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E2-85-763

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TOTAL α_s-CORRECTIONS TO PROCESSES $\gamma^* \gamma^* \to \pi^0$ AND $\gamma^* \pi \to \pi$ IN A PERTURBATIVE QCD

Submitted to "Ядерная физика".

1985

1. Introduction

Matrix elements of hard meson processes in the lowest twist are represented by convolutions of a hard parton amplitude, coefficient function, with wave functions characterizing meson properties in hard processes. Coefficient functions are calculated in perturbation theory as series in powers of $\alpha_s(\mu)$. In particular, one-loop coefficient functions (by which the calculation of subsequent orders starts) were earlier calculated for some processes like the pion form factor $\gamma^* \pi \to \pi$ /1,2/ and process $\gamma^* \gamma^* \to \pi^0$ /3/. The wave functions are determined by physics at long distances and cannot be calculated by a perturbative QCD. However, perturbation theory may predict their evolution with Q^2 (the kernel of the evolution equation is also a series in powers of $\alpha_s(\mu)$) . Therefore, recent two-loop calculations of the evolution kernel for a nonsinglet pion wave function (performed in a light-like gauge/4,5/ and in Feynman gauge/6/) allow us to complete the analysis of exclusive processes including pions in the next-to-leading approximation of QCD. To this end, based on the calculations/5/ and/6/ we shell construct (sect. 2) a solution to the evolution equation for the pion wave function and analyse this solution. Note that in the case of QCD unlike a more simple scalar model, see/7/ the evolution law for the pion wave function can be obtained only numerically. (The Table of coefficient determining the evolution is given in Appendix 2).The obtained solution may be used for calculating amplitudes of any exclusive processes including pions.

A concluding stage is the compilation of contributions from coefficient functions and from a two-loop evolution of the pion wave functions to the total α_s - correction. In sect. 3 we calculate and analyse the total α_s -correction to the process $\gamma^* \gamma' \to \pi^0$, in sect. 4 we evaluate evolution corrections to the pion electromagnetic form factor $\gamma^* \pi \to \pi$. Basic results are summarized in the Conclusion.

2. Evolution of the Nonsinglet Wave Function of a Pion in the Two-Loop Approximation

The pion wave function may be determined by its moments[8]

$$f_\pi \int_0^1 \Phi(x,\mu^2)\, x^N dx = \frac{i^N}{2(pn)^{N+1}} \langle 0|\bar d \gamma_5 \hat n (n\vec D)^N u|P\rangle, \qquad (1)$$

where n_ν is a light-like vector ($n^2 = 0$) introduced to pick out a symmetric traceless part of the local operator $O_N = \bar d \gamma_5 \gamma_\nu \vec D_{\nu_1} \ldots \vec D_{\nu_N} u$, $\vec D_\nu$ is a covariant derivative; $|P\rangle$, a one-pion state with momentum P; μ^2, the renormalization parameter of composite operators (or in other words, $1/\mu$ determines the boundary between "long" and "short" distances[1,8]); $f_\pi = 133$ MeV and zeroth moment of $\Phi(x,\mu_0^2)$ equals unity. Qualitatively, $\Phi(x,\mu^2)$ represents the probability amplitude of transition of the pion into two massless collinear quarks with momentum fractions xP and $(1-x)P \equiv \bar x P$ *) (in the infinite-momentum frame). The change of $\Phi(x,\mu^2)$ versus μ^2 is described by the evolution equation[8,9]:

$$\begin{cases} \mu^2 \dfrac{d}{d\mu^2}\Phi(x,\mu^2) = \displaystyle\int_0^1 V(x,y;g)\,\Phi(y,\mu^2)\,dy \equiv V \otimes \Phi \\ \Phi(x,\mu_0^2) = \varphi_0(x). \end{cases} \qquad (2)$$

Here $V = \frac{\alpha_s(\mu)}{4\pi} V_1(x,y) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 V_2(x,y)$ is the evolution kernel for the pion wave function in QCD in the two-loop approximation[4-6];

$$V_2 = 2 C_F N_f V_{N+} + 2 C_F C_A V_{G+} + C_F^2 V_{F+}. \qquad (3)$$

The functions $V_{(N,G,F)+}$ for group factors in (3) are written out in Appendix 1, formulae (AI-A3). (As usual, $C_F = \frac{4}{3}$, $C_A = 3$, and N_f is the number of flavours of light quarks). The function $\varphi_0(x)$ defines the form of the pion wave function in a low-energy region. Note that the result for function V_F has recently[10] been verified by a direct numerical calculation, in Feynman gauge, of its matrix elements in the basis of Gegenbauer polynomials, and the agreement has been established with the results found in refs.[4-6].

*) Hereafter we shall use the notation $\bar x = 1 - x$, $\bar y = 1 - y$.

