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FOUR-LEPTON DECAYS AND ELECTROMAGNETIC RADIUS OF K-MESON

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1. Introduction

A significant increase of beam intensities and great progress in development of detecting instruments have allowed a good success in studying rare processes. For instance, experiments were carried out to search for rare decays of K-mesons to the branching ratios level up to 10^{-9}. It is probable that after K-meson factories become operational one will be able to investigate K-decays to the level $10^{-10} - 10^{-12}$. Similar results may perhaps be achieved in future Brookhaven K-decay experiments/1/ and in Serpukhov with the help of the intensive beam of charged K -mesons intended for tagged neutrino experiments/2/.

The study of rare K-meson decays allows one to obtain information on the structure of particles through analysis of momentum dependence of the corresponding form factors in the decay region. The processes due to joint manifestation of weak and electromagnetic interactions are of interest. Among many weak-electromagnetic processes lepton decays are of special interest for the study of the hadron structure.

The paper deals with the processes

$$K^+ \to e'\nu_1 e^+e^-, \tag{1}$$

where e', e are electrons or muons. Investigation of those processes yields information on the weak-electromagnetic structure of pseudoscalar mesons in its purest form.

The decays (1) were observed for the first time in the CERN electron experiment/3/ in 1976. The branching ratio was found to be

$$R(K^+ \to \mu\nu e^+e^-) = \frac{W(K^+ \to \mu\nu e^+e^-)}{W_K} = (1.23 \pm 0.32) \cdot 10^{-7} \tag{2}$$

$$R(K^+ \to e\nu e^+e^-) = \frac{W(K^+ \to e\nu e^+e^-)}{W_K} = (0.28^{+0.28}_{-0.14}) \times 10^{-7} \tag{3}$$

for $|K^2|_{e^+e^-} \geqslant (140\ \text{MeV})^2$, where K^2 is the invariant mass of an electron-positron pair.

In Section 2 the theory is considered and branching ratios of the processes are calculated; in Section 3 the calculations are compared with the experimental data [3].

2. Calculation of $K^{+}\to\mu\nu e^{+}e^{-}$, $K^{+}\to e\nu e^{+}e^{-}$ Decay Branching Ratios

The processes (1) were investigated theoretically in several papers [4-7].

Analysis of numerical results obtained in those papers revealed a sharp difference between them. Therefore we have calculated branching ratios of those processes again. Let us summarize the theory of processes (1) mainly according to papers [7,6]. The matrix element of processes (1) may be represented as a sum of two terms corresponding to the diagram in Fig. 1.

The expression for the matrix element has the form

$$
\begin{aligned}
M = {} & \frac{e^2 G f_K}{\sqrt{2}} \; \frac{\mathcal{E}_\alpha}{K^2}\Big(m_e \bar{u}(p)(1-\gamma_5)\Big[\frac{2Q_\alpha + K_\alpha}{2(QK)+K^2} - \\
& \frac{2P_{1\alpha} + \hat{K}\gamma_\alpha}{2(P_1K)+K^2}\Big] u(-P_1) + \frac{i}{M^2}\Big\{-\mathcal{E}_{\alpha\beta\rho\sigma} \times K_\rho Q_\sigma \times \\
& \times a(K^2, Q^2) + \big(Q_\alpha K_\beta - (QK)\delta_{\alpha\beta}\big) \times b(K^2, Q^2) + \\
& + \big(K_\alpha K_\beta - K^2\delta_{\alpha\beta}\big) \times c(K^2, Q^2) + \big(K^2 Q_\alpha - (QK)K_\alpha\big) \times \\
& \times Q_\beta \times \Big[\frac{2M^2}{Q^2+M^2} - \frac{1-F(K^2)}{K^2} + h'(K^2, Q^2)\Big]\Big\} \times \ell_\beta\Big).
\end{aligned} \tag{4}
$$

The first term of expression (4) has no unknown quantities. It is completely defined by the amplitude of the radiationless decay $K\to\mu\nu$. A contribution of this term to the amplitude is usually called a contribution of the inner bremsstrahlung (IB). Contributions of other terms, characterising the K-meson structure with respect to the weak-electromagnetic interaction, are called structure-dependent contributions (SD). Here m_e is a mass of the lepton; $K = P_2 + P_1$; $Q = P_1 + P_0$; $G = 10^{-5}/m_p^2$; f_K is the constant for the $K\to\mu\nu$ decay; $\mathcal{E}_\alpha = \bar{u}(P_1)\gamma_\alpha u(-P_2)$; $\ell_\beta = \bar{u}(p)\gamma_\beta(1+\gamma_5)u(-P_1)$; $K^2 d^2 - \frac{1}{2}[Q^2+M^2+K^2] = [1-F(K^2)]M^2$; $h(K^2,Q^2) = K^2 h'(K^2,Q^2)$; $F(K^2)$ is the electromagnetic form factor of K-meson, which depends on the invariant K^2 and meets the normalisation requirement $F(0)=1$; a, b, c, d, h are the form factors characterising the decay. It is shown in papers

[6,7] that contributions with form factors h and d are negligible, therefore we took $h = d = 0$.

