сообщения
объединенного
института
ядерных
исследований
дубна

E2-85-678

Th.Braunschweig*, B.Geyer*, J.Hořejší,
D.Robaschik*

# HADRON OPERATORS ON THE LIGHT CONE

* Sektion Physik der Karl-Marx-Universität Leipzig, DDR

1985

## 1. INTRODUCTION

Physical hadrons are extended objects and therefore it is natural to describe them in terms of appropriate nonlocal operators. Such operators are important both for our understanding of QCD and for practical computations. Possible candidates for such operators have been discussed in the literature [1-3]. For example, a (bilocal) QCD meson operator may be written as

$$M(x_1, x_2) = :\bar{\psi}(x_1)\,\Gamma U(x_1, x_2)\,\psi(x_2):, \tag{1.1}$$

where $\psi(x)$ is the quark field, $\Gamma$ represents the $\gamma$-matrix structure of (1.1) as well as the flavour content of the considered meson and $U(x_1, x_2)$ is the standard path-ordered phase factor which ensures the gauge invariance of $M(x_1, x_2)$

$$U(x_1, x_2) = P\exp\left(-ig\int_{x_2}^{x_1} dx^{\mu} A_{\mu}(x)\right), \tag{1.2}$$

$A_{\mu}(x)$ is the gluon field. The short-distance properties and renormalization of the hadron operators such as (1.1) have been discussed, e.g., in [1,2].

In the present paper we will deal mainly with the operators which emerge in the operator product expansion (OPE) of the hadron operators of the type (1.1) near the light-cone . (The term OPE is frequently used in a somewhat vague sense. Throughout this paper we shall always have in mind the nonlocal light-cone expansion in the sense of ref. [4] unless otherwise stated). The local operators obtained from (1.1) by an additional Taylor expansion have been considered, e.g., in refs. [3,5,6,7] .Examples of such (nonlocal) light-cone operators (which have been already employed for some practical calculations in the earlier papers [8-11]) are

$$O^{\kappa}(\kappa_1, \kappa_2, \tilde{x}) = :\bar{\psi}(k_1\tilde{x})\,\Gamma U(\kappa_1\tilde{x}, \kappa_2\tilde{x})\,\psi(\kappa_2\tilde{x}): \tag{1.3}$$

or*

$$O^{t}(\kappa, t; \tilde{x}) = \partial(ik)\int_{-\infty}^{+\infty}\frac{d\lambda}{2\pi}\,O(\lambda(t-1)+\kappa, \lambda(t+1)+\kappa; \tilde{x}). \tag{1.4}$$

---

* Let us remark that the definition of the derivative operator $\partial(ik)$ employed in (1.4) requires special care; for details, see Appendix C.

where $\kappa_1$, $\kappa_2$ and $\kappa'$, $t$ are some real parameters and $\tilde{x}$ is a light-like vector, $\tilde{x}^2 = 0$.

The operator (1.3) appears in the OPE derived in[4,8] (cf. also [9]). It can be used straightforwardly for the description of deep inelastic inclusive scattering, since its anomalous dimension corresponding to forward matrix elements coincides with the Altarelli-Parisi evolution kernel[8,9]. The operator of type (1.3) has been used also to define light-cone wave function[12,13] of a meson participating in an exclusive scattering process with a large momentum transfer (see, e.g.[12,14,15]). In the latter context, the main disadvantage of (1.3) is the nondiagonality of its anomalous dimension $\gamma(\kappa_1,\kappa_2;\kappa_1',\kappa_2')$ for the nonforward matrix elements[16].

The operator (1.4) emerges in a variant of the OPE proposed in[10]. Its matrix element between vacuum and a one-meson state turns out to be nonzero for $|t| \leq 1$ only and coincides essentially with the meson wave function of the variable $t$. It is then natural that its anomalous dimension corresponding to nonforward matrix elements is diagonal with respect to $\kappa$ and the $t$-dependent part $\gamma(t,t')$ coincides for $|t| \leq 1$, $|t'| \leq 1$ just with the Brodsky-Lepage evolution kernel[10]. Therefore, from the technical point of view, (1.4) is preferred to (1.3) as a light-cone meson operator. On the other hand, (1.4) cannot be used directly for the description of deep inelastic inclusive scattering, since the forward matrix elements of (1.4) vanish for finite $t$. A solution of this problem takes into account, that the operator (1.4) has contributions to the anomalous dimensions also for $|t| > 1$, $|t'| > 1$.

From the methodical point of view, it is of course very important that the anomalous dimensions of nonlocal light-cone hadron operators may be calculated in a straightforward and compact way and that they coincide with the relevant evolution kernels of perturbative QCD. Our method is technically very close to those developed independently in the papers[18].

The aim of the present paper is twofold. First, we stress the importance of the operators of the type (1.4) as light-cone meson operators. We complete our previous investigations[8-11] by considering the quark-gluon mixing which is relevant for the wave functions of the flavour singlet mesons. We discuss in more detail the light-cone baryon operator introduced in[11]. Emphasizing the methodical virtues and practical aspects of our approach based on the operators (1.4), we summarize the salient features of the calculation of the anomalous dimensions of the relevant operators. Second, in view of the evident advantages of our approach, we reinforce our earlier arguments that the traditional OPE written in terms of local operators may be in fact completely superseded by expansions based on the operators like (1.3) or (1.4).



## 2. MATRIX ELEMENTS OF HADRON OPERATORS FOR LARGE MOMENTUM TRANSFER

In this section we study the behaviour of the meson wave function in the limit of large momentum transfer in the framework of (nonlocal) OPE. To simplify our considerations we restrict ourselves here to the flavour nonsinglet meson operator. The path, included in the definition (1.1), (1.2) of the meson operator, we fix to a straight line.

Let us now discuss the matrix element of the meson operator (1.1) between the vacuum and a one-meson state of momentum $p$; according to translation covariance we have

$$\langle 0 | M(x_1,x_2) | p \rangle = W(x_1,x_2;p) = W\left(\frac{x_-}{2}, -\frac{x_-}{2}, p\right) e^{ipx_+}, \qquad (2.1)$$

where $x_- = x_1 - x_2$, $x_+ = \frac{1}{2}(x_1 + x_2)$. Thus the meson wave function which depends on the relative coordinate only is $W(\frac{x_-}{2}, -\frac{x_-}{2}; p)$. Its Fourier transform

$$\tilde{W}(Q,p) = \int dx_- \, e^{iQx_-} W\left(\frac{x_-}{2}, -\frac{x_-}{2}; p\right) \qquad (2.2)$$

depends on the momentum transfer $Q$ between quark and antiquark. As is well known the large momentum transfer region $|Q^2| \to \infty$ and $|Qp| \to \infty$ corresponds to the light-cone $x_-^2 > 0$ in coordinate space. It is therefore adequate to study the meson operator near the light-cone with the help of an OPE. For this purpose we summarize some essential points of this approach.

According to the general procedure[4,8] the x-proper functional with the meson operator insertion has to be determined

$$RT\left(\mathfrak{M}\left(\frac{x_-}{2}, -\frac{x_-}{2}; p\right)S\right)^{x\text{-proper}} = \sum_\ell \int dz_1 dz_2 \, G_\ell(x_-, z_1, z_2). \qquad (2.3)$$

$:\bar{\psi}(z_1)\Gamma_\ell U(z_1,z_2)\psi(z_2): +$ operators of higher dimension.