In a one-loop approximation $V \to \frac{\alpha_s(\mu)}{4\pi} V_1$ the solution to eq. (2) is well known[9]: since the variables are separable, the problem reduces to finding eigenfunctions ψ_n for the equation with kernel V_1. These are Gegenbauer (G.p.) polynomials with the weight $y\bar y$, i.e. $\psi_n = y\bar y\, C_n^{3/2}(y-\bar y)$ ($y\bar y$ is the weight with which the Gegenbauer polynomials of index 3/2 are orthogonal). In what follows we shall also need conjugated Gegenbauer polynomials $\tilde\psi_n = \frac{4(2n+3)}{(n+1)(n+2)} C_n^{3/2}(y-\bar y)$ and $\tilde\psi_n \otimes \psi_m = \delta_{mn}$. The solution to eq.(2) then is a series over functions ψ_n (see, e.g.,[13]):

$$\Phi(x,\mu^2) = \sum_n b_n(\mu_0^2) \exp\left\{-\int_{\ln(\mu_0^2)}^{\ln(\mu^2)} dt\, \frac{\alpha_s(t)}{4\pi}\gamma_n^{(1)}\right\}\psi_n(x) \equiv \sum_n b_n(\mu_0^2)\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)}\right]^{-\gamma_n^{(1)}/b_0} \psi_n(x). \qquad (4)$$

The coefficients $b_n(\mu_0^2)$ represent Gegenbauer moments of $\varphi_0(x)$: $b_n(\mu_0^2) = \varphi_0 \otimes \tilde\psi_n$; $\gamma_n^{(1)}$ are coefficients of $\frac{\alpha_s}{4\pi}$ in the anomalous dimension of the O_n operator: $\gamma_n = \frac{\alpha_s}{4\pi}\gamma_n^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \gamma_n^{(2)}$ [12]; $-b_0$ is the first expansion coefficient of β-function.

Let us proceed to construct a 2-loop solution of eq.(2). The total evolution kernel in the 2-loop approximation can be represented by a sum of two terms, a part of the kernel diagonal in the basis $\{\psi_n\}$, $V^{(D)}$ and a nondiagonal part, $V^{(ND)}$. We shall write a partial solution Φ_n (corresponding to the $b_n(\mu_0^2)$ coefficient) in that form for which the contribution from the kernel diagonal part will be taken into account exactly:

$$\Phi_n = \exp\left\{-\int_{\ln(\mu_0^2)}^{\ln(\mu^2)} dt\, \gamma_n(\bar\alpha(t))\right\} \Phi_n^{(ND)}(x,\mu^2) \qquad (5)$$

and the function $\Phi_n^{(ND)}$ to the $O(\alpha_s)$ order is determined by the nondiagonal part $V^{(ND)}$. In (5) we made use of the result for matrix elements V in the Gegenbauer basis[6]:

$$\tilde\psi_n \otimes V \otimes \psi_n = -\gamma_n. \qquad (6)$$

Substituting (5) into (2) and incorporating (6) gives the evolution equation for $\Phi_n^{(ND)}$:

$$\left(\mu^2\partial_{\mu^2} + \beta(\alpha)\partial_\alpha - V_n^{(ND)}\right)\otimes \Phi_n^{(ND)} = 0;\quad V^{(ND)} = V + \gamma_n. \qquad (7)$$

If $V^{(ND)} = 0$, then, obviously, $\Phi_n^{(ND)} = \psi_n(x)$. The nondiagonal part contributes to $\varphi_n^{(ND)}$ in the $O(\alpha_s)$ order. Therefore, an approximate solution for $\varphi_n^{(ND)}$ will be looked as the series/11/:

$$\varphi_n^{(ND)} = \psi_n + \frac{\alpha_s(\mu)}{4\pi} \sum_{k=0}^{\infty} d_n^k(\mu^2)\, \psi_k(x) \tag{8}$$

with unknown coefficients d_n^k. Requiring radiation corrections to be absent at the normalization point μ_0 gives the initial condition $d_n^k(\mu_0^2) = 0$. Inserting (8) into (7) and using the initial condition we arrive at the expression for d_n^k:

$$d_n^k = \frac{V_{2(kn)}^{(ND)}}{\gamma_k^{(1)} - \gamma_n^{(1)} - \beta_0} \left(1 - \left[\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right]^{(\beta_0 - \gamma_k^{(1)} + \gamma_n^{(1)})/\beta_0}\right) \tag{9a}$$

$$V_{2(kn)}^{(ND)} = \tilde{\psi}_k \otimes V_2^{(ND)} \otimes \psi_n . \tag{9b}$$

Expression (9) for coefficients d_n^k was first derived in/11/. However, the authors have used a two-loop kernel V_2 reproduced from first principles like "residual conformal symmetry on the light cone"/10,11/ accepted a priori. As a result, they obtained V_2 (and consequently, d_n^k) that contradict direct calculations/4-6,10/. It is interesting to note that such an approach is valid for the scalar model $\varphi^3_{(6)}$ /7,10/, in which it provides uniquely structure of the evolution-kernel nondiagonal part. The structure thus obtained for the evolution kernel allows d_n^k to be computed entirely analytically (see Appendix 1).

