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PHASE-STRUCTURE OF SU(3) LATTICE GAUGE-HIGGS MODEL

1985

In recent years many papers have been devoted to the investigation of phase transitions in the gauge-Higgs models (see, for instance, papers[1,2,4-12]). First, the case of a "frozen" mode of the Higgs field has been treated[1,2]. Later, however, it has been found out that the radial fluctuations of the Higgs field affect essentially the form of phase diagrams which are very important for studying the continuum limit of lattice theories[3]. The first investigations of a "defrost" radial mode of the Higgs field have been made in papers[4] devoted to Z_2 -symmetric model. Then, it has been used in various models (Z_n , U (1), SU (2))[5-10], SU(3)[11] and SU(5)[12] for an adjoint representation).

The present paper is a sequel to a series of papers[5,6] aimed at studying the phase structure of the lattice gauge-Higgs theories with different symmetry groups. We consider a model with SU(3) symmetry in which the Higgs fields are transformed by the fundamental representation of SU(3) group and their radial mode is assumed to be defrost. Especially interesting this model is for the investigation of the dependence of the phase structure of the model with SU(N) symmetry on N and elucidation of the spontaneous symmetry breaking mechanism in the scalar QCD. We have found that the phase structures of SU(3) and SU(2) symmetries are similar [6]. In particular, there are two different phases which are often named "Higgs" and "confinement" ones and are divided by the line of the first order phase transitions in accordance with the Coleman-Weinberg mechanism [13].

We choose the action for a gauge field with the symmetry group SU(3) interacting with Higgs fields in the fundamental representation of the gauge group in the form

$$S = \beta \sum_{\square} S_{\square} + \sum_{L} S_{L}, \qquad (1)$$

where

$$S_{\square} = 1 - \tfrac{1}{3} \mathrm{Re\,Tr}\, U_{\square}, \qquad (2)$$

$U_{\square} = U_{ij} U_{jk} U_{kl} U_{li}$ and the gauge field $U_{ij} \equiv U_L$ is defined on the link $L = (i,j)$ outgoing from the site i and ending on the site $j = i+\mu$. The second term in (1) has the form

$$S_L = \tfrac{1}{4} \left(\tfrac{m^2}{2} \phi_i^* \phi_i + \lambda (\phi_i^* \phi_i)^2 \right) + \phi_i^* \phi_i - \mathrm{Re}\, \phi_i^+ U_{i,i+\mu} \phi_{i+\mu}. \qquad (3)$$

The Higgs fields ϕ_i are defined in each site i and ϕ_i is the column of three rows. The partition function Z is

$$Z = \int \prod_i d^6\phi_i \prod_L dU_L\, e^{-S\{U_L, \phi_i\}}, \qquad (4)$$

where dU_L is the Haar measure on group SU(3) and $d^6\phi_i = \prod_{k=1}^{3} d\,\mathrm{Re}\,\phi_i^{(k)} d\,\mathrm{Im}\,\phi_i^{(k)}$.
In our paper we calculated the following order parameters

$$\langle R^2 \rangle \equiv Z^{-1} \int \prod_i d^6\phi_i \cdot \prod_L dU_L \cdot R_i^2 e^{-S}, \qquad (5)$$

$$\langle 1-\square \rangle \equiv Z^{-1} \int \prod_i d^6\phi_i \cdot \prod_L dU_L\, \mathrm{Re}\, (1 - \tfrac{1}{3} \mathrm{Tr}\, U_{\square}) e^{-S}. \qquad (6)$$

If $m^2 \to \infty$ or $\lambda \to \infty$ the radial fluctuations of the Higgs field become negligible and we are left with a pure SU(3) gauge theory with a crossover in the order parameter $\langle 1-\square \rangle$ at $\beta \approx 6$ [14].

3. The model was numerically investigated on the cubic lattice with dimensions 3^4 and 4^4 and periodic boundary conditions. An equilibrium was achieved by the Markovian procedure realized in different ways for the gauge and Higgs fields. For the gauge fields the heat bath method with an algorithm similar to that of ref.[15] was used. The values of the Higgs field ϕ were optimized by the Metropolis algorithm[16]. 10-15 updates of the ϕ field values in each site and on each link of the lattice were optimized. Further updates do not make the convergence better in the phase transition region.