The differential branching ratio was calculated with the SCHOONSHIP programme [8]. To check the expression obtained, we calculated values of various contributions to the branching ratio at some incidental values of momenta of particles produced in decays, satisfying kinematics of processes (1). The result was compared with the contributions, calculated with the help of the expression for the differential branching ratio from Ref. [6]. The results coincided. This proves identity of both formulae.

We assume the T-invariance here, therefore form factors are real functions of K^2 and Q^2.

In papers [9] devoted to the theoretical study of the amplitude of decays

$$K^{+}\to e\nu\gamma \tag{5}$$

and (1), dominance of low-lying resonances, K^*, K_A, ρ and ω was assumed. Thus, for the form factor a, characterising the contribution of vector current, the formula

$$a(K^2,Q^2) = a_0\Big[\Big(1+\frac{K^2}{m_\rho^2}\Big)\Big(1+\frac{Q^2}{m_{K^*}^2}\Big)\Big]^{-1} \tag{6}$$

holds good. To calculate $a_0 = a(0,0)$ one uses various theoretical models [9], differing from each other by no more than 20%. Within the dominance $a(K^2,Q^2)$ is known better than a_0, therefore it is reasonable to consider the formula as a representation of a with the only unknown parameter a_0.

In the model with "hard meson" [10,11] a similar situation also takes place for four axial form factors b, c, d and h, therefore the processes under consideration can be described with three parameters a_0, b_0 and c_0, characterising values of form factors a, b and c at $K^2 = Q^2 = 0$.

In paper [6] branching ratios of the processes are calculated for two cases.

1. Formulae are used for form factors, where they are functions of $Q^2, M_\rho^2, K^2, M_{K^*}^2$.

2. Form factors are considered constant.

The results differ by 20%, so the approach of constant form factors to decays may be employed in analysis of the data with few events, which is the case we consider. At $K^2 = 0$ the form factors a and b should coincide with the corresponding form factors for the $K\to e\nu\gamma$ decay.

From the experiment [12] on investigation of the $K\to e\nu\gamma$ decay one has found a limitation

$$|a_0| \leq 0.45. \tag{7}$$

In the papers [9], analysing the amplitude of the decay (5), $|a_0| = 0.3$ was obtained.

Within the exact SU(3)-symmetry a_K can be related in the model-independent way to the corresponding form factor for the π -meson decay on the basis of the Cabibbo theory [13], which is a natural generalization of the hypothesis of isovector current conservation IVC in the case of SU(3)-symmetry

$$0.27 \leq |a_K(0,0)| \leq 0.35 \tag{8}$$

For the violated SU(3)-symmetry [14]

$$0.13 \leq |a_0| \leq 0.31. \tag{9}$$

For b_0 different values have been found in the interval [10,6] $0 \leq b_0 \leq 0.6|a_0|$, but in the model with the hard kaon technique

$$|b_0| = 0.14|a_0|. \tag{10}$$

The "hard kaon" technique [15] predicts that

$$c_0 = 2d_0 = \frac{1}{3}\langle r_K^2\rangle M_K^2 \tag{11}$$

The experimental measurement of c_0 allow one to check relation (11) if the value of $\langle r^2\rangle$ is known from other experiments.

Separate contributions to branching ratios of decays (1) were calculated with the accuracy not worse than 2%. When calculating, we, like the authors of Ref. [6], took the following values of form factors: $a_0 = 0.35$; $b_0 = 0.048$; $c_0 = 0.62$. This value of c_0 corresponds to $\sqrt{\langle r^2\rangle} = 0.55$ Fm.

The results are in good agreement with the calculations in [6] and do not agree with [4].

