объединенный
институт
ядерных
исследований
дубна

E2-85-794

A.V.Ermushev*, D.I.Kazakov, O.V.Tarasov

CONSTRUCTION OF FINITE N=1 SUPERSYMMETRIC YANG-MILLS THEORIES

Submitted to "Письма в ЖЭТФ"

* Moscow State University

1985

1. During many years the quantum field theory without ultraviolet divergences remained an unsolved theoretical problem. At last a unique example of N=4 extended supersymmetric gauge theory finite in all orders of perturbation theory has been found[1]. Later on it has been realized that there exists a wide class of N=2 supersymmetric finite models[2]. Unfortunately, they are unsatisfactory from a phenomenological point of view. The search for finite theories was continued. It has been shown[3] that N=1 supersymmetric Yang-Mills theories finite at a one-loop level can be constructed. They are described in terms of N=1 superfields by a real superfield V together with a set of matter chiral superfields Φ^a. The requirement of one-loop finiteness restricts the number of fields and their gauge group representations. It turns out [3] that if a theory is one-loop finite, it is also finite at a two-loop level. A three-loop investigation has also been performed[4] and it has been found that the gauge field propagator is finite but those of matter fields are in general divergent. Imposing further constraints on higher group Casimirs one can achieve finiteness at the three-loop level, however, nothing can be said about higher orders.

In the present paper we propose an algorithm to construct an N=1 supersymmetric gauge theory finite to all orders of perturbation theory. We show that necessary and sufficient conditions for finiteness are exhausted already at the one-loop level.

2. The most general action for the N=1 renormalizable supersymmetric gauge theory is of the form (we use the notation of ref.[3])

$$S = \int d^4x \Big[\int d^4\theta\, \bar{\Phi}_a (e^{gV})^a{}_b \Phi^b - \frac{tr}{g^2 C_G} \int d^2\theta\, W^\alpha W_\alpha + \Big(\int d^2\theta\, \frac{1}{3!} d_{abc} \Phi^a \Phi^b \Phi^c + h.c. \Big) \Big] \tag{1}$$

$$+ \text{gauge-fixing} + \text{ghost} \; .$$

Chiral superfield Φ^a is in a reducible representation R of the gauge group G. The index a is a multiindex, it runs over irreducible representations A and members of a given irreducible representation s, i.e. $a = \{A, s\}$. Here $V^a{}_b = V^i (R_i)^a{}_b$ and $(R_i)^a{}_b = (R_i^A)^s{}_t$. Matrices of irreducible representation satisfy the following conditions

$$[R_i, R_j] = i f_{ijk} R_k \, , \quad R^a_{i\,b} R^b_{i\,c} = C_A \delta^a_c \, ,$$
$$R^a_{i\,b} R^b_{j\,a} = \delta_{ij} \sum_A T_A \, , \quad f_{ijk} f_{ljk} = C_G \delta_{il} \, . \tag{2}$$

The action (1) is invariant under G if

$$d_{abc} (R_i)^c{}_d + d_{dac} (R_i)^c{}_b + d_{bdc} (R_i)^c{}_a = 0 \, , \tag{3}$$

where d_{abc} is totally symmetric in a, b and c.

The theory is finite in the one-loop approximation if the following constraints are fulfilled

$$\sum_A T_A = 3 C_G \qquad \text{and} \qquad S^E_A = \delta^E_A C_A \, , \tag{4}$$

where S^E_A, is defined by

$$d_{abc} \bar{d}^{ebc} = 2 S^e_a g^2 = 2 \delta^t_s S^E_A g^2 \, . \tag{5}$$

We note that these results were obtained by using N=1 superfields and background-gauge formalism. In this case the problem is reduced only to the calculation of propagators of the fields. Finiteness of propagators then means finiteness of the charge renormalization.

As follows from eqs.(4),(5), the one-loop finite theory is in fact a theory with a single coupling constant. Picking out a purely tensorial structure, we get from eqs.(4) and (5) that scalar couplings d_i are proportional to the gauge coupling g. However, at the three-loop level this proportionality fails due to different renormalization properties of various charges. In one case the β-function is zero while in the other it is not[4-6]. Hence, we come to a theory with several coupling constants.