The elements d_n^k are only nonzero for k and n of the same parity and $k > n$. This is a consequence of the general properties of matrix V_{kn}: the first results from a "geometric" symmetry of the evolution kernel, $V(x,y) = V(\bar{x},\bar{y})$, and the second from a triangular shape of the renormalization matrix (see/6/). Allowing for this remark and substituting (8) into (5) we get the following solution to the evolution equation:

$$\Phi(x,\mu^2) = \sum_n b_n(\mu_0^2)\, \varphi_n(x,\mu^2) \tag{10a}$$

$$\varphi_n(x,\mu^2) = \exp\left\{-\int_{\ln(\mu_0^2)}^{\ln(\mu^2)} dt\, \gamma_n[\bar{\alpha}(t)]\right\}\left(\psi_n(x) + \frac{\alpha_s}{4\pi}\sum_{k>n} d_n^k(\mu^2)\,\psi_k(x)\right). \tag{10b}$$

A practical value of a solution of that sort is defined by that how rapid is the convergence of the series in (8) (see paragraph 2 at the end of the section) and how accurate are the calculations of d_n^k. Expressions for anomalous dimensionalities ${}^{(ND)}\gamma_n^{(1,2)}$ can be found, e.g., in/12/. The calculation of elements $V_{2(kn)}$ determining d_n^k cannot be accomplished in a complete analytic form because of a highly complicated structure of V_F. However, the matrix elements for $2C_F N_f V_N + 2C_F C_A V_G$ can be computed analytically by formula (A7) of Appendix 1:

$$2C_F\left(N_f V_N + C_A V_G\right)_{(kn)}^{(ND)} = \beta_0\left(\gamma_k^{(1)} - \gamma_n^{(1)}\right) a_{nk},$$

where a_{nk} are given by (A6). The matrix elements of V_{F+} were found by numerical integration. The properties of solution (10) thus established can be formulated in terms of partial wave functions φ_n with even n, which corresponds to the symmetry of the pion wave function. So, we write φ_n in the form

$$\varphi_n = \varphi_{0(n)} + \frac{\alpha_s(\mu)}{4\pi}\,\varphi_{1(n)}, \tag{11}$$

where $\varphi_{1(n)}$ represents all corrections from the 2-loop kernel, and summarize the results of calculations (here and below $\mu_0^2 = 1\,GeV^2$; $\Lambda_{QCD} = 0.1$ GeV).

1. For $n > 2$ and $\mu^2 \leq 125$ GeV corrections from the nondiagonal part V_2 are about by an order smaller than those from the diagonal part accumulated in the exponential factor. In other words, the V_2 kernel appears "quasidiagonal" in the Gegenbauer basis (corrections at $n = 0$, i.e. for the asymptotic function, are also very small and are considered below).

2. Corrections of higher harmonics to ψ_n are mainly determined by the first term $d_n^{n+2}\psi_{n+2}$ in the sum over k (10b). Subsequent coefficients decrease rapidly with growing k, and it suffices to take only a few first terms into account (see Appendix 2).

3. The total contribution of 2-loop corrections, $\frac{\alpha_s(\mu)}{4\pi}\varphi_{1(n)}$, increases as n increases. At $n = 6$ this contribution is as much as 6% for $x = 0.5$ and $\mu^2 = 125$ GeV^2.

So, for a popular choice $\varphi_0 = 6x\bar{x}$, corresponding to the asymptotic wave function in LLA, corrections come only from $V_2^{(ND)}$:

$$\varphi_{AS}(x,\mu^2) = 6x\bar{x}\left\{1 + \frac{\alpha_s(\mu)}{4\pi}\sum_{k\geq 2} d_0^k\, C_k^{3/2}(x-\bar{x})\right\}. \tag{12}$$

Calculations show that these corrections are less than 0.4% (at $x = 0.5$) up to $\mu^2 = 6\ 10^3\ \mathrm{GeV}^2$.

Another example of the low-energy pion wave function was proposed in ref./13/:

$$\varphi_{c.z.}(x,\mu_0^2) = 30\, x\bar{x}\,(1-4x\bar{x}). \qquad (13)$$

In this case a relative contribution of the 2-loop correction at $\mu^2 = 125\ \mathrm{GeV}^2$ amounts to about 2% at maxima of the function. By using d_n^k tabulated in Appendix 2 the evolution can be calculated (at $\mu^2 = 125\ \mathrm{GeV}^2$) for any wave functions that at $\mu_0^2 \simeq 1\ \mathrm{GeV}^2$ can be represented by a sum of the Gegenbauer polynomials Ψ_n with $n \leq 10$.

We have also solved equation (12) by a direct computed algebra for $n = 0,2,4$. However, this way of calculations is very time-consuming and makes sense only as a test for the solution proposed here. The results of integration are consistent with solution (9), (10) and with conclusions 1,2,.

