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DECAYS $K^{\circ}(\overline{K^{\circ}})\to\gamma\gamma$ IN THE QUARK-LOOP MODEL

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The quark-loop model (the QL-model) [1,2] well describes two-photon decays $P\to\gamma\gamma$ of pseudoscalar mesons $P=\pi^0$, η and η' [3]. In the paper, using the QL-model we consider two-photon decays of neutral K-mesons: K_L and K_S. Wave functions of K_L and K_S mesons are linear superpositions of K^0 and $\bar{K}^0$-meson wave functions [4]:

$$|K_L\rangle=[(1+\varepsilon)|K^0\rangle+(1-\varepsilon)|\bar{K}^0\rangle][2(1+|\varepsilon|^2)]^{-1/2},$$
$$|K_S\rangle=[(1+\varepsilon)|K^0\rangle-(1-\varepsilon)|\bar{K}^0\rangle][2(1+|\varepsilon|^2)]^{-1/2}. \quad (1)$$

Wave functions of K^0 and $\bar{K}^0$ mesons are connected by the CP-parity transformation: $CP|K^0\rangle=-|\bar{K}^0\rangle$. A complex parameter ε characterizes the magnitude of the CP-violation in neutral kaon decays. With $\varepsilon=0$ wave functions $|K_L\rangle$ and $|K_S\rangle$ are eigenstates of the CP-parity operator: $CP|K_L\rangle=-|K_L\rangle$ and $CP|K_S\rangle=|K_S\rangle$.

In the decay $K^0(\bar{K}^0)\to\gamma\gamma$ the strangeness changes by unity: $|\Delta S|=1$. Following [5,6] let us define the effective Lagrangian describing transitions with $|\Delta S|=1$ in the free-quark approach and consider the Kobayashi-Maskawa six-quark variant [7] of the standard electroweak model [8]:

$$\mathcal{L}_{eff}^{|\Delta S|=1}=-\frac{G_F}{\sqrt{2}}\sum_{q=u,c,t}V^*_{qd}V_{qs}[\bar{q}\gamma^\mu(1-\gamma^5)s][\bar{d}\gamma_\mu(1-\gamma^5)q]+h.c., \quad (2)$$

where G_F = 1,17x10^{-5}(GeV)$^{-2}$ is the Fermi constant, V_{qd} and V_{qs} are Kobayashi-Makawa unitary matrix elements $\left(\sum_{q=u,c,t}V^*_{qd}V_{qs}=0\right)$.

The decay $K^0(\bar{K}^0)\to\gamma\gamma$ amplitude is defined by contact and pole Feynman diagrams in Figs. 1 and 2. In the QL-model the contribution of contact diagrams in Fig. 1 equals zero. Diagrams in Fig. 1a equal zero because of their independence on q-quark mass $(q=u,c,t)$ and the unitarity of the Kobayashi-Maskawa matrix, i.e. $\sum_{q=u,c,t}V^*_{qd}V_{qs}=0$. The zero contribution of diagrams in Fig. 1b is due to the gauge invariance and takes place on photon mass shell. Indeed, diagrams in Fig. 1b contain, for example, multiplier $(k^2g^{\mu\nu}-k^\mu k^\nu)$, where k^μ is 4-momentum of photon and μ is its Lorentz index. The scalar product of the photon vector polarization ε_μ and the multiplier $(k^2g^{\mu\nu}-k^\mu k^\nu)$ equals zero on photon mass shell ($k^2=0$ and $k\varepsilon=0$). On

