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E2-85-617

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ELECTROWEAK ONE-LOOP CORRECTIONS TO THE DECAY OF THE NEUTRAL VECTOR BOSON

Submitted to "Nuclear Physics"

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1985

1. Introduction

In the near future, the new generation of e^+e^- - accelerators SLC and LEP will give the opportunity to test the Glashow-Weinberg-Salam (GWS) - or standard theory with up to now never reached precision. Among the most interesting reactions is the creation of fermion pairs around the Z -boson pole. This process allows the detailed study of the most frequent decay branches of the weak neutral gauge boson Z :

$$Z \to e^+e^-, \ \mu^+\mu^-, \ \tau^+\tau^-, \tag{1}$$

$$Z \to \bar{\nu}_e \nu_e, \ \bar{\nu}_\mu \nu_\mu, \ \bar{\nu}_\tau \nu_\tau, \tag{2}$$

$$Z \to \bar{u}u, \ \bar{c}c, \tag{3}$$

$$Z \to \bar{d}d, \ \bar{s}s, \ \bar{b}b. \tag{4}$$

If the t-quark mass m_t is not too large ($m_t < M_Z/2$), yet another decay channel exists: $Z \to \bar{t}t$. The experimental study of decays (1)-(4) will allow a 10-30 times better determination of the coupling constants of the weak neutral current in the standard theory, so this theory may be verified including one-loop effects. We will not discuss here more fundamental problems as being connected with the number of fermion generations or the existence of non-standard contributions from supersymmetric particles or compositeness and others, on which Z -decays may shed some light too.

Radiative corrections to the partial widths of the leptonic decays (1) and (2) have been studied in the standard theory some time ago by Consoli et al. /1/ . Unfortunately, the calculational scheme they used is somewhat complicated. After the Trieste Conference on electroweak radiative corrections in 1983 /2/ Sirlins's renormalization scheme /3/ became accepted by the physical community as the most satisfying in many respects. It is characterized by renormalization on mass shell (renormalization with the use of α , M_Z , M_W , M_H , m_f as parameters) and a certain choice of parameters in actual calculations: α , G_μ (the Fermi constant in muon decay), M_Z, M_H, m_f. As has been shown in /4-7/, the complete electroweak one-loop corrections to a variety of neutrino-induced processes (νe, νN - scattering) may be taken into account through the introduction of only two form factors $\rho(q^2)$, $\kappa(q^2)$ if the condition $m_f^2/q^2 \ll 1$ is valid (q^2 -squared momentum transfer). Here ρ and κ have a simple physical interpretation: $\rho\, G_\mu \equiv G_F^{eff}(q^2)$ is the effective Fermi constant, and $\kappa \cdot \sin^2\theta_W \equiv \sin^2\theta_W^{eff}(q^2)$ the effective mixing parameter for the given process with neutral current exchange. Here and henceforth we use

$$\sin^2\theta_W = 1 - M_W^2 / M_Z^2 . \tag{5}$$

An immediate consequence of the developed framework is the possibility to think in terms of a "corrected Born-amplitude". Furthermore, it became relatively easy to compare in detail the results of different authors which is no simple task sometimes (e.g., for the above-mentioned νe -scattering of /6/ and /7/ this has been done successfully).

In this article, the approach developed in /4-7/ will be used to discuss the electroweak one-loop corrections of Fig.1 to the

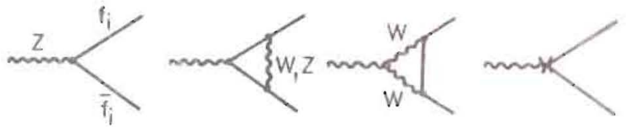

Fig.1. Born and one-loop electroweak contributions in the unitary gauge to the partial width Γ_i^{ew} of the decay $Z \to \bar{f}_i f_i$.

partial decay widths of the Z-boson for the processes (1-4) in the framework of the standard theory *. For each of the decay channels the corresponding electroweak form factors $\rho(-M_Z^2)$ and $\kappa(-M_Z^2)$ are determined. The article is organized as follows:

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* Our approximation $m_f^2/M_Z^2 \ll 1$ in the process $Z \to \bar{f}f$ does not apply to the t-quark with mass $m_t \gtrsim$ 30 GeV /8/. Nevertheless, if $m_t < M_Z/2$, our results for up-quarks combined with the mass-corrected phase space factor may serve as a rather good approximation to a much more complicated exact calculation.