3. The Total Correction to the $\gamma^*\gamma^* \to \pi^0$ Amplitude

In this section we shall calculate the total α_s -correction to the amplitude T of the process

$\gamma^*(q_1) + \gamma^*(q_2) \longrightarrow M(P)$ - spinless pseudoscalar nonsinglet (in flavour) meson

to the next-to-logarithmic order; here it is convenient to express the amplitude T in terms of the transition form factor $F_{M\gamma}$ /14/:

$$T = 4\ \frac{e^{\mu\nu\lambda\rho} P_\lambda Q_\rho}{Q^2}\, \varepsilon_{1\mu}\, \varepsilon_{2\nu}\, F_{M\gamma}$$

where $Q = \frac{1}{2}(q_1 - q_2)$; $P = q_1 + q_2$; $-q_1^2$ is large and positive; $-q_2^2 \geq 0$; $\varepsilon_{1\mu}$ and $\varepsilon_{2\nu}$ are polarizations of colliding photons. The form factor $F_{M\gamma}$ in the lowest twist can be represented by an integral convolution of the coefficient function $C(x,\omega)$ determined from a hard parton subprocess with the wave function $\varphi(x,Q^2)$ /9,3/, the bourdary between long and short distances being defined by the quantity $1/|Q|$, i.e. $\mu^2 = Q^2$ (see sect.1)/1/:

$$F_{M\gamma}(\omega) = N\, C(x,\omega) \otimes \varphi(x,Q^2), \qquad (14)$$

where $\omega = \frac{PQ}{Q^2} \leq 1$, the coefficient N for π^0 equals $f_\pi(e_u^2 - e_d^2)$ and e_q are charges of quark flavours.

The parameter ω characterizes the degree of asymmetry of colliding photons. In a symmetric case (at $q_1^2 = q_2^2$) $\omega = 0$; when one of the photons is real, $\omega = 1$.

In our approximation formula (14) becomes

$$N^{-1}F_{M\gamma} = F_0(\omega) + \frac{\alpha_s}{4\pi}F_1(\omega) = \Big(C_0(x,\omega) + \frac{\alpha_s}{4\pi}C_1(x,\omega)\Big)\otimes\Big(\varphi_0(x,Q^2) + \frac{\alpha_s}{4\pi}\varphi_1(x,Q^2)\Big), \qquad (15)$$

where $\varphi_0(x,Q^2)$ is a solution of the evolution equation (4) in LLA, and the Born term $C_0(x,\omega)$ of the pseudoscalar coefficient function equals/3,9/

$$C_0(x,\omega) = \frac{1}{1+\omega[\bar{x}-x]} + \ x \leftrightarrow \bar{x}\,.$$

So, to establish the α_s -correction to the form factor - F_1 , it is necessary to know $C_1(x,\omega)$, the coefficient function of the process in order $O(\alpha_s)$, and φ_1 , process-independent correction due to evolution of the pion wave function. (The latter was analysed in detail in sect. 2).

The coefficient function C_1 was first calculated in ref./3/ both for scalar and pseudoscalar cases. However, when calculating in a dimensional regularization there appears a difficulty caused by uncertainty of commutation properties of γ_5 in a space of dimensionality $D \neq 4$. This uncertainty was removed in paper/14/ where C_1 was calculated in the limiting case $\omega = 1$ and contribution of separate diagrams were given. We have reproduced the result for C_1 in the $\overline{MS}$ scheme of dimensional regularization, and contributions of separate diagrams (Feynman gauge) are tabulated in Appendix 3. Our final result coincides with C_1 found in/3/, and the limiting case $\omega = 1$ for each diagram separately is in agreement with the results of ref./14/.

In the Table for C_1 of Appendix 3 the scales for collinear-μ and ultraviolet- μ_R regularizations are taken different. It is seen that for the contributions of diagrams a and b the parts proportional to $\ln[Q^2/\mu^2]$ are given, respectively, by $C_0 \otimes V_a$ and $C_0 \otimes V_b +$, where are parts of the one-loop evolution kernel $V_{(0)} = V_a + V_b$; $V_1 = V_{(0)} +$ (see Appendix 1, formula (A3)). This result, in fact, emerges from the factorization theorem for the coefficient of collinear divergence of C_1 /2,8,14/. To simplify further analysis, we set $\mu^2 = \mu_R^2 = Q^2$.

The total correction to the form factor $\frac{\alpha_s}{4\pi} F_{1(h)}(\omega)$

$$F_{1(h)}(\omega) = C_1(x,\omega) \otimes \varphi_{0(h)}(x,Q^2) + C_0(x,\omega) \otimes \varphi_{1(h)}(x,Q^2) \qquad (16)$$

will be treated, as before, with the use of partial wave functions (11). The calculation by formula (16) has been made numerically. The quantities $\frac{\alpha_s}{4\pi} F_{1(h)}(\omega)$ as functions of ω are plotted in Fig. 1. In Table 1 contributions of different corrections to $\frac{\alpha_s}{4\pi} F_{1(h)}$ are given for $\omega = 1$.