3. Comparison with the Experimental Data

In the experiment [3] 14 $K \to \mu\nu e^+e^-$ events and 4 $K \to e\nu e^+e^-$ events were found for $|K^2| \geq (140\ \mathrm{MeV})^2$ cut off. This region was chosen because it is free of the background decay $K \to \mu\nu\pi^0 \to e^+e^-\gamma\mu\nu$

The branching ratio R we present in the following form

$$R = \frac{W(K \to e\nu e^+e^-)}{W_K} = IB + A^2 + B^2 + AB + C^2 + AC + BC. \tag{12}$$

Here AB, for example, is the total contribution (including the contribution of exchange diagrams for process $K \to e\nu e^+e^-$) to R of interference of amplitude terms containing form factors a and b; A is the contribution to R from interference of the amplitude part, containing the form factor a with the bremsstrahlung amplitude; IB is the contribution to R from the squared amplitude of the bremsstrahlung, etc. The Table lists numerical values of contributions to the total probability from significant terms in formula (12).

Figures in the Table can be easily recalculated for any other a_0', b_0', c_0' different from a_0, b_0, c_0 (e.g. the contribution $A'^2 = \frac{a_0'^2}{a_0^2}A^2$, etc.). The decay $K \to \mu\nu e^+e^-$ receives the main contributions from the terms IB, c_0, a_0, c_0^2, a_0^2. The experimental values of the branching ratio obtained in [3], allow the following equation for $K \to \mu\nu e^+e^-$ with respect to the values of the form factors

$$R(K \to \mu\nu e^+e^-) = (1.63a_0^2 - 2.23a_0 - 1.59c_0^2 + 2.5c_0 + 5.72) \times 10^{-8} = (12.3 \pm 3.2) \times 10^{-8}. \tag{13}$$

As is seen from the Table and relation (13), in the region $|K^2| \geq (140\ \mathrm{MeV})^2$ this decay is of great interest for the study of form factors, characterising the structure-dependent radiation, since at $|K^2| \geq (140\ \mathrm{MeV})^2$ in the $K \to \mu\nu e^+e^-$ decay the bremsstrahlung is suppressed, but the structure-dependent radiation is still large.

For the process $K \to e\nu e^+e^-$ the region $|K^2| \geq (140\ \mathrm{MeV})^2$ is also free of the background $K \to \mu\nu\pi^0$. The contributions c_0^2 and a_0^2 dominate in this region. An equation for the form factors taking into account the experimental value of the branching ratio has the following form:

$$R(K \to e\nu e^+e^-) = (1.31a_0^2 + 3.64c_0^2) \times 10^{-8} = 2.8^{+2.2}_{-1.4} \times 10^{-8}. \tag{14}$$

In Fig. 1 the ellipses of errors C' and B' limit the experimental values of a_0 and c_0 obtained from (14) for the decay $K \to e\nu e^+e^-$. Similarly, the ellipses C'' and B'' limit the values a_0 and c_0 from (13) for $K \to \mu\nu e^+e^-$.

Using the experimental limitation (7) for the form factor a_0 obtained from $K \to e\nu\gamma$, one can find the value of the form factor c_0 from (13) and (14)

$$c_0 = 0.93 \pm 0.18. \tag{15}$$

We notice that the data on $K \to e\nu e^+e^-$ and $K \to e\nu\gamma$ yield information on the module c_0, but only the joint analysis of three processes $K \to e\nu e^+e^-$, $K \to \mu\nu e^+e^-$ and $K \to e\nu\gamma$ allows one to determine the sign of c_0, which is well seen in Fig. 2.

Table

Significent contributions to the branching ratio in units 10^{-8} in the region $|K^2| \geqslant (140\ \mathrm{MeV})^2$ for decays $K \to \mu\nu e^+e^-$ and $K \to e\nu e^+e^-$. ($a_0 = 0.35$, $b_0 = 0.048$, $c_0 = 0.62$).

Type of contribution \ Process	$K^+ \to e\nu e^+e^-$	$K^+ \to \mu\nu e^+e^-$
IB	–	5.72
A^2	0.16	0.20
A	–	-0.80
B^2	–	0.01
B	–	0.21
C^2	1.4	0.61
C	–	1.55
BC	0.12	0.09
Sum	1.68	7.59
Error	0.02	0.08

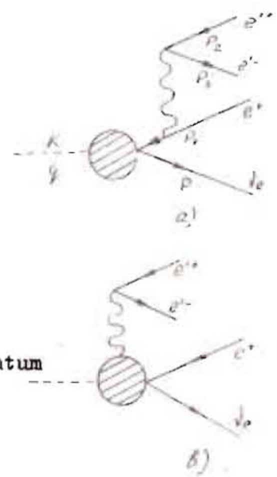

Fig. 1.