3. Consider the renormalization-group equations for the theory with several couplings. For the squares of the couplings $a = g^2$ and $h_i = d_i^2$, we have

$$\mu \frac{da}{d\mu} = \beta_a(a, h_i) = A_{10} a^2 + A_{20} a^3 + \sum_i A_{2i} h_i a^2 + A_{30} a^4 + \sum_i A_{3i} h_i a^3 + \sum_{i,j} A_{3ij} h_i h_j a^2 + \cdots , \tag{6}$$

$$\mu \frac{dh_i}{d\mu} = \beta_{h_i}(a, h_i) = h_i B^i_{10} a + h_i \sum_j B^i_{1j} h_j + h_i B^i_{20} a^2 + h_i \sum_j B^i_{2j} h_j a + h_i \sum_{j,k} B^i_{2jk} h_j h_k + h_i B^i_{30} a^3 + h_i \sum_j B^i_{3j} h_j a^2 + h_i \sum_{j,k} B^i_{3jk} h_j h_k a + h_i \sum_{j,k,l} B^i_{3jkl} h_j h_k h_l + \ldots$$

Starting from the two-loop order the coefficients of eqs.(6) are scheme-dependent. This is also true when due to eqs.(4) and (5) the two-loop β-functions vanish. This fact allows us to express h_i as some functions of a

$$h_i = \alpha_i a + \beta_i a^2 + \gamma_i a^3 + \cdots \tag{7}$$

in order to provide vanishing of the β-functions.

Substituting now eqs.(7) into eqs.(6) we have

$$\beta_a = A_{10} a^2 + \Big(A_{20} + \sum_i A_{2i} \alpha_i \Big) a^3 + \Big(A_{30} + \sum_i A_{3i} \alpha_i + \sum_{i,j} A_{3ij} \alpha_i \alpha_j + \sum_i A_{2i} \beta_i \Big) a^4 + \cdots ,$$

$$\begin{aligned} \beta_{h_i} = {} & \alpha_i \Big(B^i_{10} + \sum_j B^i_{1j} \alpha_j \Big) a^2 + \alpha_i \Big(B^i_{20} + \sum_j B^i_{2j} \alpha_j + \sum_{j,k} B^i_{2jk} \alpha_j \alpha_k + \sum_j B^i_{1j} \beta_j \Big) a^3 + \beta_i \Big(B^i_{10} + \sum_j B^i_{1j} \alpha_j \Big) a^3 \\ & + \alpha_i \Big(B^i_{30} + \sum_j B^i_{3j} \alpha_j + \sum_{j,k} B^i_{3jk} \alpha_j \alpha_k + \sum_{j,k,l} B^i_{3jkl} \alpha_j \alpha_k \alpha_l + \sum_j B^i_{2j} \beta_j + \sum_{j,k} B^i_{2jk} (\alpha_j \beta_k + \alpha_k \beta_j) + \sum_j B^i_{1j} \gamma_j \Big) a^4 \\ & + \beta_i \Big(B^i_{20} + \sum_j B^i_{2j} \alpha_j + \sum_{j,k} B^i_{2jk} \alpha_j \alpha_k + \sum_j B^i_{1j} \beta_j \Big) a^4 + \gamma_i \Big(B^i_{10} + \sum_j B^i_{1j} \alpha_j \Big) a^4 \\ & + \ldots \end{aligned} \tag{8}$$

The requirement for β-functions to vanish leads now to the following equations:

1-loop:

$$A_{10} = 0 , \tag{9a}$$

$$B^i_{10} + \sum_j B^i_{1j} \alpha_j = 0 . \tag{9b}$$

Eq.(9a) is satisfied by an appropriate choice of matter fields and their representations (eq.(4)). Eqs.(9b) determine the values of (eq.(5)). This system of eqs. has a unique solution if the matrix B_1 is nonsingular, which does usually happen. To provide finiteness of all propagators, one should also require that the number of couplings be not less than the number of independently renormalized fields.

2 loops:

$$A_{20} + \sum_i A_{2i} \alpha_i = 0 , \tag{10a}$$

$$B^i_{20} + \sum_j B^i_{2j} \alpha_j + \sum_{j,k} B^i_{2jk} \alpha_j \alpha_k + \sum_j B^i_{1j} \beta_j = 0 . \tag{10b}$$

Eq. (10a) looks like a new constraint on α_i, but it is not so. It is satisfied for α_i determined from eq.(9b), which has been verified explicitly[3]. Eqs.(10b) determine the values of β_i. The solution is also unique due to the same matrix B_1 in the homogeneous part. It turns out that all $\beta_i = 0$ [3].