Section 2 contains definitions, the calculational scheme and general formulae used in the following. In Section 3 the t-quark mass dependence is analyzed. Numerical results are presented in Section 4 and discussed as a function of the Higgs boson mass M_H and of m_t. Some explicit expressions are given in the Appendix.

2. Amplitudes, Partial Widths, Renormalization

As has been shown in /9/, the one-loop corrected matrix element for the decay $Z \to \bar{f}_i f_i$ may be written in strong resemblance to the Born approximation by introduction of two constant form factors $\mathcal{F}_{1i}$, $\mathcal{F}_{2i}$ (eqs. (B.9) and (B.10) of ref./9/):

$$\mathcal{M}_i = \frac{g}{4c_W} \varepsilon_\alpha \bar{u} \left[\gamma_\alpha (1+\gamma_5) \mathcal{F}_{1i} - 4 s_W^2 |Q_i| \gamma_\alpha \mathcal{F}_{2i} \right] u, \tag{6}$$

$$s_W^2 = 1 - c_W^2 = \sin^2\theta_W, \quad g^2 \equiv 4\pi\alpha / s_W^2, \tag{7}$$

and Q_i -charge of fermion f_i (Q_e = -1). The Born amplitude corresponds to $\mathcal{F}_{1i} = \mathcal{F}_{2i} = 1$. The partial decay width derived from (6) is

$$\Gamma_i = \frac{g^2}{8M_W^2} \frac{M_Z^3}{12\pi} |\mathcal{F}_{1i}|^2 \left[1 - 4 s_W^2 |Q_i| \kappa_i + 8 (s_W^2 |Q_i| \kappa_i)^2 \right] C_i, \tag{8}$$

where $\kappa_i = \mathrm{Re}(\mathcal{F}_{2i}/\mathcal{F}_{1i})$, and C_i is the color factor: C_i = 1(3) for leptons (quarks).

In the calculational scheme of Sirlin /3/, as independent input parameters of the GWS-theory one uses α, M_Z, M_W, M_H and the fermion masses m_f (on mass shell renormalization). So, the partial widths (8) are well-defined expressions if read in terms of (5) and (7). But, a further ingredient of the approach chosen is the use in actual calculations of the Fermi constant from muon decay,

$$G_\mu = (1.16634 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}, \tag{9}$$

instead of the W-boson mass M_W. One immediate advantage of that choice lies in the high-precision knowledge of G_μ. Furthermore, using

$$\frac{g^2}{8M_W^2} = \frac{\pi\alpha}{2 s_W^2 M_W^2} = \frac{G_\mu}{\sqrt{2}} \left[1 + O(\alpha) \right], \tag{10}$$

the Born amplitudes are scaled by $G_\mu/\sqrt{2}$ instead of the normalization in (8). This, in fact is a finite renormalization of the Born term of the order of one-loop corrections. As has been extensively discussed in the literature cited, this takes away part of the constant electroweak corrections being not directly connected with the process under consideration; among them large contributions of fermions to the vacuum polarization $\sim \Sigma Q_f^2 \ln (m_f^2 / M_W)^2$.

The necessary calculation of M_W has been done iteratively from the following formulae /10/ *:

$$M_W = M_Z \left[\tfrac{1}{2} + \tfrac{1}{2} \left(1 - \frac{4A^2}{M_Z^2} \right)^{1/2} \right]^{1/2}, \tag{11}$$

$$A \equiv S_W \cdot M_W = \frac{A_0}{(1-\Delta r)^{1/2}}, \tag{12}$$

$$A_0 = \left(\frac{\pi \alpha}{\sqrt{2}\, G_\mu} \right)^{1/2} = (37.2810 \pm 0.0003)\ \mathrm{GeV}. \tag{13}$$

Here, Δr is a calculable from muon decay expression:

$$\Delta r = \frac{\alpha}{4\pi} X(M_W, M_Z, M_H, m_f, \alpha) \tag{14}$$

where X may be taken from Eq. (E.8) of /9/ **. Correction terms arizing from a nonzero t-quark mass (vanishing for $m_t = 0$) may be found in /7/; see also this Appendix.