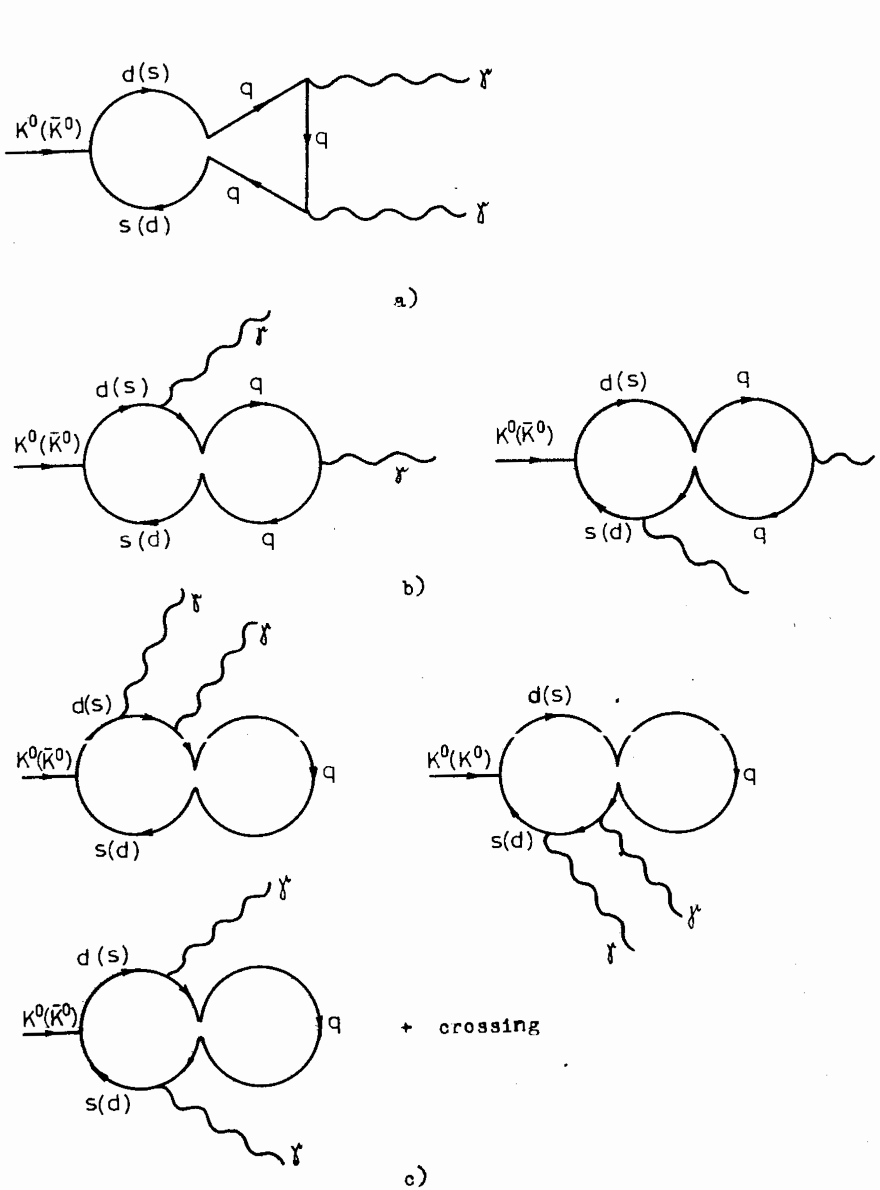

Fig. I

the other hand, diagrams in Fig. 1c equal zero owing to the left-helicity projector $\gamma^\mu(1-\gamma^5)$ in the effective Lagrangian (2). It should be emphasized that the absence of the contact diagram contributions in the QL-model agrees with estimates of these diagrams carried out in [5,6]. Thus, in the QL-model the decay $K^0(\bar K^0)\to\gamma\gamma$ amplitude is defined only by pole diagram in Fig. 2. The transitions $K^0(\bar K^0)\leftrightarrow P$, where $P=\pi^0$, η and η', are contained in the effective Lagrangian (2).