The OPE is then given by

$$RT\left(M\left(\frac{x_-}{2}, -\frac{x_-}{2}\right)S\right) \underset{x_-^2 \to 0}{\approx} \int d\kappa_1 d\kappa_2 \, F^\kappa(x_-^2, \kappa_1, \kappa_2) RT(O^\kappa(\kappa_1,\kappa_2;\tilde{x})S) + \dots \qquad (2.4)$$

with the help of the nonlocal expansion operator

$$O^\kappa(\kappa_1,\kappa_2,\tilde{x}) = :\bar{\psi}(\kappa_1\tilde{x})(\gamma\tilde{x}) U(\kappa_1\tilde{x},\kappa_2\tilde{x}) \psi(\kappa_2\tilde{x}):. \qquad (2.5)$$

Here, $R$ denotes an R-operation, $S$ is the unrenormalized S-matrix, and $\tilde{x}$ is a light-like vector related to $x_-$ such that the latter approaches $\tilde{x}$ in the limit $x_-^2 \to 0$. The coefficient function $F$ is determined from $G_\ell$ with $\Gamma_\ell = \gamma x_-$ by the following procedure

$$\tilde{G}(x^2, xq_1, xq_2, q_i q_j) = \int dz_1 dz_2 \, e^{iq_1 z_1 + iq_2 z_2} \, G_{\gamma x_-}(x_-, z_1, z_2). \qquad (2.6)$$

$$F^{\kappa}(x^2,\kappa_1,\kappa_2) = \int \frac{d(xq_1)}{(2\pi)^2} d(xq_2) e^{-i\kappa_1 xq_1 - i\kappa_2 xq_2} \tilde{G}(x^2, xq_i, q_i q_j = \mu_{ij}). \quad (2.7)$$

$\mu_{ij}$ is the renormalization point. Let us remark some important points:

(i) The construction of the x-proper functional (in axial gauge) with a gauge invariant but path-dependent operator insertion makes no difficulties; the path-dependence of the meson operator is contained in the coefficient function $G_\ell(x_-, z_1, z_2)$; the path-dependence contained in $U(z_1, z_2)$ of (2.3) results from the gauge invariant choice of the expansion operators. Our choice of straight lines simplifies all constructions.

(ii) Already from dimensional arguments it is clear that the operator with minimal dimension plays a universal role as leading operator in all OPE's.

In the following we use also the variables $\kappa_\pm = \frac{1}{2}(\kappa_1 \pm \kappa_2)$ as parameters and denote $O^{\kappa}(\kappa_+ + \kappa_-, \kappa_+ - \kappa_-; \tilde{x})$ simply by $O(\kappa_+, \kappa_-)$ and $F^{\kappa}(x^2, \kappa_+ \pm \kappa_-)$ by $F(x^2, \kappa_+, \kappa_-)$. On the other hand it has been shown /10/ that another operator basis will be much more effective, namely

$$RT(M(\frac{x_-}{2}, -\frac{x_-}{2})S) \underset{x_-^2 \to 0}{\approx} \int d\kappa\, dt\, F^t(x^2, \kappa, t) RT(O^t(\kappa, t)S) + \ldots \quad (2.8)$$

with

$$O^t(\kappa, t) = \partial(i\kappa) \int \frac{d\lambda}{2\pi} O(\kappa + \lambda t, -\lambda; \tilde{x}), \quad (2.9)$$

$$F^t(x^2, \kappa, t) = \int \frac{d\tilde{x}q_+}{2\pi} e^{-i\kappa \tilde{x} q_+} \hat{G}(x^2, \tilde{x}q_+, t), \quad (2.10)$$

$$\hat{G}(x^2, \tilde{x}q_+, \frac{\tilde{x}q_-}{\tilde{x}q_+}) = \tilde{G}(x^2, \tilde{x}q_i, q_i q_j = \mu_{ij})|_{q_{1,2} = q_+ \pm q_-}.$$

Now we are able to discuss the meson wave function (2.1) near the light cone. Taking into account the OPE (2.4) and (2.5) we obtain

$$W(\frac{x_-}{2}, -\frac{x_-}{2}, p) \underset{x_-^2 \to 0}{\approx} \int d\kappa_1 d\kappa_2\, F(x^2, \kappa_+, \kappa_-) \langle 0 | RTO(\kappa_+, \kappa_-, \tilde{x}) S | p \rangle + \ldots \quad (2.11)$$

For the matrix element of the operator (2.5) we receive

$$\langle 0 | RTO(\kappa_+, \kappa_-, \tilde{x}) S | p \rangle = e^{i\kappa_+ \tilde{x}p} \langle 0 | RT\bar{\psi}(\kappa_- \tilde{x}) \gamma \tilde{x} U \psi(-\kappa_- \tilde{x}) S | p \rangle =$$

$$= e^{i\kappa_+ \tilde{x}p}\, \tilde{x}p \int d\xi\, e^{i\xi\kappa_- \tilde{x}p}\, \Phi^q(\xi, \mu^2). \quad (2.12)$$



In (2.12) we used the definition of the distribution amplitude $\Phi^q(\xi, \mu^2)$ for a quark-antiquark pair with relative momentum $\xi p$ inside a meson with momentum p /12-15/:

$$\langle 0 | RT\bar{\psi}(\kappa_- \tilde{x}) \gamma \tilde{x} U \psi(-\kappa_- \tilde{x}) S | p \rangle = \tilde{x}p \int d\xi\, e^{i\xi\kappa_- \tilde{x}p}\, \Phi^q(\xi, \mu^2). \quad (2.13)$$

Returning to eq. (2.11) we get

$$W(\frac{x_-}{2}, -\frac{x_-}{2}, p) \underset{x_-^2 \approx 0}{\approx} \int d\kappa_1 d\kappa_2\, F(x_-^2, \kappa_+, \kappa_-)\, \tilde{x}p\, e^{i\kappa_+ \tilde{x}p} \int d\xi\, e^{i\xi\kappa_- \tilde{x}p}\, \Phi^q(\xi, \mu^2) \approx \quad (2.14)$$

$$\approx \int d\xi\, \tilde{G}(x_-^2, \frac{1}{2}(1+\xi) p\tilde{x}, \frac{1}{2}(1-\xi) p\tilde{x}; \mu_{ij})\, \tilde{x}p\, \Phi^q(\xi, \mu^2).$$

Of course the same result can be reproduced with the help of the OPE (2.8) - (2.10). The most interesting point here is that the matrix element of the operator (2.9) is directly related to the quark distribution amplitude

$$\langle 0 | RTO^t(\kappa, t) S | p \rangle = \partial(i\kappa) \int \frac{d\lambda}{2\pi} \langle 0 | RTO(\kappa + \lambda t, -\lambda; \tilde{x}) S | p \rangle =$$

$$= \int d\xi \int \frac{d\lambda}{2\pi} |\tilde{x}p|\, e^{i(\kappa + \lambda t - \lambda\xi)\tilde{x}p}\, \Phi^q(\xi, \mu^2) \tilde{x}p = \tilde{x}p\, e^{i\kappa\tilde{x}p}\, \Phi^q(t, \mu^2). \quad (2.15)$$

Straightforward calculation leads back to (2.14).

$$W(\frac{x_-}{2}, -\frac{x_-}{2}, p) \underset{x_-^2 \approx 0}{=} \int d\kappa\, dt\, F^t(x^2, \kappa, t) \langle 0 | RTO^t(\kappa, t) S | p \rangle + \ldots$$

$$\approx \int dt\, \tilde{G}(x_-^2, \frac{1}{2}(1+t) p\tilde{x}, \frac{1}{2}(1-t) p\tilde{x}; \mu_{ij})\, \tilde{x}p\, \Phi^q(t, \mu^2). \quad (2.16)$$