The calculations performed allow the following conclusions about the magnitude of α_s -correction and about the contributions of various sources to the correction (in what follows $Q^2 = 125$ GeV^2):

1. A relative contribution of α_s -corrections grows with h (see Table 1) reaching at $h = 8$ 27% (for $\omega \simeq 1$) of the Born-term contribution.

2. As is seen from Fig.1, $F_{1(h)}(\omega)$ as a function of ω with increasing h gets concentrated near $\omega = 1$. It is really not difficult to establish that $C_0 \otimes \varphi_{1(h)} \sim \omega^{h+2}$ and $C_1 \otimes \varphi_{0(h)} \sim \omega^{h}$. Therefore, the higher-harmonic corrections are most important when one of the photons in the process is near to real.

3. A dominating contribution to $F_{1(h)}(\omega)$ at $h = 0$ comes from the convolution $C_1 \otimes \varphi_{0(0)}$ (see Table I) as $\varphi_{(0)}$ evolves weakly. For subsequent harmonics $h = 2,4,6,8$ the corrections from coefficient function C_1 are comparable with those from the pion wave function φ_1. Thus, the latter are to be taken into account for a correct estimate of the amplitudes of processes with a real photon.

It is to be noted that authors of refs./3/ and/14/ are mistaken assuming a complete absence of the evolution of φ_{AS} (see (12)) in the 2-loop approximation. In this case F_1 is determined only by the coefficient function; but this holds valid only when the combination $y\bar{y}\, V_2(x,y)$ is symmetric under the change $x \leftrightarrow y$. However, both particular computations/4,5,10/ and a general analysis of the properties of $V(x,y)$/6/ testify that this is not so.

Realistic pion wave functions are extracted from the analysis of experiment by QCD sum rules. In Fig.2 plotted are the total correc-

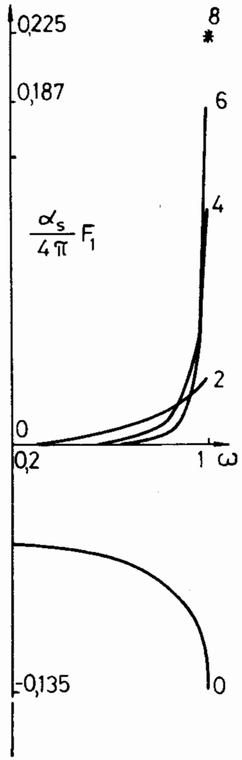


Fig. 1.

Figures on curves indicate the number h of a partial α_s -correction. The value for $\alpha_s/4\pi\, F_{1(8)}(\omega)$ is pointed out by star — * only at $\omega=1$ since the whole curve is very much pressed to point $\omega=1$.

Fig. 2.

The Born term and α_s-correction to the form factor for the Chernyak-Zhitnitsky pion wave function (13).

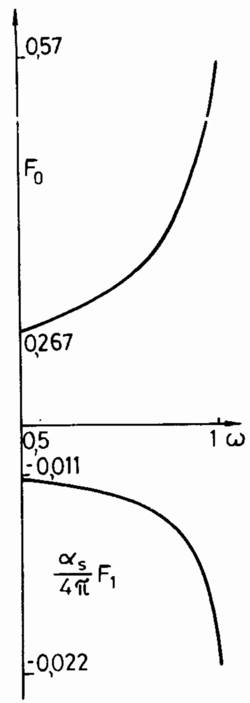


Table 1. ($\omega=1$, $Q^2=125$ GeV^2)

h		0	2	4	6	8
$F_{0(h)}(1)$		1.8	1.28	1.05	0.947	0.83
$\frac{\alpha_s}{4\pi}$	$F_{1(h)}(1)$	-0.135	0.035	0.129	0.187	0.225
	$C_1 \otimes \varphi_{0(h)}$	-0.132	0.021	0.103	0.154	0.188
	$C_0 \otimes \varphi_{1(h)}$	-0.003	0.014	0.026	0.033	0.037

tion $\frac{\alpha_s}{4\pi} F_1(\omega)_{c.z.}$ with the use of function $\Phi(X)_{c.z.}$ (13)[13] (the lower part as the correction is negative) and the Born term $F_0(\omega)_{c.z.}$ (the upper part). As is seen, the correction is small: smaller than 4% of the Born term for all ω. This is a consequence of a partial compensation between contributions of 0 -th and subsequent harmonics giving corrections with opposite sign (see Fig. 1, Table 1). For more wide wave functions the compensation of contribution will be still more strong, and hence, the correction will be still smaller in magnitude. However, for narrow wave functions and functions with a great amount of higher harmonics the correction may be significant.

