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CALCULATION OF THE STATIC
INTERQUARK POTENTIAL
IN THE STRING MODEL
IN THE TIME-LIKE GAUGE

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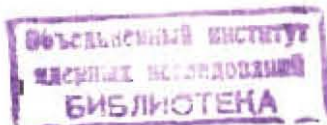
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I. I n t r o d u c t i o n

The relativistic string model^{/1,2/} provides a clear picture of the quarks confinement in hadrons. The string describes the flux tube of the gluon field connecting quark-antiquark^{/3,4/}. Most probably, just such one-dimensional configurations of the gluon field dominate when the distances between quark-antiquarks approach the hadron size.

The string energy is proportional to its length. Hence, the relativistic string connecting quarks leads to a potential linearly rising with distance. It appears that in the framework of the string model the static potential between quarks at rest can be calculated consistently not only in the nonrelativistic limit^{/5/} but also in the relativistic quantum theory^{/6-10/}.

The most straightforward way of such calculations is the investigation of the model of the relativistic string with fixed ends (the approximation of infinitely heavy quarks). This problem was considered in papers^{/11-13/}, the light-like gauge popular in the string models being used. However, this gauge as it has been noted repeatedly^{/14/}, restricts the class of admissible motions of the string, therefore in the present paper the static quark-antiquark potential will be calculated in the timelike gauge in which there is no difficulty noted above. This gauge will be introduced immediately in the reparametrization-invariant action of the string. As a result, the problem of the choice of boundary conditions for the time coordinate of the string with fixed ends is avoided in contrast with papers^{/11-13/}.



The material is arranged as follows. The second section is devoted to the Hamiltonian dynamics of the relativistic string with fixed ends in the timelike gauge. In the third section the quantization of this model is proposed. Neither restrictions on the space-time dimension nor tachyon states appear in this approach. Just here the static interquark potential is calculated. In conclusion the obtained results are briefly discussed. In the Appendix a simple but consistent derivation is given in the nonrelativistic approximation of the potential, linearly rising with distance generated by a string.

2. Hamiltonian Dynamics of the Relativistic String in the Timelike Gauge

In the relativistic string theory the light-like gauge

$$n \cdot \dot{x} = \frac{n^0}{\gamma \bar{n}} \tau + n^1 Q \quad (2.1)$$

is popular^{11,21}. Here $x^\mu(\tau, \sigma)$ are the string coordinates, n^μ is a constant light-like vector $n^2 = 0$ independent of τ and σ , P^μ is the total string momentum, Q^μ are the coordinates of "the center of mass" of the string at $\tau = 0$. The basic advantage of this gauge is that it enables one to resolve in a polynomial form the subsidiary conditions on the string coordinates

$$(\dot{x}^0 \pm \dot{x}^1)^2 = 0, \quad \dot{x}^0 = \frac{\partial x^0}{\partial \tau}, \quad \dot{x}^1 = \frac{\partial x^1}{\partial \sigma}. \quad (2.2)$$

As a result, the dependent components of the radius-vector of the world sheet of the string are expressed by quadratic combinations of the independent (transverse) components of this vector.

The light-like gauge (2.1) does not allow one to describe such motions of the string when the conditions

$$n(\dot{x}^0 \pm \dot{x}^1) = 0 \quad (2.3)$$

are fulfilled^{12,14/}. This drawback is absent obviously in the timelike gauge that can be defined by (2.1) with $n^2 = (n^0)^2 - (\vec{n})^2 > 0$. In this gauge in the Lorentz reference frame where $\vec{n} = 0$ the evolution parameter τ becomes proportional to time t .