Let us discuss the obtained results. At first we see, that both operator product expansions reproduce in a compact way the standard diagrammatic calculation scheme. Obviously the coefficient function $\tilde{G}(x_-^2, \frac{1}{2}(1+\xi)p\tilde{x}, \frac{1}{2}(1-\xi)p\tilde{x}; \mu_{ij})$ describes a process in which participates a quark-antiquark pair carrying momenta $q_1 = (1+\xi)p/2$ and $q_2 = (1-\xi)p/2$, resp., analogously to the usual diagrammatic calculation. A Fourier transformation, for which the essential contributions originate from the neighbourhood of the light cone leads to the meson wave function (2.2). In this sense both OPE's are equivalent. On the other hand it has been shown that the anomalous dimensions $\gamma(\kappa_+, \kappa_-, \kappa_+', \kappa_-')$ of the operator $O(\kappa_+, \kappa_-)$ are completely nondiagonal /16/. A more detailed investigation shows, however, that it contains the same eigenvalues as the Brodsky-Lepage kernel, so that it is in fact a nondiagonal version of this kernel. The most important point is, that the anomalous dimensions of the operator (2.9) are partially diagonal. By direct calculation /10/ it has been shown that these anomalous dimensions in the nonsinglet case are simply related to the Brodsky-Lepage kernel. In the next section we shall extend this relationship to the flavour singlet case. Furthermore we investigate a baryon operator which is simply related to baryon wave functions and evolution kernels.

## 3. QCD EVOLUTION KERNELS AS ANOMALOUS DIMENSIONS OF NONLOCAL LIGHT-CONE OPERATORS

After the general exposition of the preceding section, let us now concentrate on the meson and baryon light-cone operators of the type (1.4). Let us recall the standard definition of the parton distribution amplitude of a scalar, flavour singlet meson in terms of gauge invariant bilocal light-cone operators of the type (1.3) (cf. [14,15]) renormalized at a scale $\mu^2$ :

$$<0|RT\bar{\psi}(\kappa_-\tilde{x})\gamma\tilde{x}U^q\psi(-\kappa_-\tilde{x})S|M;p> = \tilde{x}p \int_{-1}^{+1} d\xi e^{i\xi\kappa_-\tilde{x}p} \Phi^q(\xi,\mu^2), \quad (3.1a)$$

$$<0|RT\tilde{x}^\mu F_{\mu\lambda}(\kappa_-\tilde{x})U^G F_\nu^\lambda(-\kappa_-\tilde{x})\tilde{x}^\nu S|M;p> =$$

$$= (\tilde{x}p)^2 \int_{-1}^{+1} d\xi e^{i\xi\kappa_-\tilde{x}p} \Phi^G(\xi,\mu^2). \quad (3.1b)$$

where $U^q$ and $U^G$ denote symbolically the path-ordered phase factors in the fundamental and adjoint representations resp. (cf. (1.2) and [2,9] ). In eq. (3.1) M is a scalar flavour singlet meson with momentum p ; $\Phi^q$ and $\Phi^G$ denote the quark and gluon components of the parton distribution amplitude respectively. In analogy with ref. [10] we introduce the following operators

$$O^{tq}(\kappa,t) = \frac{\partial}{\partial i\kappa}\partial(i\kappa)\int\frac{d\lambda}{2\pi} O^q(\kappa+\lambda t,-\lambda), \quad (3.2a)$$

$$O^{tG}(\kappa,t) = \partial(i\kappa)\int\frac{d\lambda}{2\pi} O^G(\kappa+\lambda t,-\lambda), \quad (3.2b)$$

where $O^q$, $O^G$ are operators of the type (1.3) (cf. ref. [9])

$$O^q(\kappa_+,\kappa_-) = :\bar{\psi}((\kappa_+ + \kappa_-)\tilde{x})\gamma\tilde{x}U^q\psi((\kappa_+ - \kappa_-)\tilde{x}):,$$

$$O^G(\kappa_+,\kappa_-) = :\tilde{x}^\mu\tilde{x}^\nu F_{\mu\lambda}((\kappa_+ + \kappa_-)\tilde{x})U^G F_\nu^\lambda((\kappa_+ - \kappa_-)\tilde{x}):.$$



It is then easy to verify (see eq. (2.15))

$$<0|RT(O^{tq}(\kappa,t)S|M;p> = (\tilde{x}p)^2 e^{i\kappa\tilde{x}p}\Phi^q(t,\mu^2), \quad (3.3a)$$

$$<0|RT(O^{tG}(\kappa,t)S|M;p> = (\tilde{x}p)^2 e^{i\kappa\tilde{x}p}\Phi^G(t,\mu^2). \quad (3.3b)$$

Notice that (3.3a) contains an extra factor $\tilde{x}p$ in comparison with formula (2.15). This is due to the extra derivative with respect to $\kappa$ in (3.2a). Such a derivative is useful if we want to deal simply with the quark-gluon mixing, since $O^G$ involves one extra factor $\tilde{x}$ in comparison with $O^q$ (see also ref. [9] ). On the basis of the arguments given in [10] we expect that the matrix elements of the operators (3.2) between vacuum and one-meson states are nonzero for $|t|\le 1$ only.

We may now consider the mixing of the operators (3.2) under renormalization. One expects that the corresponding matrix of anomalous dimensions should reproduce the evolution kernels for the scalar parton distribution amplitude obtained earlier by different methods [14,19] , since the renormalization group equation for the operators (3.2) may be reinterpreted in view of (3.3), as an evolution equation for $\Phi^q(t,\mu^2)$ and $\Phi^G(t,\mu^2)$ with respect to $\mu^2$. The main virtue of our approach is that an explicit calculation of the anomalous dimensions of the nonlocal operators (3.2) is indeed feasible. We thus arrive at an alternative and efficient method of direct calculation of QCD evolution kernels. In technical sense, our procedure turns out to be very close (though not identical) to that developed in the papers [18]*.

In the sequel we shall formulate the salient features of our method (a sample of one-loop calculation is given in Appendix A):

1. The anomalous dimensions of the operators (3.2) are generalized functions $\gamma^{AB}(\kappa,t,\kappa',t')$ (where A,B denote q or G) and may be defined in terms of the corresponding Z-factors as follows

$$\mu\frac{\partial}{\partial\mu}Z^{AB}(\kappa,t;\kappa',t') = \int d\kappa'' dt'' Z^{AC}(\kappa,t,\kappa'',t'')\gamma^{CB}(\kappa'',t'',\kappa',t'), \quad (3.4)$$

---

* We stress that in all calculations we restrict ourselves to the interval $|t|\le 1$ , $|t'|\le 1$ . In principle it is possible to calculate formally the anomalous dimensions in question for $t,t'\in(-\infty,\infty)$.

where in shorthand notation

$$\begin{pmatrix} O^{tq} \\ O^{tG} \end{pmatrix}^{unren.}_{1PI} = \begin{pmatrix} Z_q^{-1} Z^{qq} & Z^{qG} \\ Z^{Gq} & Z^{-1} Z^{GG} \end{pmatrix} \circledast \begin{pmatrix} O^{tq} \\ O^{tG} \end{pmatrix}^{ren.}_{1PI} . \qquad (3.5)$$

In (3.5) the symbol $\circledast$ means matrix multiplication combined with the integration over $\kappa', t'$, $Z_q$ and $Z_G$ are the standard Z-factors for quark and gluon fields respectively and the symbol 1PI denotes the sum of all relevant one-particle irreducible graphs.