Diagrams of processes $K^+ \to e'\nu_e e^+e^-$.

a) corresponds to emission of a virtual γ -quantum by the lepton e;

b) describes radiation from hadrons.

Note that variation of $|a_0|$ in the interval (7) changes c_0 only by 5%, and by 8-10% in the interval $|a_0| \leqslant 0.7$, i.e. c_0 is practically insensitive to variations of a_0 in a reasonable interval.

Using relation (11), one can find the root-mean-square radius of the kaon from (15)

$$\sqrt{\langle r_K^2 \rangle} = (0.67 \pm 0.07)\,\mathrm{Fm}. \qquad (16)$$

The value (16) agrees within the errors with the experimental result obtained in /16/ on $K-e$ scattering (0.53 ± 0.05) Fm, so validity of relation (11) is proved. The results of the paper have shown also that, despite very poor statistics, it is possible to obtain independent information on the electromagnetic radius of the K-meson from the available experimental data on decays (1). We notice that the statistical accuracy of the result for the radius is comparable with the accuracy of the special experiment on $K-e$ scattering. This is due to strong dependence decays (2) and especially (3) on the term c_0^2 which is proportional to the fourth power of the radius.

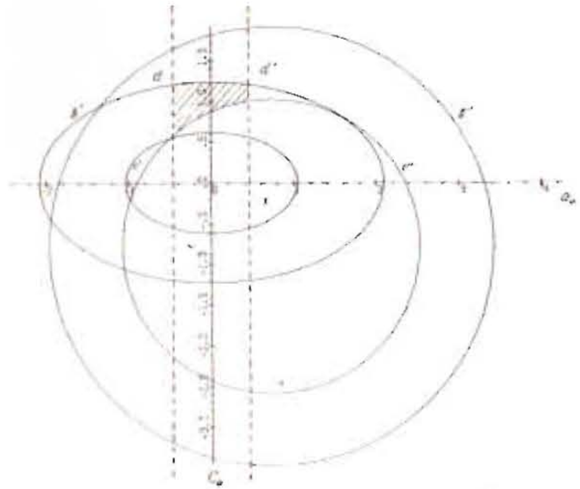

Fig. 2.

Limitations for allowed values of form factors a_0 and c_0: the region between C' and B' error ellipses - from the $K \to e\nu e^+e^-$ data; C'' and B'' - from the $K \to \mu\nu e^+e^-$ data. The straight lines show the region $|a_0| \colon 0.45$, limiting allowed values of the form factor a_0 according to the experimental data on $K \to e\nu\gamma$. The hatched region shows the values of the form factors, being fitted for the three processes.

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Четырехлептонные распады и электромагнитный радиус K-мезона

Исследованы процессы редких четырехфермионных распадов K-мезонов. Эти исследования дают информацию о слабой электромагнитной структуре псевдоскалярных мезонов. Приведены расчеты вероятностей распадов $K^+ \to e\nu e^+ e^-$ и $K^+ \to \mu\nu e^+ e^-$ с помощью программы SCHOONSHIP. На основе экспериментального материала, полученного в CERN-е, при использовании интенсиеного пучка заряженных K-мезонов найдено значение аксиального формфактора Co, характеризующего эти распады, и проверено соотношение $C_0 = \frac{1}{3} \langle r_k^2 \rangle M_k^2$, связывающее Co с электромагнитным радиусом каона. Определено среднеквадратическое значение радиуса K-мезона $\sqrt{\langle r_k^2 \rangle}$ = /0,67±0,07/ Фм, которое в пределах ошибок согласуется с экспериментальным результатом по прямому определению этой величины.

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Four-Lepton Decays and Electromagnetic Radius of K-Meson

Processes of rare four-lepton decays of K-mesons are investigated. This investigations yields information on weak-electromagnetic structure of pseudoscalar mesons. The branching ratios $K^+ \to e\nu e^+ e$ and $K^+ \to \mu\nu e^+ e^-$ have been calculated using the SCHOONSHIP Programme. On the basis of experimental data obtained in CERN with the help of the intensive beam of charged K-mesons. a value of the axial form factor Co, characterising those decays, has been found. The relation $C_0 = \frac{1}{3} \langle r_k^2 \rangle M_k^2$ between Co and the electromagnetic radius of the kaon has been checked. The rootmean-square radius of the K-meson was determined $\sqrt{\langle r_k^2 \rangle}$ = (0.67±0.07) Fm, which agrees within the errors with the direct experimental result.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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