In practice it is convenient to introduce immediately this gauge in the reparametrization-invariant action of the relativistic string

$$S = -\gamma \int_{\tau_1}^{\tau_2} d\tau \int_0^{\bar{\tau}} d\sigma \sqrt{(\dot{x}^0)^2 - \dot{\vec{x}}^2}. \quad (2.4)$$

As a result, we obtain

$$S = -\gamma \int_{t_1}^{t_2} dt \int_0^{\bar{\tau}} d\sigma \sqrt{\dot{\vec{x}}^2 (1 - \dot{\vec{x}}^2) + (\dot{x}^0 \dot{\vec{x}})^2}, \quad (2.5)$$

where $\vec{x} \equiv \vec{x}(t, \sigma)$ and the dot denotes the differentiation with respect to time t .

The transition from (2.4) to (2.5) is completely analogous to the transition from the reparametrization-invariant action of a point-like particle

$$S_p = -m \int_{\tau_1}^{\tau_2} d\tau \sqrt{\dot{z}^2}, \quad z^\mu = z^\mu(\tau) \quad (2.6)$$

to the action written in the gauge $z^0(\tau) = \tau$

$$S_p = -m \int_{t_1}^{t_2} dt \sqrt{1 - \left(\frac{d\vec{z}}{dt}\right)^2}. \quad (2.7)$$

The string action (2.5) is invariant under arbitrary transformations of the parameter σ

$$\sigma \rightarrow \bar{\sigma} = \bar{\sigma}(\tau, \sigma) \quad (2.8)$$

Hence, the Lagrangian (2.5) in contrast with (2.7) is singular^{15,16/}

In the theory there is the primary constraint, i.e. the relation among the canonically conjugate variables $\vec{x}(t, \sigma)$ and $\vec{p}(t, \sigma)$

$$\varphi(\sigma) = \vec{x}(t, \sigma) \cdot \vec{p}(t, \sigma) = 0, \quad (2.9)$$

where

$$\vec{p}(t, \sigma) = \frac{\partial \mathcal{L}}{\partial \dot{\vec{x}}} = \frac{\gamma^2}{\mathcal{L}} [\dot{\vec{x}} \cdot (\dot{\vec{x}} \dot{\vec{x}}) - \dot{\vec{x}} (\dot{\vec{x}})^2], \quad (2.10)$$

$$\mathcal{L} = -\gamma \sqrt{\dot{\vec{x}}^2 (1 - \dot{\vec{x}}^2) + (\dot{\vec{x}} \cdot \dot{\vec{x}})^2}. \quad (2.11)$$

The identities^{/17/}

$$\vec{p} \cdot \dot{\vec{x}} - \mathcal{L} = -\frac{\gamma^2}{\mathcal{L}} \dot{\vec{x}}^2, \quad (2.12)$$

$$\vec{p}^2 + \gamma^2 \dot{\vec{x}}^2 = \left(-\frac{\gamma^2}{\mathcal{L}} \dot{\vec{x}}^2 \right)^2$$

result immediately in the following expression for the canonical Hamiltonian

$$H = \int_0^{\pi} (\dot{\vec{x}} \cdot \vec{p} - \mathcal{L}) d\delta = \int_0^{\pi} d\delta \sqrt{\vec{p}^2 + \gamma^2 \dot{\vec{x}}^2}. \quad (2.13)$$

The constraints (2.9) are of the first-class^{/16,18/}, as it must be in the reparametrization-invariant theory,

$$\{\varphi(\delta), \varphi(\delta')\} = -(\varphi(\delta) + \varphi(\delta')) \frac{\partial}{\partial \delta'} \Delta(\delta, \delta'). \quad (2.14)$$

Here $\{\dots, \dots\}$ are the Poisson brackets

$$\{F, G\} = \int_0^{\pi} \left(\frac{\delta F}{\delta \vec{x}} \cdot \frac{\delta G}{\delta \vec{p}} - \frac{\delta F}{\delta \vec{p}} \cdot \frac{\delta G}{\delta \vec{x}} \right) d\delta \quad (2.15)$$

and $\Delta(\delta, \delta')$ is the δ -function that takes into account the boundary conditions for canonical variables $\vec{x}(t, \delta)$, $\vec{p}(t, \delta)$ in the problem under consideration.