4. Total Correction to the Pion Electromagnetic Form Factor

In this section we shall analyse the evolution α_s -corrections to the pion form factor $F_\pi(Q^2)$ in the lowest twist. The form factor at sufficiently large transfer momenta Q^2 is factorized in all orders and for all logarithms of perturbation theory[1]:

$$F_\pi(Q^2) = \frac{1}{Q^2}\, \Phi(x,Q^2) \otimes E(x,y;\alpha_s(Q)) \otimes \Phi(y,Q^2)\left\{1 + O\left(\frac{1}{Q^2}\right)\right\}. \tag{17}$$

Here the coefficient function $E(x,y;\alpha_s(Q))$ is the amplitude of hard parton subprocess $q_1 \bar{q}_2 \gamma^* \to q_1' \bar{q}_2'$, whose second order in $\alpha_s(Q)$ is given by[1]:

$$E(x,y;\alpha_s(Q)) = E_0(x,y;\alpha_s(Q))\left(1 + \frac{\alpha_s(Q)}{\pi} E_1(x,y)\right). \tag{18}$$

The coefficient function E_0 was computed in pioneering works[15] and[16]. The first correct calculation of E_1 was made in[1] in Feynman gauge. This result has recently been verified by calculations in axial and Feynman gauges in paper[2] in which the dependence of E_1 on the choice of renormalization scheme has also been studied. Throughout we set scales of ultraviolet μ_R^2 and collinear μ^2 regularizations equal Q^2, therefore all logarithms of scale ratios are zero, and E depends on Q^2 only through $\alpha_s(Q)$ (see[1] and[2]).

For analysing the role of evolution corrections we made numerical estimations of the form factor $Q^2 F_{\pi(n)}(Q^2)$ (17) using the partial wave functions φ_n both taking account of the two loops evolution of the latter (according to formula (11)) and in the LLA ($\varphi_n \to \varphi_{0(n)}$). The results are presented in Table 2 in terms of the total α_s -cor-

rection $T_{1(n)}$ to the Born term $T_{0(n)}$ and the partial α_s -correction orignating only from the coefficient function E_1:

$$N\, T_{0(n)}\left(1 + \frac{\alpha_s(Q)}{\pi}\left\{\begin{matrix} T_n \\ \tilde{T}_n \end{matrix}\right\}\right) = \left\{\begin{matrix} Q^2 F_{\pi(n)}(Q^2) \\ Q^2 \tilde{F}_{\pi(n)}(Q^2) \end{matrix}\right\}, \tag{19}$$

Here

$$N = \frac{2\pi\alpha_s(Q)}{N_c} C_F f_\pi^2\,; \quad N\, T_{0(n)} = \Phi_{0(n)} \otimes E_0 \otimes \Phi_{0(n)}\,; \quad E_0 = \frac{N}{xy}.$$

As is seen from Table 2, the radiation corrections are rapidly increasing with n, and starting from the fourth harmonics perturbation theory breaks. Thus, for the pion wave functions dressed with high harmonics the validity of perturbation theory is extended into the region of several hundreds (GeV)^2; a final result for $T_{1(n)}$ is slightly influenced by the evolution α_s -corrections.

Table 2. ($Q^2 = 33$ GeV^2; $\alpha_s(Q^2) = 0.14$)

n	0	2	4	6	8
$T_{0(n)}$	0.25	0.124	0.09	0.073	0.062
$\tilde{T}_{1(n)}$	7.22	19.3	29.1	37.0	43.9
$T_{1(n)}$	7.06	18.9	28.4	34.2	45.5

5. Conclusion

In this paper, we found an approximate solution to the evolution equation in the 2-loop approximation of QCD and numerically constructed the Q^2– evolution of the pion wave function. A solution like (10) may be used for computing amplitudes of any exclusive processes including pions.

Then, based on solution (10) we calculated the total α_s -correction to the amplitude of process $\gamma^* \gamma^* \to \pi^0$ in the next-to-leading approximation and numerically analysed the role of evolution α_s -corrections for various types of the wave functions. It is shown that the α_s -corrections are important when one of the photons is nearly real.

A numerical estimation is also made for the contribution of evolution α_s -corrections to the pion electromagnetic form factor $\gamma^* \pi \to \pi$ and it is found to be small.

APPENDIX 1

Here structures $V_{(F,G,N)}$ of the 2-loop kernel V_2 are presented, and expressions are derived for nondiagonal matrix elements V_N and V_G in the $\{\psi_n\}$ basis. So,

$$V_{F+} = 4\mathcal{C}\Big\{\theta(y>x)\Big[-\frac{\pi^2}{3}F + \frac{x}{y} - \Big(\frac{3}{2}F - \frac{x}{2\bar{y}}\Big)\ln\Big(\frac{x}{y}\Big) - (F-\bar{F})\ln\frac{x}{y}\ln\Big(1-\frac{x}{y}\Big) + \Big(F + \frac{x}{2\bar{y}}\Big)\ln^2\Big(\frac{x}{y}\Big)\Big] - \frac{x}{2\bar{y}}\ln x\,(1+\ln x - 2\ln\bar{x}) - H(x,y)\Big\}_+ \tag{A1}$$

$$H(x,y) = \theta(x>\bar{y})\big[2(\bar{F}-F)\,Li_2\Big(1-\frac{x}{y}\Big) - 2F\ln x\ln y + (F-\bar{F})\ln^2 y\big] + 2F\,Li_2(\bar{y})\big[\theta(x>\bar{y}) - \theta(y>x)\big] - 2F\,Li_2(x)\big[\theta(x>\bar{y}) - \theta(x>y)\big] + \theta(y>x)\,2\bar{F}\ln y\ln\bar{x}. \tag{A2}$$