Inserting (12) and (13) into (10), the above-mentioned finite renormalization of the original Born widths of (8), the partial widths Γ_i^{ew} to be used in the following are obtained:

$$\Gamma_i^{ew} = \frac{G_\mu M_Z^3}{\sqrt{2} \cdot 12\pi} \rho_i \left[1 - 4 (S_W^2 |Q_i| \cdot K_i) + 8 (S_W^2 |Q_i| K_i)^2 \right] C_i, \tag{15}$$

where the electroweak corrections of Fig.1 are contained in

$$\rho_i \equiv (1-\Delta r) |\tilde{\mathcal{F}}_{1i}|^2 \simeq 1 - \Delta r + 2 \,\mathrm{Re} (\tilde{\mathcal{F}}_{1i} - 1), \tag{16}$$

$$K_i \equiv \mathrm{Re} (\tilde{\mathcal{F}}_{2i} / \tilde{\mathcal{F}}_{1i}) \simeq 1 + \mathrm{Re} (\tilde{\mathcal{F}}_{2i} - \tilde{\mathcal{F}}_{1i}). \tag{17}$$

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*Concerning the calculation of M_W from α, M_Z, M_H and m_f, we would like to remark that we got an impressive agreement of our results already used in /11/ with those of /12/ where in Table II the M_W has been given with a four-digits precision.

**The hadronic vacuum polarization influences the results of this article only through the calculation of M_W. Differing from /9/, here we take it from the cross-section of e^+e^--annihilation into hadrons /3,13,12/.

The $\tilde{\mathcal{F}}_{1,2i}$ differ from $\mathcal{F}_{1,2i}$ of /9/ by the exclusion of some pure QED - terms which together with Bremsstrahlung lead to the well-known correction factor /14/. The analogue real and virtual gluon Bremsstrahlung adds up too /14/, thus yielding the partial widths containing all one-loop corrections of the standard theory,

$$\Gamma_i = \Gamma_i^{ew} \left(1 + \frac{3}{4} \frac{\alpha}{\pi} Q_i^2 \right) \left(1 + \frac{\alpha_s(M_Z)}{\pi} D_i \right), \tag{18}$$

where $D_i = 0(1)$ for lepton (quark) production. The QED-correction in (18) does not exceed 0.17%, whereas the QCD-correction may be estimated to be about 4% /15/. Of course, both of them one could include into the definition of ρ_i; being interested in the pure electroweak corrections of Fig.1, we will not do so. The explicit expressions for ρ_i and K_i are given in the Appendix.

3. Influence of the t-Quark Mass

In the standard model there are two mass parameters not yet fixed - m_t and M_H -, so they should be varied in estimating one-loop effects. The influence of the Higgs mass may easily be dirived from ref. /9/ so we don't discuss here any details.

The existing experimental findings on the t-quark mass, $m_t = (40 \pm 10)$ GeV /8/, are rather preliminary. Nevertheless, they show the necessity of taking into account m_t in precision calculations. In the following, we will take for granted $m_t \geq 30$ GeV.

The t-quark mass shows up in several ways: (i) The calculation of M_W from the measured parameters α, G_μ, M_Z through (11) and (14) depends on the fermion masses including m_t. (ii) The definitions of X, $\mathcal{F}_{1i}$, $\mathcal{F}_{2i}$ or, after the finite renormalisation, of the form factors ρ_i, K_i explicitly depend on m_t.

From the technical point of view, all the quantities M_W, X, ..., K_i depend on fermion masses through self-energy diagrams (via the counter term), or 1-fold integrals, which are easily calculated for nonvanishing m_t. In case of Z-boson decays into down-type quarks, m_t leads to additional corrections resulting from charged current loops in the diagrams of Fig.1. They necessitate the calculation of 2-fold integrals with three different nonvanishing masses M_Z, M_W, m_t. These vertex corrections are proportional to $|V_{tq}|^2$, where V_{tq} is the Kobayashi-Maskawa matrix element for t-q transitions. The t-quark has small mixing with the light quarks: $0 \leq |V_{td}| \leq 0.024$, $0.036 \leq |V_{ts}| \leq 0.069$, $0.997 \leq |V_{tb}| \leq 0.99$ /16/. Thus, one may expect numerical m_t-dependent vertex corrections only for the decay $Z \to \bar{b} b$. Formulae for flavor - changing Z-boson decays have been given in /17/ and refs. cited therein. Those results, being obtained in the 't Hooft-Feynman gauge, are applicable in the present context, too, since the piece containing the m_t-dependence, say $V(m_t) - V(0)$, is gauge-invariant. Nevertheless, we independently recalculated these terms including the corresponding counter terms /18/ in the unitary gauge used here and got excellent numerical agreement with the analytical result of /17/. The exact expressions of ρ_i and κ_i are given in the Appendix. The leading terms in the limit $m_t^2 \gg M_W^2$ are:

$$\delta\rho_i^t = \delta\rho_i + d_i \cdot \delta\rho_i^V , \qquad \delta\kappa_i^t = \delta\kappa_i + d_i \cdot \delta\kappa_i^V , \tag{19}$$

$$\delta\rho_i \simeq \frac{\alpha}{4\pi s_W^2} \frac{3}{4} \frac{m_t^2}{M_W^2} , \tag{20}$$

$$\delta\kappa_i \simeq \frac{\alpha}{4\pi s_W^2} \left[\frac{c_W^2}{s_W^2} \frac{3}{4} \frac{m_t^2}{M_W^2} + \left(\frac{1}{2 s_W^2} - \frac{1}{3} \right) \ln \frac{m_t^2}{M_W^2} \right] , \tag{21}$$

$$\delta\rho_i^V = -2\,\delta\kappa_i^V \simeq \frac{\alpha}{4\pi s_W^2} |V_{ti}|^2 \left[-\frac{m_t^2}{M_W^2} - \left(\frac{8}{3} + \frac{1}{6 c_W^2} \right) \ln \frac{m_t^2}{M_W^2} \right] , \tag{22}$$

where $d_i = 1$ for down-type quarks and $d_i = 0$ for all other fermions. Formulae (19-22) remain true for a fourth-generation sequential quark doublet with large top-bottom mass splitting.

4. Results

For the Z-boson decays (1) and (3)-(4), the one-loop electroweak form factors ρ_i and κ_i have been exhibited in Tables 1,2 as functions of M_Z, M_H and m_t with four-digits precision. In Table 3, the percentage corrections are presented to the partial widths Γ_{i0}^{ew},

$$\delta_i^{ew} = \frac{\Gamma_i^{ew} - \Gamma_{i0}^{ew}}{\Gamma_{i0}^{ew}} , \tag{23}$$

where $\Gamma_{i0}^{ew} \equiv \Gamma_i^{ew}(\rho_i = \kappa_i = 0)$ is the Born approximation. For decays into electrically neutral neutrinos is $\kappa_\nu \cdot |Q_\nu| = 0$ so that $\rho_\nu = 1 + \delta_\nu^{ew}$ may be taken from Table 3.

The form factor ρ_i contains the difference between the coupling constant acting in the decay $Z \to \bar{f}_i f_i$ and the Fermi constant G_μ, whereas κ_i measures the deviation of $\sin^2\theta_{W,i}^{eff} = \kappa_i \sin^2\theta_W$

from the mixing parameter $\sin^2\theta_W$ which is fixed in our scheme by the gauge boson masses. The existence of and only of ρ and κ has a far-reaching consequence - experiments may be analyzed in terms of usual coupling constants, e.g. vector and axial vector couplings, being corrected for electroweak one-loop effects in a simple manner:

$$a_i = \rho_i^{1/2} \cdot S_i , \qquad v_i = \rho_i^{1/2} \cdot S_i \left(1 - 4 s_W^2 |Q_i| \kappa_i \right) , \tag{24}$$

where S_i = +(-) for up(down)-type fermions. This explicitly shows that data on Z-boson decays may be conveniently described in terms of "Born-like" expressions forgetting the complicated details of one-loop calculations.

We would like to begin the discussion of numerical results with a rough estimate of the anticipated experimental accuracy. Based on the large statistics of Z-boson physics at resonance of about 1 million of Z-decays per experiment, one may expect width measurements with an accuracy reaching or exceeding the 1% level. For $\bar{b}b$-production, e.g., the corresponding tagging efficiency is about 10^4 $\bar{b}b$-pairs /15/. So, it seems to be worth to try a more detailed discussion of the tables. We remark that for the leptonic channels (1,2) our réults coincide with those of /1/ within the precision of their figures. To start with, for small m_t (~ 30 GeV) we get for all channels:

$$|\rho - 1| \leq 0.5\% , \qquad |\kappa - 1| \lesssim 1.1\% , \qquad |\delta^{ew}| \lesssim 0.5\% ,$$

implying that a simple interpretation of data in terms of a Born approximation using G_μ as coupling constant and $\sin^2\theta_W$ from (5) is a rather good approximation. Of course, this may be traced back to the calculational scheme chosen as explained in Sect.2.