In the theory of the relativistic string with fixed ends the boundary conditions are

$$\vec{x}(t, 0) = 0, \quad \vec{x}(t, \pi) = \vec{R}, \quad (2.16)$$

$$\vec{p}(t, 0) = \vec{p}(t, \pi) = 0. \quad (2.17)$$

According to the Dirac procedure^{/16/} the generalized Hamiltonian, generating the equations of motion in the phase space is

$$H_T = H + \int_0^{\pi} \lambda(t, \delta) \varphi(\delta) d\delta = \int_0^{\pi} d\delta \sqrt{\vec{p}^2 + \gamma^2 \dot{\vec{x}}^2} + \int_0^{\pi} \lambda(t, \delta) \dot{\vec{x}} \cdot \vec{p} d\delta. \quad (2.18)$$

To avoid the functional freedom in the theory and to fix the Lagrange multiplier $\lambda(t, \delta)$, it is necessary to impose, in addition to the constraints (2.9), the gauge condition^{/19/}. It is convenient to choose the gauge in the following form

$$\chi(\delta) = \vec{p}^2 + \gamma^2 \dot{\vec{x}}^2 - \gamma = 0. \quad (2.19)$$

The physical dynamics will develop not in the whole phase space Γ but only on its submanifold Γ^* defined by (2.9) and (2.19). It is easy to calculate the Poisson brackets between χ and φ on Γ^*

$$\{\chi(\delta), \varphi(\delta')\}|_{\Gamma^*} = \gamma^2 \frac{\partial}{\partial \delta'} \Delta(\delta, \delta'). \quad (2.20)$$

Thus, (2.19) does fix the gauge.

Demanding that

$$\frac{d}{dt} \chi(\delta)|_{\Gamma^*} = \{\chi, H_T\}|_{\Gamma^*} = 0 \quad (2.21)$$

one determines the Lagrange multiplier

$$\begin{aligned} \{\chi, H_T\}|_{\Gamma^*} &= \int_0^{\pi} \lambda(t, \delta') \{\chi, \varphi\} d\delta' = \\ &= \int_0^{\pi} \lambda(t, \delta') \frac{\partial}{\partial \delta'} \Delta(\delta, \delta') d\delta' = \lambda'(t, \delta) = 0. \end{aligned} \quad (2.22)$$

Hence

$$\lambda(t, \delta) \equiv \lambda(t). \quad (2.23)$$

The equations of motion generated on Γ^* by the generalized Hamiltonian (2.18) have the form

$$\ddot{\vec{x}} = \gamma^{-1/2} \vec{p} + \lambda(t) \vec{R}, \quad (2.24)$$

$$\dot{\vec{p}} = \gamma^{3/2} \ddot{\vec{x}} + \lambda(t) \vec{p}^i \quad (2.25)$$

To agree (2.24) with the boundary conditions (2.16), (2.17), one must put $\lambda(t) = 0$. Finally, the equations of motion are

$$\dot{\vec{x}} = \gamma^{-1/2} \vec{p}, \quad \dot{\vec{p}} = \gamma^{3/2} \ddot{\vec{x}} \quad (2.26)$$

The solution of these equations obeying the boundary conditions (2.16), (2.17) can be represented by the Fourier series

$$\vec{x}(t, b) = R \frac{b}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi} \gamma} \sum_{n \neq 0} \frac{\vec{\alpha}_n}{n} \sin(nb) \exp(-in\sqrt{\gamma} t) \quad (2.27)$$

As $\vec{x}(t, b)$ is real, the amplitudes $\vec{\alpha}_n$ obey the condition

$$\vec{\alpha}_n^* = \vec{\alpha}_{-n} \quad (2.28)$$

The Poisson brackets of $\vec{x}(t, b)$ and $\vec{p}(t, b)$ are

$$\{x^i(t, b), p^j(t, b')\} = \delta^{ij} \Delta(b, b'), \quad (2.29)$$

where $\Delta(b, b')$ is the antiperiodical δ -function

$$\Delta(b, b') = \sum_{n=-\infty}^{+\infty} [\delta(b - b' + 2\pi n) - \delta(b + b' + 2\pi n)] = \frac{1}{\sqrt{\pi}} \sum_{n=-\infty}^{+\infty} \sin(nb) \sin(nb') \quad (2.30)$$