A phase diagram in the plane (β , m^2) at fixed λ can be constructed by two methods: thermal cycles and the method of different starts. At each step of thermal cycle the averaging

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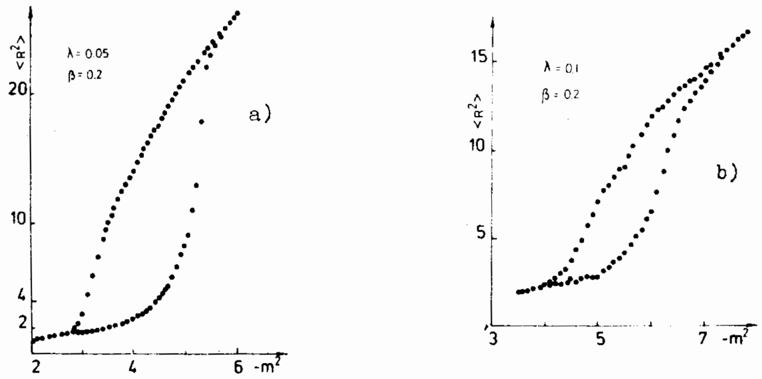
was performed over 1-2 iterations. The typical step of changing the parameter was equal to 0.05-0.2. The typical shape of a hysteresis loop which then may appear is shown in figs. 1a,b and 2a,b. The de-



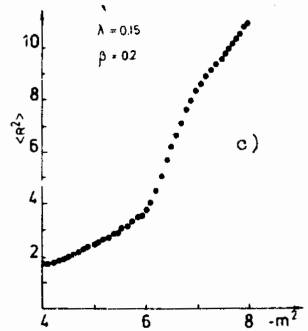
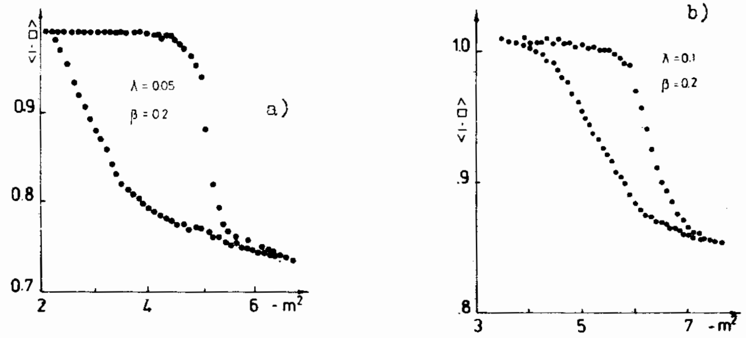


Fig.1. Thermal cycles in m^2 for order parameter $\langle R^2 \rangle$ at β = 0.2 and different values of λ.

pendence of the running average order parameter $\langle R^2 \rangle$ on the number of iterations is shown in fig.3. At λ = 0.1, β = -0.2 and m^2 = -5.3 the stable phase is the one corresponding to a lower start (fig.3a) whereas at λ = 0.1, β = -0.2 and m^2= -5.4 - to an upper start (fig.3c). At a medium value of m^2 (m^2= -5.35) there are two "long-lived" phases: the one apparently being stable and the other metastable (fig.3b). This means that the first order phase transition occurs near the point (λ; β; m^2) = (0.1;- 0.2; -5.35). The thermal cycles in β for the order parameter $\langle 1 - \Box \rangle$ at λ = 0,1 and different values of m^2 (m^2 = -1; -4; -7) are shown in fig.4. At m^2 = -4 the thermal cycle results in a hysteresis corresponding to the first order phase transition.



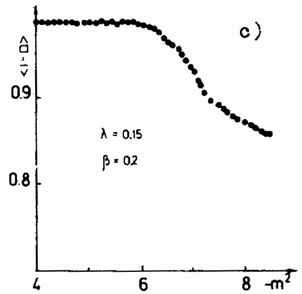
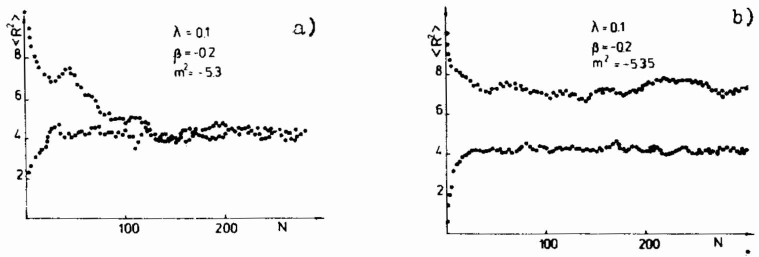


Fig.2. Thermal cycles in m^2 for the order parameter $\langle 1-\Box \rangle$ at β = 0.2 and different values of λ.