The t-quark mass (or, analogously, a fourth-generation sequential fermion doublet with large mass splitting) has only a small influence on ρ. A Δm_t of 200 GeV leads to $0 \lesssim \pm \Delta\rho_i \lesssim 1.5\%$; the minus sign to be taken for b-quark production as a consequence of the dominating in that case vertex corrections in accordance with eq.(22). Stated another way, $1.1\% < |\rho - 1| < 1.6\%$ for $m_t = 230$ GeV in all channels. For the width corrections δ_i^{ew} the change with m_t is more channel-dependent: $|\Delta \delta_d^{ew}| \lesssim 0.8\%$, $|\Delta \delta_\nu^{ew}| \lesssim 1.5\%$, $|\Delta \delta_{e,u}^{ew}| \lesssim 2.5\%$, $|\Delta \delta_b^{ew}| \lesssim 3.4\%$. The most sensitive with respect to m_t quantity is κ. In the large m_t limit considered, κ also becomes large: $6\% \lesssim (\kappa - 1) \lesssim 12.4\%$. As a consequence of m_t, a difference arises between the b-quark and other down-quark channels (see eq. (22)) which already influenced the discussion. For $m_t = 230$ GeV the following values may be quoted:

$$(\rho_b - \rho_d) \sim -2.5\%, \quad (K_b - K_d) \sim 1.2\%, \quad (\delta_b^{ew} - \delta_d^{ew}) \sim -3\%.$$

All tabulated values show some dependence on M_Z and M_H. The detailed M_Z -dependence is not so important at the time Z-decay experiments will be done, since then M_Z will be known with excellent accuracy. The variation with M_H is more interesting since one could hope to derive from radiative corrections an estimate on the Higgs boson mass. This has been proposed and discussed in more detail in /13/ in a related context (combined fit to M_Z^2 and $\sin^2\theta_W$). Unfortunately, the variations being found here are small. For m_t = 30 GeV, a change of M_H from 100 GeV to 1000 GeV results in:

$$|\Delta\rho_i| \approx 0.2\%, \quad |\Delta\delta_i^{ew}| \lesssim 0.15\% \quad |\Delta K_i| \lesssim 0.7\%$$

These numbers show that it would be necessary to reach a sensitivity to K (as the most sensitive parameter) which is clearly below the percentage level. This seems hardly to be reachable.

From our numerical analysis one may conclude that perhaps the most interesting quantity out of ρ, K, δ^{ew} is (for all channels) the co-factor K with its largest percentage corrections and sensitivity to the Higgs and fermion masses. Since it is K which scales the effective weak mixing parameter, one has to determine it with high accuracy if one is interested in a precise determination of $\sin^2\theta_W$ from the partial widths of the Z-boson.

To conclude, we determined the electroweak one-loop corrections to the partial widths $\Gamma(Z \to \bar{f}_i f_i)$ to be of an order of at most some percents in the scheme used and in dependence on M_Z, M_H, m_t. For the anticipated precision experiments, they should be taken into account numerically in the analysis of data.

We would like to thank Proofs. G.B.Abdullaev, F.Kaschluhn, and D.V.Shirkov for interest in our work and support.

Table I. The electroweak form factors ρ and K for the decays $Z \to \bar{\ell}\ell$, $\bar{q}_u q_u$ ($q_u = u, c$) as functions of M_Z, M_H, m_t (all masses in GeV)