For the Poisson brackets of the amplitudes $\vec{\alpha}_n$ one gets from (2.29)

$$\{\alpha_n^k, \alpha_m^l\} = -in \delta_{kl} \delta_{n,-m}, \quad (2.31)$$

$k, l = 1, 2, \dots, D-1$, D is the dimension of space-time.

Substitution of the expansion (2.27) into the constraint equation (2.9) and into the gauge condition (2.19) results, by virtue of (2.26), in the following constraints on the Fourier amplitudes $\vec{\alpha}_n$

$$L = \frac{1}{2} \sum_{n=-\infty}^{+\infty} \vec{\alpha}_{n-m} \cdot \vec{\alpha}_m = 0, \quad n = \pm 1, \pm 2, \dots, \quad (2.32)$$

$$L_0 = \frac{1}{2} \sum_{m=-\infty}^{+\infty} \vec{\alpha}_{-m} \cdot \vec{\alpha}_m - \frac{\pi}{2} = 0 \quad (2.33)$$

Here the amplitude with $n=0$ is

$$\vec{\alpha}_0 = (\gamma/\pi)^{1/2} \vec{R} \quad (2.34)$$

The algebra of the Poisson brackets of the constraints L_n is defined by

$$\{L_n, L_m\} = -i(n-m) L_{n+m} - i \frac{\pi}{2} (n-m) \delta_{n,-m}, \quad (2.35)$$

$n, m = 0, \pm 1, \dots$

Even at the classical level this algebra is not closed as the gauge freedom is completely fixed by (2.32), (2.33).

3. Static Interquark Potential Generated by a String

A remarkable peculiarity of the relativistic string model is that there arises the potential linearly rising with a distance between the string ends.

One would think that this potential makes sense only in the nonrelativistic approximation, but it is reasonable at the quantum-relativistic level as well. In the Nambu-Goto string model with massive ends the dynamical string variables disappear in the nonrelativistic approximation (see the Appendix). As a result, the problem reduces to the two-body dynamics with the central potential linearly rising with distance.

In the relativistic case it is natural to consider, as the static potential, the minimal value of the energy E of the string with fixed ends as a function of the distance R between the string ends. As the canonical Hamiltonian (2.13) is time-independent, then

$$E = H. \quad (3.1)$$

Now the physical meaning of the gauge condition (2.19) is clear: the parameter b is chosen so that the energy density is constant along the string, i.e., it is independent of b .

Let us express the energy E in terms of the Fourier amplitudes taking into account the constraints (2.32), (2.33)

$$E(R) = \sqrt{\gamma^2 R^2 + \pi \gamma \sum_{m \neq 0} \vec{\alpha}_{-m} \cdot \vec{\alpha}_m} \quad (3.2)$$

At the classical level we get from (3.2) the potential linearly rising with distance

$$\bar{V}_d(R) = \bar{E}(R) = \gamma R. \quad (3.3)$$

In the quantum case $\bar{E}(R)$ in (3.2) is an operator. Hence,

$$V_d(R) = \langle \psi_0 | \bar{E}(R) | \psi_0 \rangle, \quad (3.4)$$

where ψ_0 is the wave function of the ground state of the string. A straightforward way to quantize this model is to use the Dirac method^{/16/}. After imposing the gauge condition (2.19) we obtain the Hamiltonian system with the second-class constraints (2.9), (2.19). To co-ordinate the commutators of the quantum operators, for example $\vec{\alpha}_n$, with the constraints equations, it is necessary to employ the rule

$$[\alpha_n^k, \alpha_m^j] = i \{ \alpha_n^k, \alpha_m^j \}^*, \quad (3.5)$$

where $\{ \dots \}^*$ are the Dirac brackets^{/16/} for the set of constraints (2.32), (2.33). Very complicated expressions thus obtained for the basic commutators (3.5) do not enable one to realize directly this program even in the theory of a free relativistic string in the timelike gauge^{/20/}.