2. Arguments given in [10] indicate that $\gamma^{AB}(\kappa,t;\kappa',t')$ is diagonal with respect to the variable $\kappa$, i.e.,

$$\gamma^{AB}(\kappa,t,\kappa',t') = \delta(\kappa-\kappa')\,\gamma^{AB}(t,t') . \qquad (3.6)$$

We shall verify (3.6) by an explicit one-loop calculation.

3. It turns out that the mixing (3.5) takes place for properly symmetrized operators only. The gluon operator $O^G(\kappa_1,\kappa_2)$ is symmetric under the exchange $\kappa_1 \to \kappa_2$ or $\kappa_+ \to \kappa_+$, $\kappa_- \to \kappa_-$ and $O^{tG}(\kappa,t)$ is symmetric under the exchange $\kappa \to \kappa$, $t \to -t$. On the other hand the quark operator $O^q(\kappa_1,\kappa_2)$ has a more involved symmetry behaviour; its anti-symmetric part mixes under renormalization with the gluon operator whereas the symmetric part decouples completely. The operators to be considered are therefore

$$O_a^{tq}(\kappa,t) = \partial(i\kappa)\frac{\partial}{\partial i\kappa}\int \frac{d\lambda}{4\pi}[O^q(\kappa+\lambda t,-\lambda) - O^q(\kappa+\lambda t,\lambda)] . \qquad (3.7a)$$

$$O_s^{tG}(\kappa,t) = \partial(i\kappa)\int \frac{d\lambda}{4\pi}[O^G(\kappa+\lambda t,-\lambda) + O^G(\kappa+\lambda t,\lambda)] . \qquad (3.7b)$$

Evidently, (3.7) implies $\Phi^q(t) = -\Phi^q(-t)$ while $\Phi^G(-t) = \Phi^G(t)$. This agrees with the symmetry properties of the scalar-meson parton distribution amplitudes established on physical grounds [14].

4. The explicit one-loop calculation of the matrix $\gamma^{AB}(\kappa,t,\kappa',t')$ amounts to the evaluation of the uv-divergent parts of the Z-factors in (3.5). The relevant one-loop diagrams describing the quark-gluon mixing are familiar and have been exploited in many places (see, e.g., [9,21] ). The Feynman rules for the operator vertices corresponding to (3.7) are obtained in standard way. For an explicit example see formulae (A.1) through (A.4).

5. To evaluate the uv-divergent part of a relevant diagram we employ covariant gauge and employ dimensional regularization. It is convenient to start with the exponential form of the operator vertices (see (A.1) or (A.3)). After introducing Feynman



parameters and shifting the loop momentum k, the exponentials from operator vertices are expanded in Taylor series in powers of $\bar{k} = k\tilde{x}$. It turns out, that such an expansion terminates after the first few terms when the integrations are performed, owing to $\tilde{x}^2 = 0$. This simple observation is crucial and facilitates greatly the whole calculation.

6. Ultimately, one is left with an integral over $\lambda$ and over Feynman parameters. These parameters may be appropriately transformed in order to recover the correct parametric dependence of the lowest operator vertex.

Although it is clear that the one-loop calculation is particularly simple, we believe that our method is applicable to higher-order calculations as well, similarly to the methods of ref.[18]. The results of our one-loop calculation are the following (cf. (3.6))

$$\gamma^{qq}(t,t') = -C_F\frac{g^2}{8\pi^2}\Big[\theta(t-t')\Big(-\frac{1+t}{1-t'} + \frac{2}{(t-t')_+}\Big) + \frac{3}{2}\delta(t-t') + \begin{pmatrix} t\to -t \\ t'\to -t' \end{pmatrix}\Big] \qquad (3.8)$$

$$\gamma^{Gq}(t,t') = -C_F\frac{g^2}{32\pi^2}\Big[\theta(t-t')\frac{1-t}{1-t'}(1+t-2t') - \begin{pmatrix} t\to -t \\ t'\to -t' \end{pmatrix}\Big] \qquad (3.9)$$

$$\gamma^{qG}(t,t') = -n_f\frac{g^2}{2\pi^2}\Big[\theta(t-t')\frac{(1-t)(1-t'+2t)}{(1-t')(1-t'^2)} - \begin{pmatrix} t\to -t \\ t'\to -t' \end{pmatrix}\Big] \qquad (3.10)$$

$$\gamma^{GG}(t,t') = C_A\frac{g^2}{4\pi^2}\Big[\theta(t-t')\Big(\frac{t^2+t'^2}{(1+t)(1-t')} - \frac{1}{(t-t')_+} + \frac{1}{2}\Big)\frac{1-t^2}{1-t'^2}$$
$$- \frac{1}{2}\delta(t-t')\Big(\frac{11}{6} - \frac{1}{3}\frac{n_f}{C_A}\Big) + \begin{pmatrix} t\to -t \\ t'\to -t' \end{pmatrix}\Big] \qquad (3.11)$$

where the distribution $(x-y)_+^{-1}$ is defined by

$$\int \frac{1}{(x-y)_+}\phi(y)\,dy = \int \frac{\phi(y)-\phi(x)}{x-y}\,dy$$

and $C_A$, $C_F$ denote the quadratic Casimir invariant for SU(3) colour in adjoint and fundamental representation resp.; $n_f$ is the number of quark flavours. Of course, (3.8) coincides with the corresponding result for the flavour nonsinglet case [10]. The formulae (3.9) through (3.11) may be compared with the evolution kernels derived in refs.[14,19]. Our result coincides (up to inessential overall factors) with those of Baier and Grozin[14] if in the latter formulae one sets $2x-1=t'$, $2y-1=t$. The formulae of Chase[19] may be recovered from ours by taking $t\to y$, $t'\to x$ and multiplying (3.8) through (3.11) by $(1-t'^2)/(1-t^2)$ (this is due to a different definition of the parton distribution functions in[19]).

Let us now turn to baryon operators. A trilocal baryon operator analogous to (1.1) has been proposed earlier (cf.[2] and references therein, see also[20])

$$B(x_1,x_2,x_3) = \psi_{a_1}(x_1)\psi_{a_2}(x_2)\psi_{a_3}(x_3)\cdot\epsilon_{\beta_1\beta_2\beta_3}\cdot U^q_{\beta_1 a_1}(x_0,x_1)\,U^q_{\beta_2 a_2}(x_0,x_2)\,U^q_{\beta_3 a_3}(x_0,x_3). \tag{3.12}$$