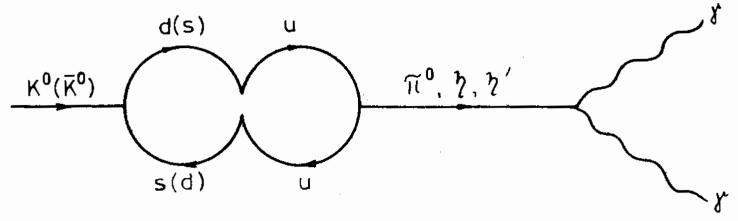

Fig. 2

It should be noted that on the above level the decay $K^0(\bar K^0)\to\gamma\gamma$ amplitude is defined by quark diagrams with usual quarks u, d and s. The contribution of heavy flavours c and t is displayed only in the vanishing of contact diagrams in Fig. 1a. However it may take place just in the GIM variant [9] of the standard electroweak model with four quarks u, d, s and c. Thus, the six-quark structure of the electroweak model doesn't display in decays $K^0(\bar K^0)\to\gamma\gamma$. It may be nonvalid for the decay $K^0(\bar K^0)\to\mu^+\mu^-$ where photons are virtuals and therefore diagrams in Fig. 1b depending on c and t -quark masses don't come to zero.

Let us write down the matrix element $\langle P|\mathcal{L}_{eff}^{|\Delta S|=1}|K^0\rangle$ calculated within the QL-model [10]:

$$\langle\pi^0;\eta;\eta'|\mathcal{L}_{eff}^{|\Delta S|=1}|K^0\rangle =$$

$$= -\frac{G_F}{\sqrt{2}}\sum_{q=u,c,t} V^*_{qd}V_{qs}\langle\pi^0;\eta;\eta'|[\bar q\gamma^\mu(1-\gamma^5)s][\bar d\gamma_\mu(1-\gamma^5)q]|K^0\rangle =$$

$$=(1;\sin(\theta_0-\theta_p);\cos(\theta_0-\theta_p))\times\left(-\frac{G_F}{\sqrt{2}}V^*_{ud}V_{us}\right)\frac{\sqrt{2}}{3}F_\pi(m_K^2)F_K(m_K^2,\lambda) \quad (3)$$

We have denoted

$$F_\pi(m_K^2) = F_\pi\left(1 + m_K^2/8\pi^2 F_\pi^2 Z\right), \tag{4a}$$

$$F_K(m_K^2,\lambda) = F_\pi\left(\frac{1+\lambda}{2}\right)\left[1-\frac{\alpha_\rho}{2\pi}\left(\frac{\lambda^2\ln\lambda^2}{\lambda^2-1}-1\right)\right]^{1/2}\cdot$$

$$\cdot\left\{1+\frac{2}{1+\lambda}\,\frac{3}{Z}\left(\frac{m_K}{2\pi F_\pi}\right)^2\left[1-\frac{\alpha_\rho}{2\pi}\left(\frac{\lambda^2\ln\lambda^2}{\lambda^2-1}-1\right)\right]^{-1}\cdot\right. \tag{4b}$$

$$\left.\cdot\left[\frac{\lambda^2+1}{2(\lambda^2-1)^2}+\frac{\lambda^4-5\lambda^2-2}{6(\lambda+1)(\lambda^2-1)^2}-\frac{\lambda^2\ln\lambda^2}{(\lambda+1)(\lambda^2-1)^3}\right]\right\}.$$

Here m_K =498 MeV is the neutral kaon mass, $\lambda = m_s/m_u$ is the parameter of the unitary symmetry breakdown, $\alpha_\rho \approx 3$ is the decay coupling constant, F_π =93 MeV is the decay $\pi \to \mu\nu$ constant, θ_P is a mixing angle of the nonet of pseudoscalar mesons and $tg\,\theta_0 = 1/\sqrt{2}$. The multipier $Z^{-1} = 1 - 6m_u^2/m_{A_1}^2 = 0.72$, where m_u =276 MeV is a u -quark mass and m_{A_1} =1275 MeV is the A_1 -axial meson mass, takes into account the finite renormalization of the nonet-pseudoscalar-meson wave functions, and it is a consequence of transitions $A \leftrightarrow P$ /11/. The Z magnitude is the same for all nonet components /12/.

Then m_K^2-terms in (4) are so-called q^2-terms /1,2/ that describe the q^2-dependence of the matrix element $K^0(\bar{K}^0) \leftrightarrow P$ transition. On the kaon mass shell q^2 equals m_K^2 ($q^2 = m_K^2$). The account of q^2-terms increases the magnitude of the $K^0(\bar{K}^0) \leftrightarrow P$ transition matrix element by 1.5 times. This seems to be very important for comparing theoretical computations with experimental data.