More convenient for our purpose is another approach analogue to the quantization of the free string in the covariant form^{/1,2/}. We shall interpret the constraints (2.32), (2.33) at the quantum level as the conditions for the physical state vectors, despite that these constraints are the second-class ones. Remember that the Virasoro conditions in the covariant quantization of the free relativistic string are the first-class constraints at the classical level only. In the quantum case the algebra of the Virasoro operators is not closed due to the anomalous Schwinger terms that appear as a result of the normal ordering of the operators in the Virasoro conditions. This drawback by interpreting the Virasoro operators as the conditions on the physical state vectors is easily got over: it is sufficient to impose on the state vectors only "the positive frequency parts" of these operators.

So, we shall consider the amplitudes α_n^i in the expansion (2.27) as the usual harmonic oscillator operators with the commutation relations

$$[\alpha_n^k, \alpha_m^j] = i \{ \alpha_n^k, \alpha_m^j \} = n \delta_{kj} \delta_{n,-m}. \quad (3.6)$$

Further in L_0 we postulate the normal product of $\vec{\alpha}_m$

$$L_0 = \frac{1}{2} \sum_{m=-\infty}^{+\infty} : \vec{\alpha}_{-m} \cdot \vec{\alpha}_m : - \frac{\pi}{2}. \quad (3.7)$$

The operators L_n obey the following algebra

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{\pi}{2} (n-m) \delta_{n,-m} + \frac{D-1}{12} n(n^2-1) \delta_{n,-m}. \quad (3.8)$$

If $n, m \geq 0$, two last terms in (3.8) vanish and the algebra of L_n is closed. This enables one to impose the following conditions on the physical state vectors

$$L_n |\psi\rangle = 0, \quad n = 1, 2, \dots, \quad L_0 |\psi\rangle = \alpha(0) |\psi\rangle, \quad (3.9)$$

where $\alpha(0)$ is a constant that can be introduced into the classical expression (3.7) by passing to the quantum theory. As $L_{-n} = L_n^\dagger$, the conditions (3.9) are sufficient that all the constraint equations will be satisfied in the quantum case at the level of matrix elements with respect to the physical states.

In the model under consideration there appear only the space-like vectors $\vec{\alpha}_n$, therefore the problem of state vectors with a negative norm does not arise here. Hence the mechanism that fixes the dimension of space-time D and the constant $\alpha(0)$ in the covariant quantum theory of the free relativistic string^{/1,2/} does not work here. The constant $\alpha(0)$ in our case can be determined only from physical considerations.

In paper^{/11/} the attempt was made to fix the constants D and $\alpha(0)$ in this problem demanding the fulfilment of the Poincaré algebra at the quantum level. But this approach is obviously inconsistent because the formulation of the model of the relativistic string with fixed ends assumes immediately the loss of Lorentz invariance.

The same constant $\alpha(0)$ can be introduced into the quantum expression for the string energy

$$E(R) = \left(\gamma^2 R^2 + \pi \gamma \sum_{m \neq 0} : \vec{\alpha}_{-m} \cdot \vec{\alpha}_m : - 2\pi \gamma \alpha(0) \right)^{1/2}. \quad (3.10)$$

For the potential (3.4) we obtain

$$V_q(R) = \sqrt{\gamma^2 R^2 - 2\pi \gamma \alpha(0)}. \quad (3.11)$$

In principle this formula could be used for determining the constant $\alpha(0)$ if the function $V_q(R)$ is known experimentally.