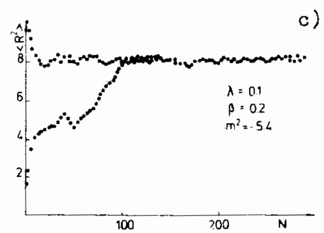


Fig.3. Behaviour of $\langle R^2 \rangle$ at different initial starts and different values of m^2.

Such calculations allow one to construct a total phase diagram in the plane (β, m^2) at fixed (value of) λ. At fixed λ in the plane (β, m^2) there exists a line of first order phase transition that breaks from the left at the end point (fig. 5). The analysis based on the use of the effective potential method (see below) allows one to conclude that second order phase transition occurs at the end point. At large enough values of β ($\beta \geq 10$) the hysteresis becomes almost unobservable. The reason may be due to the fact that as $\beta \to \infty$ instead of first order phase transition one can observe second order phase transition, though this assertion has no sufficient support.

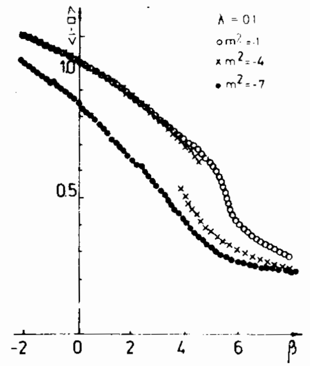


Fig.4. Dependence of $\langle 1-\square \rangle$ on β at different values of m^2. At $m^2 = -4$ the curve crosses the line of the first order phase transitions in the range of $\beta \approx 4.2$.

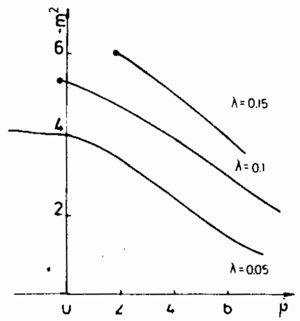


Fig.5. Lines of the first order phase transitions in the plane (β, m^2) calculated by the Monte-Carlo method. For $\lambda = 0.1$ and $\lambda = 0.15$ the end (critical) points shown in the figure are found.

With increasing λ and fixed β the hysteresis loops become narrower and narrower until it vanishes at all (see figs.1 and 2). This fact as well as the calculations by the method of different starts indicates that the line of first order phase transitions is shifted to the right-upward with increasing λ. Finally, the phase structure of SU(3) symmetric gauge-Higgs model with a defrost radial mode of the Higgs fields is demonstrated in fig.5. It is seen that the phase structure of SU(3) symmetric theory is quite analogous to

that of SU(2) model. In particular, the "confinement" and "Higgs" phases are divided by the line of first order transitions. This fact has been established for SU(2) and U(1) groups in papers /5,6/ (see also ref. /9/). Note that the analysis of an analogous model with a "frozen" radial mode /2/ also shows the similarity of the phase diagrams of SU(3) and SU(2) symmetries but there is the line of second order phase transitions. The mechanism of phase transitions can be illustrated by using an effective potential of the Coleman--Weinberg-type /13/; we shall restrict ourselves only to the case $\beta = 0$. The linear and quadratic corrections in β will be treated elsewhere. Now we consider the unitary gauge:

$$\Phi_i^* = (0, 0, R_i).$$

Then for the partition function (4) we have

$$Z(\beta=0) \sim \int \prod_l dU_l \prod_i d\mu(R_i)\, e^{-\sum_l [(1+m^2/8)R_i^2 + \frac{\lambda}{4}R_i^4 - R_i R_{i+\mu} \mathrm{Re}(U_l)_{33}} \equiv \int \prod_i dR_i\, e^{-\tilde{S}\{R_i\}}, \quad d\mu(R_i) \sim R_i^5\, dR_i. \tag{7}$$

where

$$\tilde{S}\{R_i\} = \sum_{i,\mu} \left[\left(1 + \frac{m^2}{8}\right) R_i^2 + \frac{\lambda}{4} R_i^4 - \ln \frac{8 I_2(R_i R_{i+\mu})}{(R_i R_{i+\mu})^2} - \frac{5}{8} \ln R_i^2 \right]. \tag{8}$$

From (8) we get the following expression for the effective potential in the leading approximation:

$$V_{eff}(\bar{R}) = \left(1 + \frac{m^2}{8}\right)\bar{R}^2 + \frac{\lambda}{4}\bar{R}^4 - \ln \frac{8 I_2(\bar{R}^2)}{\bar{R}^4} - \frac{5}{8} \ln \bar{R}^2. \tag{9}$$