		$\rho-1$ (%)			$K-1$ (%)		
M_H		10	100	1000	10	100	1000
m_t	M_Z	$Z \to \bar{\ell}\ell$					
	90	-0.350	-0.I68	-0.354	0.I24	-0.I40	-0.865
	92	-0.366	-0.I68	-0.349	0.I39	-0.I36	-0.904
30	94	-0.38I	-0.I67	-0.342	0.I55	-0.I32	-0.94I
	96	-0.394	-0.I65	-0.335	0.I7I	-0.I28	-0.978
	98	-0.407	-0.I6I	-0.327	0.I88	-0.I23	-0.0I5
	90	I.I83	I.375	I.I79	7.674	7.263	6.230
	92	I.I64	I.374	I.I82	8.48I	8.037	6.9I8
230	94	I.I47	I.374	I.I86	9.326	8.848	7.637
	96	I.I30	I.377	I.I92	I0.2I5	9.699	8.39I
	98	I.II4	I.38I	I.I99	II.I53	I0.597	9.I83
		$Z \to \bar{q}_u q_u$					
	90	-0.288	-I.I06	-0.293	0.08I	-0.I82	-0.907
	92	-0.30I	-0.I03	-0.284	0.096	-0.I79	-0.946
30	94	-0.3I2	-0.099	-0.275	0.III	-0.I76	-0.985
	96	-0.323	-0.094	-0.265	0.I27	-0.I72	-0.023
	98	-0.334	-0.088	-0.255	0.I43	-0.I67	-0.060
	90	I.250	I.442	I.245	7.63I	7.220	6.I87
	92	I.235	I.444	I.25I	8.437	7.993	6.874
230	94	I.220	I.448	I.259	9.28I	6.803	7.592
	96	I.206	I.453	I.267	I0.I70	9.654	8.346
	98	I.I92	I.459	I.276	II.I07	I0.55I	9.I37

Table 2. The electroweak form factors ρ and κ for the decays $Z \to \bar{q}_d q_d, \bar{b}b$ $(q_d = d,s)$ as functions of M_Z, M_H, m_t (all masses in GeV)

	M_H	$\rho - 1$ (%)			$\kappa - 1$ (%)		
m_t	M_Z	10	100	1000	10	100	1000
		$Z \to \bar{q}_d q_d$					
30	90	-0.I99	-0.0I7	-0.205	0.030	-0.233	-0.958
	92	-0.2II	-0.0I3	-0.I95	0.045	-0.230	-0.998
	94	-0.22I	-0.008	-0.I84	0.060	-0.227	-0.036
	96	-0.23I	-0.002	-0.I74	0.076	-0.223	-0.074
	98	-0.24I	0.004	-0.I63	0.09I	-0.2I9	-I.III
230	90	I.34I	I.533	I.335	7.580	7.I68	6.I36
	92	I.327	I.536	I.343	8.385	7.942	6.823
	94	I.3I3	I.54I	I.35I	9.230	8.752	7.54I
	96	I.300	I.546	I.36I	I0.II8	9.603	8.294
	98	I.287	I.553	I.37I	II.055	I0.499	9.086
		$Z \to \bar{b}b$					
30	90	-0.287	-0.I04	-0.29I	0.074	-0.I90	-0.9I5
	92	-0.302	-0.I04	-0.284	0.09I	-0.I85	-0.953
	94	-0.3I6	-0.I02	-0.277	0.I08	-0.I80	-0.990
	96	-0.329	-0.099	-0.269	0.I25	-0.I75	-0.026
	98	-0.342	-0.095	-0.26I	0.I42	-0.I69	-I.062
230	90	-I.273	-1.067	-1.237	8.887	8.468	7.422
	92	-I.296	-1.072	-1.235	9.697	9.245	8.II2
	94	-I.3I9	-I.075	-I.232	I0.546	I0.059	8.832
	96	-I.34I	-I.077	-I.228	II.439	I0.9I4	9.588
	98	-I.363	-I.078	-I.223	I2.380	II.8I5	I0.383

Table 3. Electroweak corrections δ^{ew} (%) as defined in eq.(23) as functions of M_Z, M_H, m_t (all masses in GeV)