We used the notation: $\mathcal{C} = \mathbb{1} + x \leftrightarrow \bar{x},\ y \leftrightarrow \bar{y}$;
$F = F(x,y) = \frac{x}{y}\Big(1 + \frac{1}{y-x}\Big)$; $\bar{F} = F(\bar{x},\bar{y})$; $Li_2(x) = -\int_0^x \frac{dt}{t}\ln(1-t)$
is the Spense function. Symbol " + " in exps. (A1) and (3) signifies

$$V(x,y)_+ = V(x,y) - \delta(y-x)\int_0^1 V(z,y)\,dz$$

and the fact that in QCD $\int_0^1 V(x,y)\,dx = 0$ because of the axial-current conservation. The sum of the remaining structures may be represented in the form (see[6]):

$$2N_f C_F V_N + 2C_A C_F V_G = \beta_0 \frac{d}{d\nu} V_{(\nu)}\Big|_{\nu=0} + U_{(3)} + H. \tag{A3}$$

where $V_{(\nu)} = \mathcal{C}\,\theta(y>x)\,2C_F\Big(\frac{x}{y}\Big)^{1+\nu}\Big(1+\frac{1}{y-x}\Big)$; $V_{(0)+} = V_1$; the derivative with respect to index ν will be denoted by dot: $\frac{d}{d\nu}V_{(\nu)}\Big|_{\nu=0} = \dot{V}_{(0)}$. From direct calculations it follows that the functions H and U_{3} in (A1) and (A3) are diagonal and consequently, do not form the elements d_n^k. Let us determine matrix elements for the first term in (A3). Since eigenfuctions of the equation for eigenvalues with kernel $V_{(\nu)+}$ are given by the functions $\psi_n^\nu = (x\bar{x})^{1+\nu} C_n^{3/2+\nu}(x-\bar{x})$ $(\psi_n^0 = \psi_n)$ we have the equation

$$V_{(\nu)+} \otimes \psi_n^\nu = -\gamma_n^\nu \psi_n^\nu \tag{A4}$$

and differentiation of this equation with respect to ν at $\nu = 0$ yields:

$$\dot{V}_{(0)+} \otimes \psi_n = -(\gamma_n + V_{(0)+}) \otimes \dot{\psi}_n + \dot{\gamma}_n \psi_n. \tag{A5}$$

On the other hand, the derivative $\dot{\psi}_n$ can be expanded into a series over ψ_k (k is of the parity of n):

$$\dot{\psi}_n = \sum_{k>n} a_{nk}\psi_k, \quad a_{nk}\underset{n>k}{=} -2\frac{(n+1)(n+2)(2k+3)}{(k+1)(k+2)(k-n)(k+n+3)} \tag{A6}$$

whose coeffidients a_{nk} can easily be found with the use of formulae (A6) and (A7) of ref.[17]. Inserting (A6) into (A5) and using A4) we arrive at nondiagonal matrix elements for (A3) in the form

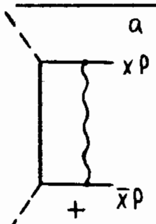
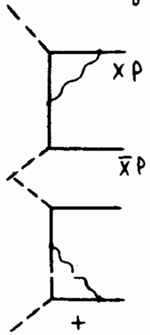
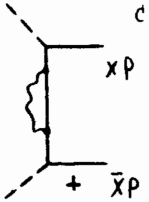
$$\beta_0 \dot{V}_{1(kn)} = \beta_0(\gamma_k^{(1)} - \gamma_n^{(1)})\,a_{nk}. \tag{A7}$$

In a completely analogous way one may determine the matrix elements for arbitrary products of powers of V_1 and $\dot{V}_1$. The nondiagonal part of the evolution kernel in the scalar model $\varphi^3_{(6)}$ (see the discussion after formula (9)) is just represented by a sum of such products[7]. Therefore, the calculation of d_n^k within that model may also be accomplished analytically.

APPENDIX 2. Coefficients d_n^k calculated at $\mu_0^2 = 1$ GeV^2, $\Lambda_{QCD} = 0.1$ GeV and $\mu^2 = 125$ GeV^2

k \ n	0	2	4	6	8	10
2	−0.28					
4	0.012	−0.89				
6	0.032	−0.26	−1			
8	0.027	−0.087	−0.4	−1		
10	0.021	−0.03	−0.19	−0.47	−0.96	
12	0.016	−0.007	−0.094	−0.25	−0.5	−0.91
14	0.012	0.002	−0.05	−0.15	−0.29	−0.51
16	0.01	0.006	−0.025	−0.09	−0.18	−0.32
18	0.009	0.007	−0.008	−0.05	−0.12	−0.21

APPENDIX 3. Contributions of separate diagrams to the coefficient function $C_1(x,\omega)$