The behaviour of the effective potential (9) allows one to understand the nature of the phase transitions observed. A qualitative behaviour of $V_{eff}(\bar{R})$ for different m^2 and an arbitrary small fixed λ is demonstrated in fig. 6. It is analogous to the behaviour of V_{eff} in

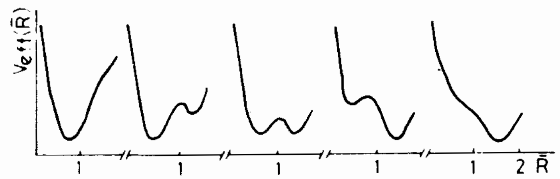


Fig.6. A qualitative form of the effective potential at $\beta = 0$, $\lambda = \mathrm{Const}$ and different m^2. Middle diagram corresponds to the point of the first order phase transition ($m^2 = m_c^2$).

case of the theory with U(1) and SU(2) symmetry groups[5,6]. As it is seen from fig.6 in a certain range of values of m^2 the effective potential has two minima: the one corresponding to a stable phase, and the other to a metastable one. At some value of $m^2 = m_c^2$ the values of Veff at both the minima coincide. This means that at $m^2 = m_c^2$ and a certain value of λ there occurs first order phase transition. With increasing λ minima approach each other and at some value of $\lambda = \lambda_c$ the effective potential cannot possess two minima at any value of m^2. The dependence of the point $m_c^2(\lambda)$ of the phase transition on λ is exemplified in fig.7. The point $\lambda = \lambda_c$,

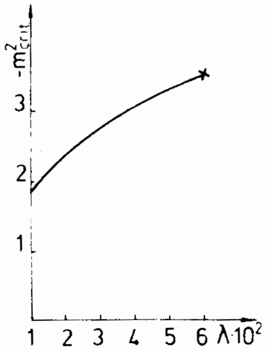


Fig.7. Dependence of m_c^2 on λ obtained by formula (9) for the effective potential at $\beta = 0$. The cross denotes the point corresponding to $\lambda = \lambda_c = 0.074$; it is the end point of the line of first order phase transitions.

$m^2 = m_c^2$ ($\beta = 0$) is the end point of the line of the first order phase transitions. It has been shown in our papers[6] that at this point a derivative of the order parameter $d\langle R^2\rangle/dm^2$ has a singularity indicating the second order phase transition at the end point.

It is interesting to trace how close quantitatively are the results for the theories with SU(3) and SU(2) groups, as their qualitative similarity has been pointed out above. It is known that a meaningful result in the theory with SU(N) symmetry in the limit $N \to \infty$ can be obtained by changing $\lambda \to \lambda/N$; $g^2 \to g^2/N$. Now we make this change for $N = 2$ and $N = 3$

$$\lambda^{SU(2)} \to \frac{\lambda_o^{SU(2)}}{2}, \qquad \lambda^{SU(3)} \to \frac{\lambda_o^{SU(3)}}{3}.$$

Then we get that $\lambda_o^{SU(2)} \approx 0.266$[5,6] and $\lambda_o^{SU(3)} \approx 0.222$ (see fig.7); the curves of the dependence $m_c^2 = m_c^2(\lambda)$ for SU(2) and SU(3) groups almost coincide in this case. Therefore, we may hope that for the

theories with SU(N) ($N > 3$) symmetry groups a similar picture will be observed.

It is to be noted that unlike our approach based on the effective Coleman-Weinberg potential[13], the mean field method provides the line of the second order phase transitions between the "confinement" and "Higgs" phases for U(1) and SU(2) models with a defrost radial mode[10]. Apparently, this is due to an inaccurate consideration of fluctuations of the gauge degrees of freedom within the lowest approximations of the average field method. But formula (9) is obtained by an accurate integration of the partition function (at $\beta = 0$) over gauge degrees of freedom.

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Фазовая структура SU(3) симметричной
хиггс-калибровочной теории на решетке

Исследована фазовая структура SU(3)-симметричной хиггс-калибровочной теории с размороженной радиальной модой. Хиггсовские поля рассмотрены в фундаментальном представлении группы SU(3). Показано, что фазовая структура SU(3)-симметричной теории качественно совпадает с фазовой структурой SU(2)-симметричной модели.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Phase-Structure of SU(3) Lattice
Gauge-Higgs Model

Phase structure is investigated of SU(3) symmetric gauge-Higgs theory with a defrost radial mode. The Higgs fields are considered in the fundamental representation of SU(3) group. It is shown that the phase structures of SU(3) and SU(2) symmetric models coincide qualitatively.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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