3 loops:

$$A_{30} + \sum_i A_{3i} \alpha_i + \sum_{i,j} A_{3ij} \alpha_i \alpha_j + \sum_i A_{2i} \beta_i = 0 , \tag{11a}$$

$$B^i_{30} + \sum_j B^i_{3j} \alpha_j + \sum_{j,k} B^i_{3jk} \alpha_j \alpha_k + \sum_{j,k,l} B^i_{3jkl} \alpha_j \alpha_k \alpha_l + \sum_j B^i_{2j} \beta_j + \sum_{j,k} B^i_{2jk} (\alpha_j \beta_k + \alpha_k \beta_j) + \sum_j B^i_{1j} \gamma_j = 0 . \tag{11b}$$

The situation here is entirely like that of two-loops. The first equation is satisfied automatically[4] and the second one allows us to determine γ_i with the same matrix B_1.

Remarkable cancellation of divergences in the gauge field propagator up to the three-loop level found in ref.[3,4] is not a matter of chance but is a consequence of a general theorem. It states[7] that if an N=1 supersymmetric gauge theory is finite up to L-loops (i.e. all the Green functions are finite), then the gauge propagator is finite up to (L+1) loops, and hence the gauge β-function vanishes at (L+1) loops. That explains why eqs.(10a) and (11a) are automatically satisfied. Obviously, the same mechanism will take place in all orders. Choosing h_i as functions of a as in eqs.(7) one can always get vanishing β-functions for <u>all</u> couplings. This in turn means finiteness of the theory in all orders in g. Therefore, one can construct a finite N=1 supersymmetric gauge theory by an appropriate choice of the couplings h_i. In the phase space of couplings $\{a, h_i\}$ our particular solution (7) represents some curve the form of which is determined order by order in perturbation theory. In presence of some additional symmetry the shape of the curve can be fixed. This happens, for instance, in cases of N=4 and N=2 supersymmetries where the curve is a ray $h_i = d_i a$ defined in one loop. However, as we have seen, this is not the only possibility to construct a finite field theory. The class of finite models is rather wide and is not exhausted by N=2,4 supersymmetries or by a special choice of the gauge group and its representations. The only restriction arises already at the one-loop level being the necessary and sufficient conditions for finiteness: eqs.(4) and (5) should be satisfied and the matrix B_1 should be nonsingular.

We hope that the procedure proposed here will be useful for constructing realistic finite field theories and may also be applied in attemps to make up a finite supergravity theory.

References

1. P.Howe, P.K.Townsend and K.Stelle, Nucl.Phys., 1983, B214, p. 519; S.Mandelstam, Nucl.Phys., 1983, B213, p. 149; L.Brink, O.Lindgren and E.W.Nillsson, Nucl.Phys., 1983, B212, p. 401.
2. P.Howe, K.Stelle and P.West, Phys.Lett., 1983, 124B, p. 55.
3. A.J.Parkes, P.C.West, Phys.Lett., 1984, 138B, p. 99.
4. A.J.Parkes, P.C.West, Nucl.Phys., 1985, B256, p. 340.
5. A.J.Parkes, Phys.Lett., 1985, 156B, p.73.
6. D.R.T.Jones, A.J.Parkes, Phys.Lett., 1985, 160B, p. 267.
7. M.T.Grisaru, B.Milewski and D.Zanon, Phys.Lett., 1985, 155B, p. 357.

Received by Publishing Department
on November 5, 1985.

Ермушев А.В., Казаков Д.И., Тарасов О.В. E2-85-794
Построение конечных N=1 суперсимметричных теорий Янга-Миллса

Предложен алгоритм построения ультрафиолетово-конечных калибровочных теорий с N=1 суперсимметрией. Показано, что необходимые и достаточные условия сокращения расходимостей во всех порядках теории возмущений определяются уже в однопетлевом приближении.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Ermushev A.V., Kazakov D.I., Tarasov O.V. E2-85-794
Construction of Finite N=1 Supersymmetric Yang-Mills Theories

We propose an algorithm to construct ultraviolet finite N=1 supersymmetric gauge theories. Necessary and sufficient conditions for divergences to cancel in all orders of perturbation theory are exhausted in one-loop order.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985