Here $a_i$, $\beta_i$ denote colour indices corresponding to the fundamental representation. The complete antisymmetric tensor $\epsilon_{\beta_1\beta_2\beta_3}$ produces a colour singlet. The operator (3.12) restricted to the light-cone may be used to define the quark distribution amplitude of the baryon similar to the meson case (see ref.[20] for more details). In direct analogy to the meson operator (1.4) the baryon operator on the light-cone is defined as[11]

$$O^{tB}(\kappa,t_1,t_2,t_3) = 3\frac{\partial^2}{\partial i\kappa^2}\int\frac{d\lambda_1 d\lambda_2 d\lambda_3}{(2\pi)^2}\,\delta(\lambda_1+\lambda_2+\lambda_3)\,O^B(\kappa+\Sigma\lambda_i t_i,-\lambda_i), \tag{3.13}$$

where

$$O^B(\kappa+\lambda_i t_i,-\lambda_i) = \xi^{a_1}\xi^{a_2}\xi^{a_3}:\psi_{a_1 a_1}(\tilde z_1)\,\psi_{a_2 a_2}(\tilde z_2)\,\psi_{a_3 a_3}(\tilde z_3)\cdot U^q_{\beta_1 a_1}(\tilde z_0,\tilde z_1)\,U^q_{\beta_2 a_2}(\tilde z_0,\tilde z_2)\,U^q_{\beta_3 a_3}(\tilde z_0,\tilde z_3):\epsilon^{\beta_1\beta_2\beta_3} \tag{3.14}$$

with

$$\tilde z_i = \tilde z_0 - \lambda_i\tilde x, \qquad \Sigma\lambda_i = 0,$$

$$\tilde z_0 = \frac{1}{3}(\tilde z_1+\tilde z_2+\tilde z_3) = (\kappa+\lambda_1 t_1+\lambda_2 t_2+\lambda_3 t_3)\,\tilde x.$$

These expressions have to be taken at $t_1+t_2+t_3=1$. All quark and gluon operators in (3.14) are defined on the same light ray fixed by $\tilde x$. For simplicity we assume that the spinor fields have mutually different flavours and all spins are parallel, i.e., we consider a baryon of spin 3/2 with the same spin projection onto the direction $\tilde x$. $\xi^a$ is a spinor related to $\tilde x$ according to $\tilde x^\mu = \xi^a\sigma^\mu_{ab}\eta^b$. The $\epsilon$-tensor produces the colour singlet structure whereas the spinor $\xi^a$ eliminates in standard manner[22,23] the nonleading twist contributions. Because we restrict ourselves to one-loop calculations and massless spinors it is convenient to use the two-component spinor formalism[22].



Let us now introduce the quark distribution amplitude $\Phi(t_1,t_2,t_3)$ for three quarks with momentum fractions $t_1,t_2$ and $t_3$ inside a baryon with momentum p

$$<0|RTO^B(0,-\lambda_1,-\lambda_2,-\lambda_3,S|p> = \int dt_1 dt_2 dt_3\,\delta(\Sigma t_i-1)\,e^{-i\Sigma t_i\lambda_i\tilde x p}\,\Phi^B(t_1,t_2,t_3,\mu^2). \tag{3.15}$$

Then the matrix element of the baryon operator (3.13) between a one-baryon state carrying momentum p is given by

$$<0|TRO^{tB}(\kappa,t_1,t_2,t_3)S|p>|_{\Sigma t_i=1} = 3\left(\frac{\partial}{\partial i\kappa}\right)^2\int\frac{d\lambda_1 d\lambda_2 d\lambda_3}{(2\pi)^2}\times\cdot\,\delta(\lambda_1+\lambda_2+\lambda_3)\,e^{i(\kappa+\Sigma\lambda_i t_i)p\tilde x}\cdot\int du_1 du_2 du_3\,\delta(\Sigma u_i-1)\,e^{-i\Sigma u_i\lambda_i p\tilde x}\,\Phi^B(u_i) = e^{i\kappa p\tilde x}\,\Phi^B(t_1,t_2,t_3,\mu^2)|_{\Sigma t_i=1}. \tag{3.16}$$

The calculation of the anomalous dimension of the operator (3.13) in a covariant gauge reproduces without further assumption the Brodsky-Lepage evolution kernel[12] (For the explicit calculation see Appendix B)

$$\gamma(\kappa,t_i,\kappa',t_i') = \delta(\kappa-\kappa')\left[\frac{1}{t_1' t_2' t_3'}V_{BL}(t_i,t_i') - 3\gamma_2\,\delta^{(3)}(t_i-t_i')\right] \tag{3.17}$$

with $(C_B = C_F/2)$

$$V_{BL}(t_i,t_i') = -\frac{g^2}{4\pi^2}C_B\sum_{\substack{\text{cyclic}\\ \text{permutations}}}\frac{t_1 t_2 t_3}{(t_i'-t_i)_+}\,\frac{t_i'}{t_i}\,\theta(t_i'-t_i)\left(\delta(t_j'-t_j)+\delta(t_k'-t_k)\right). \tag{3.18}$$

The additional term containing the anomalous dimension $\gamma_2$ of the spinor field results from the external legs. These contributions are usually included in the definition of the anomalous dimensions.

For interesting discussion we thank F.-M. Dittes and A.V. Radyushkin.

## APPENDIX A: Determination of the Anomalous Dimension $\gamma^{Gq}(\kappa,t,\kappa',t')$ in One-Loop Approximation

As an illustration of our techniques we shall describe here the essential steps leading to the one-loop result (3.9b) for the anomalous dimension of the meson operators (3.2). The relevant Feynman diagram is shown in Fig. 1. The Feynman rules

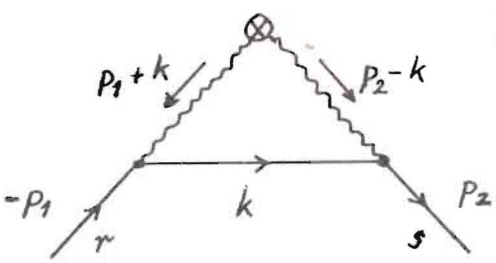



*Fig. 1. The one-loop diagram determining $y^{Gq}(\kappa,t,\kappa',t')$ according to the definitions (3.4) and (3.5).*

for the lowest operator vertices corresponding to (3.7a) and (3.7b) respectively are (all momenta outgoing)


$$\equiv O^{q}_{rs}(\tilde{x},\kappa,t;q_1,q_2) \tag{A.1}$$

$$=\delta_{rs}\tilde{\gamma}\,\epsilon(\tilde{q}_+)\tilde{q}_+^2\,e^{i\kappa\tilde{q}_+}\int\frac{d\lambda}{2\pi}\left[e^{i\lambda(t+1)\tilde{q}_1+i\lambda(t-1)\tilde{q}_2}-e^{i\lambda(t-1)\tilde{q}_1+i\lambda(t+1)\tilde{q}_2}\right]$$

$$=\delta_{rs}\tilde{\gamma}\tilde{q}_+e^{i\kappa\tilde{q}_+}[\delta(t+\tilde{q}_-/\tilde{q}_+)-\delta(t-\tilde{q}_-/\tilde{q}_+)] \tag{A.2}$$


$$\equiv O^{G}_{ab,\mu\nu}(\tilde{x},\kappa,t;q_1,q_2) \tag{A.3}$$

$$=-\delta_{ab}C_{\mu\nu}(q_1,q_2)\epsilon(\tilde{q}_+)\tilde{q}_+e^{i\kappa\tilde{q}_+}\int\frac{d\lambda}{2\pi}\left[e^{i\lambda(t+1)\tilde{q}_1+i\lambda(t-1)\tilde{q}_2}+e^{i\lambda(t-1)\tilde{q}_1+i\lambda(t+1)\tilde{q}_2}\right]$$

$$=-\delta_{ab}C_{\mu\nu}(q_1,q_2)\,e^{i\kappa\tilde{q}_+}[\delta(t+\tilde{q}_-/\tilde{q}_+)+\delta(t-\tilde{q}_-/\tilde{q}_+)], \tag{A.4}$$

where $\tilde{q}_\pm=\tilde{q}_1\pm\tilde{q}_2$ and $C_{\mu\nu}(q_1,q_2,\tilde{x})=\tilde{q}_1\tilde{q}_2g_{\mu\nu}-\tilde{q}_1q_{2\mu}\tilde{x}_\nu-\tilde{q}_2q_{2\nu}\tilde{x}_\mu+(q_1q_2)\tilde{x}_\mu\tilde{x}_\nu$.