In papers^{/6-11,13/} devoted to the calculation of the string potential the second term in (3.11) turns out to be dependent on the dimension of space-time D . A formal consideration leading to this result is as follows. The constant $\alpha(0)$ that appears in (3.9) and (3.10) can be interpreted, according to paper^{/21/}, as a contribution of the zero-point fluctuations of harmonic oscillators with amplitudes $\vec{\alpha}_m$. Really, taking into account these fluctuations one should replace the classical expression

$$\sum_{m \neq 0} \vec{\alpha}_{-m} \cdot \vec{\alpha}_m = 2 \sum_{m=1}^{\infty} \sum_{\tau=1}^{D-1} m \alpha_m^{\tau i} \alpha_m^{\tau i} \quad (3.12)$$

by the operator

$$2 \sum_{m=1}^{\infty} m \sum_{\tau=1}^{D-1} \left(\alpha_m^{\tau i} \alpha_m^{\tau i} + \frac{1}{2} \right) = (D-1) \sum_{m=1}^{\infty} m + 2 \sum_{m=1}^{\infty} \sum_{\tau=1}^{D-1} m \alpha_m^{\tau i} \alpha_m^{\tau i}. \quad (3.13)$$

Here the following notation is used:

$$\vec{\alpha}_m = \sqrt{m} \vec{a}_m, \quad \vec{\alpha}_{-m} = \sqrt{m} \vec{a}_m^{\dagger}, \quad m=1,2,3,\dots, \quad (3.14)$$

$$[a_n^i, a_m^j] = \delta_{ij} \delta_{nm}, \quad [a_n^i, a_m^{\dagger}] = [a_n^{\dagger}, a_m^i] = 0.$$

In (3.13) it is supposed that all the oscillators are independent. But in the case under consideration the amplitudes $\vec{\alpha}_m$ are subjected to the constraints (2.32), (2.33). Therefore, at fixed m we have $D-2$ independent amplitudes, instead of $D-1$. Thus,

$$\alpha(0) = - \frac{D-2}{2} \sum_{m=1}^{\infty} m. \quad (3.15)$$

The divergent series in (3.15) should be regularized. Compare it with the Riemann ζ -function

$$\zeta(s) = \sum_{m=1}^{\infty} m^{-s}, \quad \text{Re } s > 1$$

This function can be continued analytically to the point $s = -1$. This gives

$$\zeta(-1) = -\frac{1}{12}.$$

Therefore, one may assume the renormalized value of $\alpha(0)$ to be

$$\alpha(0) = \frac{D-2}{24}. \quad (3.16)$$

This procedure of renormalization of the divergent series (3.15) turns out to be in agreement with the requirement that in the noncovariant quantum theory of the free relativistic string the Poincaré algebra is fulfilled in the 26-dimensional space-time^{/23/}. However, for the string with fixed ends this consideration is absolutely formal.

The substitution of (3.16) into (3.11) leads to the static quark-antiquark potential obtained in papers^{/6-11/}

$$V_q(R) = \sqrt{\gamma^2 R^2 - \pi \gamma \frac{D-2}{12}}. \quad (3.17)$$

This formula can be used only at positive values of the expression under the radical sign.

4. Conclusion

So, using the timelike gauge we have shown that the relativistic string connecting the quark-antiquark generates between them the static potential of the form

$$V_q(R) = \sqrt{\gamma^2 R^2 + \mathcal{E}_0},$$

where the constant \mathcal{E}_0 is a free parameter in the theory that should be determined from experiment. The proposed method of quantization of the relativistic string with fixed ends does not lead to

restrictions on the space-time dimension, and there are no tachyon states. An important point in this approach is the fulfilment of the constraints and gauge conditions in quantum theory only as the matrix elements with respect to physical state vectors.