Using formula (3) we can write down the effective Lagrangian for two-photon decays of K^0 and $\bar{K}^0$ mesons

$$\mathcal{L}_{eff} = \frac{1}{\sqrt{2}}\xi(\alpha G_F)\left(V_{ud}V_{us}^* K^0 + V_{ud}^* V_{us}\bar{K}^0\right)\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}, \tag{5}$$

where $F_{\mu\nu}$ is the electromagnetic-field-strength tensor, $\alpha = e^2/4\pi = 1/137$ is the fine structure constant, and ξ is a model-dependent parameter. In the QL-model ξ takes the form

$$\xi = \frac{\sqrt{2}}{64\pi}F_K(m_K^2,\lambda)\left(1+\frac{m_K^2}{8\pi^2F_\pi^2 Z}\right)\left\{\frac{m_K^2}{m_K^2-m_{\pi^0}^2}\right. +$$

$$+\frac{m_K^2}{m_K^2-m_\eta^2}\,\frac{\sin(\theta_0-\theta_P)}{3}\left[5\sin(\theta_0-\theta_P)-\frac{\sqrt{2}}{\lambda}\cos(\theta_0-\theta_P)\right]+ \tag{6}$$

$$\left.+\frac{m_K^2}{m_K^2-m_{\eta'}^2}\,\frac{\cos(\theta_0-\theta_P)}{3}\left[5\cos(\theta_0-\theta_P)+\frac{\sqrt{2}}{\lambda}\sin(\theta_0-\theta_P)\right]\right\}.$$

Now let us rewrite the Lagrangian (5) in terms of K_L and K_S states and keep up only terms linear in ε :

$$\mathcal{L}_{eff} = \xi(\alpha G_F)V_{ud}V_{us}(K_L - \varepsilon K_S)\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{7}$$

Here we have taken into account that V_{ud} and V_{us} are real, i.e., $V_{ud}^* = V_{ud}$ and $V_{us}^* = V_{us}$ /4/.

Table. Numerical values of partial decay $K_L, K_S \to \gamma\gamma$ widths

Decay	Theory		Experiment (in units 10^{-18} MeV)
	Algebraic expression	Numerical value (in units 10^{-18} MeV)	
$K_L \to \gamma\gamma$	$\xi^2(\alpha G_F)^2 \cdot \lvert V_{ud}V_{us}\rvert^2 m_{K^0}^3$	4,79	$6{,}22 \pm 0{,}52$ /4/
$K_S \to \gamma\gamma$	$\lvert\varepsilon\rvert^2\xi^2(\alpha G_F)^2 \cdot \lvert V_{ud}V_{us}\rvert^2 m_{K^0}^3$	$(2{,}5 \pm 0{,}1)\times 10^{-6}$	$< 2{,}95\times 10^3$ /4/

Note: Numerical values of $\Gamma_{K_L\to\gamma\gamma}$ and $\Gamma_{K_S\to\gamma\gamma}$ are obtained with λ =1,6 and θ_P =-18°, V_{ud} =0,974, V_{us} =0,228 and $|\varepsilon|^2$ =(5,22±0,26)x10^{-6} /4/.

The numerical values of partial decays $K_L, K_S \to \gamma\gamma$ widths are represented in the Table. The computation is carried out with λ =1,6 and θ_P =-18° /13/. With such values of λ and θ_P we can obtain: 1) a correct mass spectrum of nonets of pseudoscalar and vector mesons [13] and 2) correct values of partial widths of decays $\eta\to\gamma\gamma$ and $\eta'\to\gamma\gamma$:

$$\Gamma_{\eta\to\gamma\gamma}=\left(\frac{m_\eta}{\pi}\right)^3\left(\frac{\alpha}{24F_\pi}\right)^2\left[5\sin(\theta_0-\theta_p)-\frac{\sqrt{2}}{\lambda}\cos(\theta_0-\theta_p)\right]^2=0.69\,KeV,$$