Let us remind that throughout this paper we employ the notation $a_\mu\tilde{x}^\mu=\tilde{a}$ for any four-vector $a^\mu$ (including the Dirac matrices). We perform our calculation in Feynman gauge. Proceeding in the manner described in Sect. 3 (cf. items 5 and 6 therein) we obtain for the uv-divergent part of the graph in Fig. 1 the following expression (we use the correspondence $\ln(\Lambda/\mu)\to(4-n)^{-1}$):

$$I^{Gq}_{div}=\delta_{rs}\tilde{\gamma}C_F\frac{g^2}{4\pi^2}\ln\frac{\Lambda}{\mu}\int\frac{d\lambda}{2\pi}\int_0^1dx\int_0^1dy\,y\,\epsilon(\tilde{p}_+)\tilde{p}_+\;\times \tag{A.5}$$

$$\times[(x\tilde{p}_1-(1-xy)\tilde{p}_2)(\bar{E}_1+\bar{E}_2)-iy\lambda(\tilde{p}_1+x\tilde{p}_2)((1-y)\tilde{p}_1+(1-xy)\tilde{p}_2)(\bar{E}_1-\bar{E}_2)],$$



where

$$\bar{E}_1\equiv\bar{E}_1(\lambda,x,y;t,\tilde{p}_1,\tilde{p}_2)=e^{i\lambda(t-1+2y)\tilde{p}_1+i\lambda(t-1+2xy)\tilde{p}_2}$$

$$\bar{E}_2\equiv\bar{E}_2(\lambda,x,y;t,\tilde{p}_1,\tilde{p}_2)=e^{i\lambda(t+1-2y)\tilde{p}_1+i\lambda(t+1-2xy)\tilde{p}_2}\,. \tag{A.6}$$

Note that in the considered case only the first two terms of the Taylor expansion of the operator vertex (A.3) in powers of $k$ give nonvanishing contributions after symmetric integrations, and these lead to the two terms in the integrand of (A.5). Now, observing that $\frac{\partial}{\partial y}(\bar{E}_1+\bar{E}_2)=2i\lambda(\tilde{p}_1+x\tilde{p}_2)(\bar{E}_1-\bar{E}_2)$ , we may perform partial integrations in the second term in (A.5). We thus get

$$I^{Gq}_{div}=\delta_{rs}\tilde{\gamma}\epsilon(\tilde{p}_+)\tilde{p}_+\,e^{i\kappa\tilde{p}_+}C_F\frac{g^2}{32\pi^2}\ln\frac{\Lambda}{\mu}(\tilde{p}_+I_++\tilde{p}_-I_-) \tag{A.7}$$

with

$$I_+=\int\frac{d\lambda}{2\pi}\int_0^1dx\int_0^1dy\,y[(2y-y^2)\delta(x)-(1-x^2)\delta(1-y)](\bar{E}_1+\bar{E}_2)$$

$$I_-=\int\frac{d\lambda}{2\pi}\int_0^1dx\int_0^1dy\,y[(2y-y^2)\delta(x)+(1-x^2)\delta(1-y)+4(y-y^2+xy^2)](\bar{E}_1+\bar{E}_2). \tag{A.8}$$

In the integrals (A.8) we may now perform a transformation of the integration variables from $(\lambda,y)$ to $(\lambda',t')$ so that $\lambda(t-1+2y)=\lambda'(t'+1)$ and $\lambda(t-1+2xy)=\lambda'(t'-1)$ for the terms involving $E_1$ , and $\lambda(t+1-2y)=\lambda'(t'-1)$ and $\lambda(t+1-2xy)=\lambda'(t'+1)$ for the terms involving $E_2$ . Thus we restore the correct parametric dependence of the exponentials corresponding to the operator vertex (A.1). After some manipulations (A.8) may be then recast as

$$I_\pm=\int\frac{d\lambda'}{2\pi}\int dt'[\theta(t-t')Z_\pm(t,t')\mp\theta(t'-t)Z_\pm(-t,-t')](E_1\mp E_2), \tag{A.9}$$

where

$$Z_+(t,t')=\frac{(1-t)(1+t-2t')}{(1-t')^3} \tag{A.10}$$

$$Z_-(t,t')=\frac{(1-t)(1+t-2t')}{(1-t')^3}-\frac{(1-t)^2}{(1-t')^2}+2\frac{1-t}{1-t'}, \tag{A.11}$$

and

$$E_1\equiv E_1(\lambda',t';\tilde{p}_1,\tilde{p}_2)=e^{i\lambda'(t'+1)\tilde{p}_1+i\lambda'(t'-1)\tilde{p}_2}$$

$$E_2 \equiv E_2(\lambda', t'; \tilde{p}_1, \tilde{p}_2) = e^{i\lambda'(t'-1)\tilde{p}_1 + i\lambda'(t'-1)\tilde{p}_2}$$

Now it is necessary to integrate in (A.9) over $\lambda'$ and employ the form (A.2) of the operator vertex. Then we have

$$I^{Gq}_{div} = C_F \frac{g^2}{32\pi^2} \ln\frac{\Lambda}{\mu} \int d\kappa' \delta(\kappa-\kappa') \int dt' \times \tag{A.12}$$

$$\times[\theta(t-t')Z(t,t') - \theta(t'-t)Z(-t,-t')]\, O^q_{rs}(\tilde{x},\kappa',t';\tilde{p}_1,\tilde{p}_2),$$

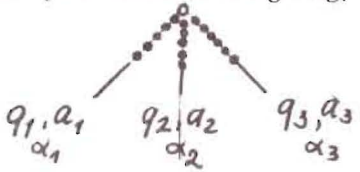
where

$$Z(t,t') = Z_+(t,t') - t' Z_-(t,t') = \frac{1-t}{1-t'}(1+t-2t'). \tag{A.13}$$

This, together with the definitions (3.4) and (3.5) gives the result (3.9b) if we take into account the antisymmetry of the operator $O^q_{rs}(\tilde{x},\kappa',t';\tilde{p}_1,\tilde{p}_2)$ under the exchange $t' \to -t'$.

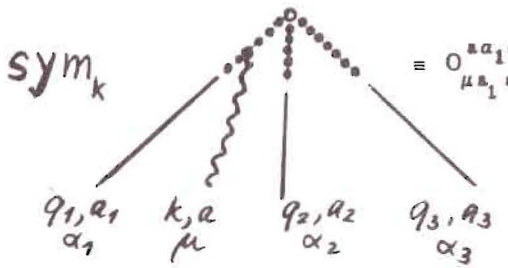
## APPENDIX B: Determination of the Anomalous Dimensions $\gamma(\kappa, t_i, \kappa', t'_i)$ in One-Loop Approximation

