей - модели главного кирального поля, в действие которой добавлено слагаемое Весса-Зумино, и модели релятивистской струны. Цель работы - изучение динамики релятивистских струн /бозонных и фермионных/ в компактных и некомпактных пространствах групп Ли. Изучение ведется методами теории поля, а также методом обратной задачи теории рассеяния. Для рассматриваемой бозонной теории в гамильтонановом подходе построена вспомогательная спектральная задача и получен производящий функционал для бесконечного числа инволютивных законов сохранения. Для фермионной теории изучена квантовая алгебра связей, для которой вычислены центральные заряды. Полученные в работе результаты свидетельствуют о том, что предложенные теории обобщают известные теории релятивистских струн и, таким образом, могут быть использованы для построения единой теории поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Isaev A.P. E2-85-82
Bosonic and Fermionic String Models in the Space of Lie Groups

A two-dimensional model is considered which has the features of the chiral field model with the Wess-Zumino term and of the relativistic string model. An auxiliary spectral problem for this model in the Hamiltonian approach is constructed and a generating function for an infinite number of involutory integrals of motion is obtained. A supersymmetric extension of this model is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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