$$\delta_i^{ew} = \frac{\Gamma_i^{ew} - \Gamma_{i0}^{ew}}{\Gamma_{i0}^{ew}}$$

		$m_t = 30$ GeV			$m_t = 230$ GeV		
	M_Z	$M_H = 10$	100	1000	$M_H = 10$	100	1000
δ_ν^{ew}	90	-0.08	0.I0	-0.09	I.46	I.65	I.45
	92	-0.I0	0.I0	-0.08	I.44	I.65	I.46
	94	-0.II	0.II	-0.07	I.43	I.65	I.46
	96	-0.I2	0.II	-0.06	I.4I	I.66	I.47
	98	-0.I3	0.I2	-0.05	I.40	I.67	I.48
δ_l^{ew}	90	-0.36	-0.I6	-0.32	-0.I0	0.22	0.28
	92	-0.39	-0.I5	-0.22	-0.90	-0.54	-0.39
	94	-0.42	-0.I4	-0.I3	-I.63	-I.23	-I.0I
	96	-0.44	-0.I3	-0.06	-2.29	-I.85	-I.56
	98	-0.47	-0.I2	0.0I	-2.88	-2.4I	-2.07
δ_u^{ew}	90	-0.32	-0.03	0.07	-I.78	-I.44	- I.23
	92	-0.34	-0.03	0.II	-2.09	-I.73	- I.50
	94	-0.36	-0.03	0.I3	-2.38	-I.99	-I.73
	96	-0.38	-0.02	0.I6	-2.63	-2.22	-I.95
	98	-0.39	-0.02	0.I8	-2.87	-2.43	-2.I4
δ_d^{ew}	90	-0.2I	0.05	0,09	-0.73	-0.44	-0.38
	92	-0.22	0.05	0,09	-0.83	-0.53	-0.45
	94	-0.24	0.05	0.I0	-0.92	-0.60	-0.52
	96	-0.25	0.06	0.II	-I.0I	-0.67	-0.59
	98	-0.26	0.06	0.II	-I.09	-0.73	-0.64
δ_b^{ew}	90	-0.3I	-0.05	-0.0I	-3.63	-3.34	-3.25
	92	-0.33	-0.05	-0.0I	-3.72	-3.40	-3.3I
	94	-0.34	-0.05	-0.0I	-3.80	-3.47	-3.37
	96	-0.36	-0.05	0.00	-3.88	-3.53	-3.42
	98	-0.38	-0.05	0.00	-3.95	-3.58	-3.46

APPENDIX

Using the expressions of ref. [9] for X, F_{1i}, F_{2i} one gets according to eqs. (16) and (17) the following expressions for ρ_i and κ_i in the approximation $m_i^2 \ll M_Z^2$.

$$\rho_i = 1 + \frac{\alpha}{4\pi(1-R)} \left\{ Z(-1) + Z_F(-1) - W(0) + \frac{5}{8} R(1+R) - \frac{11}{2} - \frac{9}{4(1-R)} \ln R + u_i \right\}, \qquad (A.1)$$

$$\kappa_i = 1 - \frac{\alpha}{4\pi(1-R)} \left\{ \frac{R}{1-R} \left[W(-1) - Z(-1) \right] - M(-1) + \frac{1}{2} u_i - \frac{(1-R)^2}{R} Q_i^2 \left[V_1(-M_Z^2, M_Z^2) + \frac{3}{2} \right] \right\}, \qquad (A.2)$$

$$u_i = \frac{1}{2R} \left[1 - 6|Q_i|(1-R) + 12 Q_i^2 (1-R)^2 \right] \cdot \left[V_1(-M_Z^2, M_Z^2) + \frac{3}{2} \right] +$$

$$+ \left[1 - 2R - 2|Q_i|(1-R) \right] \cdot \left[V_1(-M_Z^2, M_W^2) + \frac{3}{2} \right] + 2R \left[V_2(-M_Z^2, M_W^2, M_W^2) + \frac{3}{2} \right]. \qquad (A.3)$$

Further definitions used are taken from refs. [9,7]. Corrections due to nonvanishing m_t derive from (A.1-3) by the replacement of $W(0)$ through $[W(0) + W^t(0)]$ etc., using the following expressions which vanish for $m_t = 0$:

$$W^t\left(\frac{q^2}{M_W^2}\right) = 2c_t \frac{q^2}{M_W^2} \left[I_3(q^2, m_t^2, 0) - I_3(q^2, 0, 0) \right] + c_t \tau I_1(q^2, m_t^2, 0), \qquad (A.4)$$

$$W_F^t(-1) = \frac{c_t}{3} \left[\ln \tau - \frac{1}{2}\tau - \tau^2 + (1-\tau^3) \ln |1 - \tau^{-1}| \right], \qquad (A.5)$$

$$Z^t(-1) = -\frac{c_t}{R} \left[1 - 4(1-R)|Q_t| + 8(1-R)^2 Q_t^2 \right] \left[I_3(-M_Z^2, m_t^2, m_t^2) - I_3(-M_Z^2, 0, 0) \right] +$$

$$+ \frac{c_t}{2} \tau I_0(-M_Z^2, m_t^2, m_t^2), \qquad (A.6)$$

$$Z_F^t(-1) = -\frac{c_t}{2} \tau \left[1 - \tau M_W^2 J(-M_Z^2, m_t^2, m_t^2) \right] + \frac{1}{3R} c_t \left[1 - 4(1-R)|Q_t| + 8(1-R)^2 Q_t^2 \right] \cdot$$