$$\Gamma^{exp}_{\eta\to\gamma\gamma}=(0.56\pm0.12\pm0.10)\,KeV\ [14]$$

$$\Gamma_{\eta'\to\gamma\gamma}=\left(\frac{m_{\eta'}}{\pi}\right)^3\left(\frac{\alpha}{24F_\pi}\right)^2\left[5\cos(\theta_0-\theta_p)+\frac{\sqrt{2}}{\lambda}\sin(\theta_0-\theta_p)\right]^2=4.15\,KeV,$$

$$\Gamma^{exp}_{\eta'\to\gamma\gamma}=(3.80\pm0.26\pm0.43)\,KeV\ [15];\ (4.42\pm0.34)\,KeV\ [16].$$

A theoretical value of the partial decay $K_L\to\gamma\gamma$ width is less than the experimental one approximately by 30%. It is possible to get a required agreement by a suitable variation of parameters λ and θ_p. However, in this way we may get in contradiction with experimental data on pseudoscalar and vector meson mass spectra and their radiative decays. For instance, the variation of θ_p by one degree changes the magnitude of the partial decay $K_L\to\gamma\gamma$ width approximately by 10%. Thus, with $\theta_p\approx -21°$ we can get a good agreement with experimental data. However, this decrease of the mixing angle leads to undesirable increase of the partial decay $\eta\to\gamma\gamma$ width: $\Gamma_{\eta\to\gamma\gamma}=0.77$ keV. So, it is natural not to change λ and θ_p. In this case we can explain the discrepancy of theoretical and experimental values of the partial decay $K_L\to\gamma\gamma$ width by insufficient accuracy of the free-quark approach of the $\mathcal{L}^{|\Delta S|=1}_{eff}$ calculation and necessity of QCD-high-energy corrections. QCD-corrections to $\mathcal{L}^{|\Delta S|=1}_{eff}$ can change the $\Gamma_{K_L\to\gamma\gamma}$ value approximately by 25÷30%. Such an increase takes place, for example, in the $K^0\leftrightarrow\bar{K}^0$ transition matrix element [10].

Photons in decay $K^0(\bar{K}^0)\to\gamma\gamma$ are formed in a CP-odd state. Thus, in the free-quark approach the decay $K_S\to\gamma\gamma$ is only due to a CP-violating interaction. We take the latter into account by means of a phenomenological parameter $\varepsilon=(1{,}536\pm0{,}062)\times10^{-3}+i(1{,}692\pm0{,}052)\times10^{-3}$ [4]. The theoretical value of the partial decay $K_S\to\gamma\gamma$ width seems to us to be very small. In our opinion, it is possible to increase the value of $\Gamma_{K_S\to\gamma\gamma}$ by using the $\mathcal{L}^{|\Delta S|=1}_{eff}$ with QCD-high-energy corrections. In this case photons of decay $K^0(\bar{K}^0)\to\gamma\gamma$ can be formed in a CP-even state, and the decay $K_S\to\gamma\gamma$ can be allowed without CP-violation. The computation of two-photon widths of neutral kaon decays with the use of the $\mathcal{L}^{|\Delta S|=1}_{eff}$ containing QCD-high-energy corrections will be carried out in following publications.

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Иванов А.Н., Троицкая Н.И., Волков М.К. E2-85-736

Распады $K^{\circ}(\bar{K}^{\circ}) \to \gamma\gamma$ в модели кварковых петель

Вычислены парциальные ширины двухфотонных распадов нейтральных каонов в модели кварковых петель. Найдена заметная зависимость вероятности распада от угла синглет-октетного смешивания в нонете псевдоскалярных мезонов и от q^2 членов в матричных элементах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Ivanov A.N., Troitskaya N.I., Volkov M.K. E2-85-736

Decays $K^{\circ}(\bar{K}^{\circ}) \to \gamma\gamma$ in the Quark-Loop Model

Two-proton decays of neutral K-mesons are calculated in the quark-loop model. It is emphasized that partial decay widths depend on q^2 -terms and mixing angle of the nonet of pseudoscalar mesons θ_p appreciably.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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