Here we calculate explicitly the anomalous dimension of the nonlocal baryon operator (3.13). Up to first order in $g$ the Feynman rules for the operator vertices in momentum space are (all momenta outgoing)




$$\equiv O^{\alpha_1\alpha_2\alpha_3}_{a_1 a_2 a_3}(\tilde{x},\kappa,t_i;q_i) \tag{B.1}$$

$$= 3\tilde{q}^2_+ \int \frac{d\lambda_1 d\lambda_2 d\lambda_3}{(2\pi)^2} \delta(\Sigma\lambda_i)\, e^{i(\kappa+\Sigma\lambda_i t_i)\tilde{q}_+}\, e^{-i\Sigma\lambda_i q_i}\, \epsilon^{\alpha_1\alpha_2\alpha_3} \xi_{a_1}\xi_{a_2}\xi_{a_3}$$




$$\equiv O^{a\alpha_1\alpha_2\alpha_3}_{\mu a_1 a_2 a_3}(\tilde{x},\kappa,t_i;q_i,k)$$



$$= (-ig)\,\tilde{x}_\mu 3\tilde{q}^2_+ \int \frac{d\lambda_1 d\lambda_2 d\lambda_3}{(2\pi)^2} \delta(\Sigma\lambda_i)\, e^{i(\kappa+\Sigma\lambda_i t_i)\tilde{q}_+}\, e^{-i\Sigma\lambda_i q_i} \times$$

$$\times\left\{\frac{e^{-i\lambda_1\tilde{k}}-1}{i\tilde{k}}\, t^a_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2}\delta_{\beta_3\alpha_3} + \text{cyclic perm.(123)}\right\}\epsilon^{\beta_1\beta_2\beta_3}\xi_{a_1}\xi_{a_1}\xi_{a_3}\,. \tag{B.2}$$

where $q_+ = q_1 + q_2 + q_3$ and $\tilde{q} = q \cdot \tilde{x}$; the dotted lines correspond to the factors $U(z_i, z_0)$ and the symmetrization with respect to $k$ according to the different origin of the gluon line has to be taken.

To compute the anomalous dimension of the operator (3.13) in one-loop approximation the following diagrams (Fig.2) must be considered

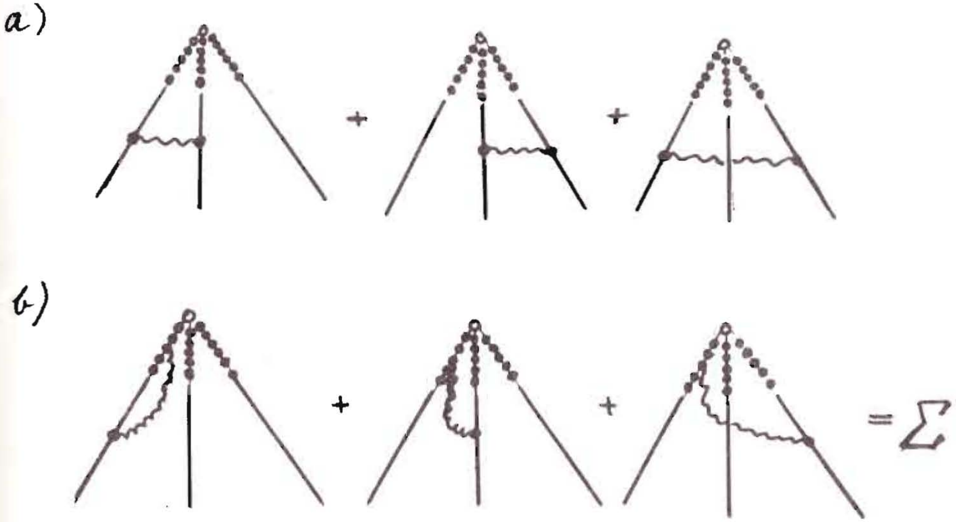



*Fig.2. One-loop diagrams contributing to the anomalous dimension of the baryon operator (3.13). a) Gluon-exchange between different quarks; b) Gluonic contributions from the U-factors approaching one quark (symmetrization included).*

The integrals corresponding to the diagrams of Fig.2a do not contribute because the gluon propagator contains a factor $\delta_{ab}$ and one then encounters the expressions such as (cf.ref. [24])

$$t^a_{\alpha_1\beta_1} t^b_{\alpha_2\beta_2} \delta_{ab}\, \epsilon^{\alpha_1\alpha_2\alpha_3} = 2C_B \delta_{\alpha_1\alpha_2}\delta_{\beta_1\beta_2}\epsilon^{\alpha_1\alpha_2\alpha_3} = 0.$$

The first step in calculating the Feynman integrals corresponding to the diagrams of Fig.2b is similar to that in Appendix A. One thus gets (note that $C_B = C_F/2$ )

$$\Sigma_{div} = \frac{g^2}{4\pi^2}\frac{C_B}{(2\pi)^2}\,3\tilde{q}_+^2 \ln\frac{\Lambda}{\mu}\int d\lambda_1 d\lambda_2 d\lambda_3\,\delta(\Sigma\,\lambda_i)\,e^{i(\kappa+\Sigma\lambda_i t_i)\tilde{q}_+} \times$$

$$\times\int_0^1 dx\frac{1-x}{(x)_+}\{e^{-i(\lambda_2\tilde{q}_2+\lambda_3\tilde{q}_3)}(e^{-i[\lambda_1(1-x)+\lambda_2 x]\tilde{q}_2}+e^{-i[\lambda_1(1-x)+\lambda_3 x]\tilde{q}_3})$$

$$+\ \text{cyclic perm.}\ (123)\}\,\epsilon^{a_1 a_2 a_3}\xi_{a_1}\xi_{a_2}\xi_{a_3}\,. \qquad (B.3)$$

Now the following substitution will be made (separately for every part of the sum in parentheses)

$$\lambda_i' = (1-x)\lambda_i + x\lambda_k, \qquad t_i' = \frac{1}{1-x}t_i$$

$$\lambda_j' = \lambda_j \qquad t_j' = t_j$$

$$\lambda_k' = \lambda_k \qquad t_k' = t_k - \frac{x}{1-x}t_i$$

such that $\Sigma t_\ell = \Sigma t_\ell'$ and

$$\frac{dx}{(x)_+} = \frac{dt_i'}{(t_i'-t_i)_+}\frac{t_i}{t_i'}\,.$$

Then the expression (B.3) takes on the following form

$$\Sigma_{div} = \frac{g^2}{4\pi^2}C_B \ln\frac{\Lambda}{\mu}\int d\kappa'\int dt_1' dt_2' dt_3'\,\delta(\Sigma\,t_i'-1)\,\delta(\kappa-\kappa')$$

$$\times[\sum_{\substack{\text{cyclic}\\ \text{perm.}\\ (123)}}\frac{1}{(t_1'-t_1)_+}\frac{t}{t_1'}\,\theta(t_1'-t_1)(\delta(t_2'-t_2)+\delta(t_3'-t_3))]\,O^B(\kappa',t_i',\tilde{x})\,. \qquad (B.4)$$

Of course, for every part of the sum in (B.4) only one of the integrations has essential meaning. From this expression we can read off the anomalous dimension (3.18).