Diagramm	$C_1(x,\omega)$	$C_1(x,1)$ $(\omega=1)$
a  + cross	$\ln\left[\frac{Q^2}{\mu^2}\right] C_0(z,\omega)\otimes V_a(z,x)$ $+\int_0^1 \frac{dz}{1+\omega[\bar z - z]} V_a'(z,x) + x\leftrightarrow\bar x$	$\left\{-\ln\left[\frac{2Q^2}{\mu^2}\right]\frac{\ln\bar x}{x} + \frac{\ln\bar x}{x} - \frac{\ln^2(\bar x)}{2x}\right\} +$ $x\leftrightarrow\bar x$
b  + cross	$\ln\left[\frac{Q^2}{\mu^2}\right] C_0(z,\omega)\otimes V_b(z,x)_+$ $- C_0(z,\omega)\otimes V_b(z,x)_+$ $+\int_0^1 \frac{dz\ V_b'(z,x)_+}{1+\omega[\bar z - z]} + x\leftrightarrow\bar x$ $+\Delta(x) + x\leftrightarrow\bar x$ $-2\ln\left[\frac{Q^2}{\mu_R^2}\right] C_0(z,\omega)\otimes\delta(z-x)$	$\ln\left[\frac{2Q^2}{\mu^2}\right]\left(\frac{2}{\bar x} + \frac{\ln\bar x}{x\bar x}\right) + x\leftrightarrow\bar x$ $+$ $\left\{\frac{\ln^2(\bar x)}{2x\bar x} + \ln(\bar x)\left[\frac{2x-3}{2x\bar x}\right] - \frac{4}{\bar x}\right\} + x\leftrightarrow\bar x$ $\ln\left[\frac{2Q^2}{\mu_R^2}\right]\frac{1}{x\bar x}$
c  + cross	$\ln\left[\frac{Q^2}{\mu_R^2}\right] C_0(z,\omega)\otimes\delta(z-x)$ $\int_0^1 \frac{dz\ \delta(z-x)}{1+\omega[\bar z - z]}\ln(1+\omega[\bar z - z]) +$ $x\leftrightarrow\bar x$ $- C_0(z,\omega)\otimes\delta(z-x)$	$\ln\left[\frac{2Q^2}{\mu_R^2}\right]\frac{1}{2x\bar x}$ $+$ $\frac{1}{2\bar x}\left(\ln(\bar x) - 1\right) + x\leftrightarrow\bar x$

The notation is here as follows: $V_a'(z,x) = \left(\left(\theta(x>z)\,2\frac{z}{x}\right)\ln(1+\omega[\bar z - z]\right)$

$$V_b'(z,x)_+ = \left\{\left(\frac{\theta(x>z)\,2\frac{z}{x}}{x-z}\right)\ln\left(1+\omega[\bar z - z]\right)\right\}_+$$

$$\Delta = -\frac{1}{1+\omega[\bar z - z]}\left(\ln(1+\omega)+\ln(1-\omega)\right) + \frac{1}{2\omega}\left(\frac{\ln(1-\omega)}{\bar x} - \frac{\ln(1+\omega)}{x} + \ln(1+\omega[\bar x - x])\right)\left(\frac{1}{x} -\right.$$

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Received by Publishing Department
on October 24,1985.

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Каданцева Е.П., Михайлов С.В., Радюшкин А.В. E2-85-763

Полные α_s-поправки к процессам $\gamma^*\gamma^* \to \pi^0$ и $\gamma^*\pi \to \pi$ в пертурбативной КХД

Методом, основанным на получисленном решении уравнения эволюции, построена эволюция по Q^2 волновой функции пиона $\Phi(x,Q^2)$ и исследованы ее свойства. С использованием этих расчетов проведен анализ полных α_s-поправок в следующем за лидирующим приближении к амплитудам процессов $\gamma^*\gamma^* \to \pi^0$ и $\gamma^*\pi \to \pi$. Установлено, что эволюционные α_s-поправки существенны для первого процесса /когда один из фотонов - реальный/ и незначительны для второго. Полученные результаты могут быть использованы при вычислении амплитуд любых эксклюзивных процессов, включающих пионы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Kadantseva E.P., Mikhailov S.V., Radyushkin A.V. E2-85-763

Total α_s-Corrections to Processes $\gamma^*\gamma^* \to \pi^0$ and $\gamma^*\pi \to \pi$ in a Perturbative QCD

A seminumerical method, within the 2-loop approximation of QCD, is applied to construct Q^2-evolution of the pion wave function $\Phi(x,Q^2)$ and to study its properties. On the basis of these calculations total α_s-corrections to the amplitudes of processes $\gamma^*\gamma^* \to \pi^0$ and $\gamma^*\pi \to \pi$ are analysed in the next-to-leading approximation. The evolution α_s-corrections are shown to be essential for the first process (when one of the photons is real) and unimportant for the second process. The methods developed can be applied to calculate amplitudes of any exclusive processes involving pions.

The investigation has been performed at the Laboratory of Theoretical Physics JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985