APPENDIX

Nonrelativistic limit in the theory of the Nambu-Goto string with massive ends. Here we represent simple but rigorous considerations that show in what way in the nonrelativistic limit the potential linearly rising with distance appears in the string model. Let us introduce in the string action (2.5) the velocity of light C and take into account that the dimension of the constant γ in this case is $[M] \cdot [T]^{-1}$:

$$S = -\gamma \int_{t_1}^{t_2} dt \int_0^{\pi} d\delta \sqrt{\dot{\vec{x}}^2 (c^2 - \dot{\vec{x}}^2) + (\dot{\vec{x}} \cdot \dot{\vec{x}})^2} - \sum_{i=1}^2 m_i c \int_{t_1}^{t_2} \sqrt{c^2 - \dot{\vec{x}}_i^2} dt. \quad (A.1)$$

Here $\vec{x}_i(t, \delta)$, $i = 1, 2$, $\delta_1 = 0$, $\delta_2 = \pi$ describe the trajectories of massive string ends. Suppose that the velocities $\dot{\vec{x}}(t, \delta)$ of all the string points are considerably less than the light velocity C

$$|\dot{\vec{x}}(t, \delta)| \ll c, \quad 0 \leq \delta \leq \pi.$$

As a result, we obtain

$$S \approx -\gamma c \int_{t_1}^{t_2} dt \int_0^{\pi} d\delta \sqrt{\dot{\vec{x}}^2} - \sum_{i=1}^2 m_i c^2 + \sum_{i=1}^2 m_i \frac{\dot{\vec{x}}_i^2}{2}. \quad (A.2)$$

The integral over $d\delta$ gives the length of the string at moment t . We suppose that there are no folds on the string. The variation of the first term in (A.2) with respect to the string coordinates $\vec{x}(t, \delta)$, $0 < \delta < \pi$ results obviously in the requirement that the string should have the form of a segment of the straight line connecting the massive points on its ends. Therefore, the effective

action \bar{S} that gives the equations of motion for the point mass on the string ends is

$$\bar{S} = -\gamma c \int_{t_1}^{t_2} |\dot{\vec{x}}_1(t) - \dot{\vec{x}}_2(t)| dt - \sum_{i=1}^2 \frac{m_i}{2} \dot{\vec{x}}_i^2. \quad (A.3)$$

Hence in the nonrelativistic limit the string generates the potential linearly rising with distance

$$V(|\vec{x}_1 - \vec{x}_2|) = \gamma c |\dot{x}_1(t) - \dot{x}_2(t)|. \quad (A.4)$$

The string coordinates disappear from the dynamics completely. The same result can be obtained by investigation of the equation of motion of the string in the nonrelativistic approximation^{15/}.

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Kolpakov I.F. In: XI Intern. Symposium on Nuclear Electronics, JINR, D13-84-53, Dubna, 1984, p.26.

Savin I.A., Smirnov G.I. In: JINR Rapid Communications, N2-84, Dubna, 1984, p.3.

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Нестеренко В.В.

E2-85-739

Расчет статического межкваркового потенциала
в струнной модели во времени-подобной калибровке

Строится квантовая теория релятивистской струны с закрепленными концами с использованием времени-подобной калибровки. В теории нет ограничений на размерность пространства-времени и тахионных состояний. Связи и калибровочные условия выполняются в квантовом случае только в среднем, на уровне матричных элементов по допустимым /физическим/ векторам состояний. Струна генерирует статический потенциал $V(R) = \sqrt{y^2 R^2 + \epsilon_0}$, где константа ϵ_0 является свободным параметром теории, определяемым из эксперимента.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Nesterenko V.V.

E2-85-739

Calculation of the Static Interquark Potential
in the String Model in the Time-Like Gauge

Quantum theory of the relativistic string with fixed ends is constructed in the timelike gauge. The gauge conditions are introduced in the reparametrization-invariant action of the string. In the theory there are no tachyon states and constraints on the dimension of space-time. The constraints and gauge conditions are fulfilled in quantum theory only as the matrix elements with respect to physical state vectors. The string generates the static interquark potential $V(R) = \sqrt{y^2 R^2 + \epsilon_0}$, where ϵ_0 is a free parameter determined in the experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985