## APPENDIX C: Definition of the Derivative Operation $\partial(i\kappa)$

For simplicity we shall explain the construction of the operation $\partial(i\kappa)$ introduced in eq.(1.4) for a scalar field where the operators $O^\kappa$ and $O^t$ read



$$O^\kappa(\kappa_1,\kappa_2;\tilde{x}) = \int dq_1 dq_2\, e^{i\kappa_1\tilde{x}q_1+i\kappa_2\tilde{x}q_2} :\phi(q_1)\,\phi(q_2): = :\phi(\kappa_1\tilde{x})\,\phi(\kappa_2\tilde{x}):$$

$$O^t(\kappa,t;\tilde{x}) = \int dq_1 dq_2\,\delta(t-\frac{\tilde{x}q_-}{\tilde{x}q_+})\,e^{i\kappa\tilde{x}q_+} :\phi(q_1)\,\phi(q_2): \ ;$$

$$q_\pm = q_2 \pm q_1\,.$$

Now it is possible to rewrite the second operator as

$$O^t(\kappa,t;\tilde{x}) = \int dq_1 dq_2\,|\tilde{x}q_+|\int\frac{d\lambda}{2\pi}\,e^{i\lambda(t\tilde{x}q_+-\tilde{x}q_-)+i\kappa\tilde{x}q_+} :\phi(q_1)\,\phi(q_2):$$

$$= \partial(i\kappa)\int\frac{d\lambda}{2\pi}\,O^\kappa(\lambda(t-1)+\kappa,\ \lambda(t+1)+\kappa;\tilde{x})\,.$$

According to the last equation, the symbol $\partial(i\kappa)$ has to be understood as a formal operation which in the Fourier representation produces the factor $|\tilde{x}q_+|$ . If we take into account that (see ref.[25] )

$$|\tilde{x}q| = \int d\sigma\,e^{-i\tilde{x}q\sigma}\,(-\frac{1}{\pi}\,\mathrm{Pf}\,\frac{1}{\sigma^2})$$

we may write the operator $O^t$ as

$$O^t(\kappa,t;\tilde{x}) = \int dq_1 dq_2\,d\lambda\,d\sigma(-\frac{1}{\pi}\mathrm{Pf}\frac{1}{\sigma^2})\,e^{i\lambda(t\tilde{x}q_+-\tilde{x}q_-)+i(\kappa-\sigma)\tilde{x}q_+} :\phi(q_1)\phi(q_2):$$

$$= -\int\frac{d\lambda}{2\pi}\int\frac{d\sigma}{\pi}\,\mathrm{Pf}\frac{1}{\sigma^2}\,O^\kappa(\lambda(t-1)+\kappa-\sigma,\ \lambda(t+1)+\kappa-\sigma;\tilde{x})\,.$$

Note that in ref.[10] we have not taken into account the factor $|\tilde{x}q|$ correctly but this did not influence any of the results.

## REFERENCES

1. Craigie N.S., Dorn H. Nucl.Phys., 1981, B185, p.204 and references therein.
2. Dorn H., Robaschik D., Wieczorek E. Ann. der Physik, 1983, 40, p.166.
3. Craigie N.S., Dobrev V.K., Todorov I.T. Ann. of Phys.,1985, 159, p.411.
4. Anikin S.A., Zavialov O.I. Ann.of Phys.,1978,116, p.135.
5. Brodsky S.J. et al. Phys.Lett., 1980, 91B, p.239.
6. Efremov A.V., Radyushkin A.V. Phys.Lett.,1980,94B,p.245; Teor.Mat.Fiz.,1980,42, p.147.
7. Craigie N.S., Stern J. Nucl.Phys., 1983, B216, p.209.
8. Bordag M. et al. Yad.Fiz.,1983,37,p.193.

9. Braunschweig Th., Horejsi J., Robaschik D. Z.Physik.,1984, C23, p.19.
10. Geyer B. et al. Z.Physik, 1985, C26, p.591.
11. Braunschweig Th. et al. In: Proc.of the XXII Conf. on High Energy Physics, Leipzig 1984,p.91.
12. Brodsky S.J., Lepage G.P. Phys.Rev.,1980,D22, p.2157.
13. Lepage G.P. et al. In: Particles and Fields 2 (Proc.Banff Summer Inst. 1981), p.83.
14. Baier V.N., Grozin A.G. Nucl.Phys., 1981, B192, p.477.
15. Chernyak V.L., Zhitnitskii A.R. Phys.Rep.,1984,112,p.174.
16. Geyer B. Czech. J.Phys.,1982, B32, p.645.
17. Dittes F.-M., Radyushkin A.V. Phys.Lett.,1984,134B,p.359; Sarmadi M.H. Phys.Lett.,1984, 143B, p.471.
18. Mikhailov S.V., Radyushkin A.V. Nucl.Phys.,1985,B254,p.89; Katz G.R. Phys.Rev., 1985, D31, p.652.
19. Chase M.K. Nucl.Phys., 1980, B174, p.109.
20. Chernyak V.L., Zhitnitskii A.R. Nucl.Phys.,1984, B246,p.52.
21. Reya E. Phys.Rep., 1981, 69, p.197; Buras A. Rev.Mod.Phys.,1980, 52, 199.
22. Peskin M. Phys.Lett.,1979, 88B, p.128.
23. Ohrndorf Th. Nucl.Phys., 1982, B198, p.26.
24. Dokshitzer Yu.L., Dyakonov D.I., Troyan S.I. Phys.Rep., 1980, 58, p.376.
25. Gelfand I.M., Shilov G.E. Generalized Functions and Operations on Them. Fizmatgiz, Moscow, 1959 (in Russian).

Received by Publishing Department
on September 18, 1985.

COMMUNICATIONS, JINR RAPID COMMUNICATIONS, PREPRINTS, AND PROCEEDINGS OF THE CONFERENCES PUBLISHED BY THE JOINT INSTITUTE FOR NUCLEAR RESEARCH HAVE THE STATUS OF OFFICIAL PUBLICATIONS.

JINR Communication and Preprint references should contain:

- names and initials of authors,
- abbreviated name of the Institute (JINR) and publication index,
- location of publisher (Dubna),
- year of publication
- page number (if necessary).

For example:

1. *Pervushin V.N. et al. JINR, P2-84-649, Dubna, 1984.*

References to concrete articles, included into the Proceedings, should contain

- names and initials of authors,
- title of Proceedings, introduced by word "In;"
- abbreviated name of the Institute (JINR) and publication index,
- location of publisher (Dubna),
- year of publication,
- page number.

For example:

*Kolpakov I.F. In: XI Intern. Symposium on Nuclear Electronics, JINR, D13-84-53, Dubna, 1984, p.26.*

*Savin I.A., Smirnov G.I. In: JINR Rapid Communications, N2-84, Dubna, 1984, p.3.*

Браунщвейг Т. и др. E2-85-678

Адронные операторы на световом конусе

Рассмотрены мезонные и барионные операторы на световом конусе, матричные элементы которых между вакуумом и одномезонным /однобарионным/ состоянием совпадают с амплитудой распределения кварков в соответствующем адроне. Наши прежние результаты обобщаются на случай кварк-глюонного смешивания, которое играет роль при описании мезонов синглетных по аромату. Аномальные размерности рассматриваемых операторов явно вычислены в однопетлевом приближении. Таким образом, восстанавливаются эволюционные ядра для мезонных и барионных волновых функций, полученные раньше другими методами. Эти результаты усиливают наши прежние аргументы в пользу того, что стандартное локальное операторное разложение можно заменить разложением по нелокальным операторам, определенным на световом конусе.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Braunschweig Th. at al. E2-85-678

Hadron Operators on the Light Cone

Meson and baryon operators on the light cone are studied which reproduce the well-known parton distribution amplitudes as matrix elements between the vacuum and a one-meson /baryon/ state. We extend our previous results to the case of the quark-gluon mixing which is relevant for the description of a flavour-singlet meson. The anomalous dimensions of the considered nonlocal light-cone operators are calculated explicitly in the one-loop approximation. We thus recover the evolution kernels for meson and baryon wave functions derived earlier by other authors using different methods. These results reinforce our earlier arguments that the standard local operator product expansion may be superseded by an expansion in terms of nonlocal light-ray operators.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985