$$\cdot \left[\frac{1}{2} \ln(\tau R) + \tau R + \left(-\frac{1}{4R} + \frac{\tau}{2} - \tau^2 R \right) M_W^2 J(-M_Z^2, m_t^2, m_t^2) \right], \qquad (A.7)$$

$$M^t(-1) = -2c_t \left[|Q_t| - 4(1-R) Q_t^2 \right] \cdot \left[I_3(-M_Z^2, m_t^2, m_t^2) - I_3(-M_Z^2, 0, 0) \right]. \qquad (A.8)$$

Here, $R = M_W^2/M_Z^2$ and $\tau = m_t^2/M_W^2$; c_t is the color factor.

Whereas Z,W,M-functions stem from the counter terms, functions V_1 and V_2 are derived from the vertex diagrams of Fig.1. Thus, when they contribute to Γ_i^{ew} they depend on m_i or on $\overline{m}_i$, the corr. weak-isospin partner masses. Consequently, through V_1, V_2 the m_t may influence only decays into down-type quarks, mainly in the b-quark channel. In that case, one has to include with weight $|V_{ti}|^2$:

$$V_1^t(-M_Z^2, M_W^2; m_t^2) = \frac{1}{R} \int_0^1 dy \left\{ \left[\frac{1}{2} - 3y(1-y) \right] \ln |r_1| + 2R\tau \ln |r_2| - R\tau + \right.$$

$$+ (2+R)\left[\overline{F}_1(\tau) - \overline{F}_1(0) \right] - \left(\frac{3}{2} + R \right) \left[\overline{F}_2(\tau) - \overline{F}_2(0) \right] + \frac{1}{2} R\tau (2+R) \overline{F}_2(\tau) - \qquad (A.9)$$

$$\left. - 2R\tau \frac{1-R}{(1-4R)} \left[1 + \ln |r_2| - 4 \overline{F}_1(\tau) + \frac{1}{2}(1-\tau) \overline{F}_2(\tau) \right] \right\},$$

$$V_2^t(-M_Z^2, M_W^2, M_W^2; m_t^2) = \frac{1}{R} \int_0^1 dy \left\{ -(2+R)\left[F_2(\tau) - F_2(0) \right] + \tau \left[2R \ln |r_3| + \right. \right. \qquad (A.10)$$

$$\left. \left. + \frac{1}{4}(1-2R)(\ln |r_4| - 1) + \frac{1}{2}(\tau - 2R\tau - 4R - 4) F_1(\tau) + \frac{1}{4}(1 - \tau + 2R + 2R\tau) F_2(\tau) \right] \right\}.$$

Here,

$$r_1 = \frac{\tau R}{y(1-y)} - 1, \qquad r_2 = \tau - y(1-y)\frac{1}{R},$$

$$r_3 = y + \tau(1-y), \qquad r_4 = 1 - y(1-y)\frac{1}{R},$$

$$F_i(\tau) = f_i(1, \tau), \qquad \overline{F}_i(\tau) = f_i(\tau, 1), \qquad i = 1, 2,$$

$$f_1(a,b) = \frac{1}{a - b - \frac{1}{R} y} \cdot \ln \frac{a - y(1-y)\frac{1}{R}}{ay + b(1-y)},$$

$$f_2(a,b) = \frac{1-y}{a - b - \frac{1}{R} y} - \left(b + y^2 \frac{1}{R} \right) \cdot f_1(a,b).$$

As explained in refs. [17,18], the vertex counter term contains a flavour-changing piece whose m_t-dependence yields one further correction, with weight $\alpha/(2\pi(1-R))$:

$$\delta\rho^t_{c.t.,i} = -2\,\delta\kappa^t_{c.t.,i} = -\frac{(1+2R)}{6} \frac{\tau}{1-\tau} \left(\frac{5\tau - 11}{2} + 3\frac{\tau - 2}{1-\tau} \tau \ln \tau \right) |V_{ti}|^2. \qquad (A.11)$$

The complete vertex correction in analytical form, not leaving one numerical integration, may be taken from ref. /17/ to replace (A.9) (A.10) and (A.11) :

$$\delta\rho_{i}^{V} = -2\,\delta k_{i}^{V} = \frac{\alpha}{4\pi(1-R)}\, s_i\, |V_{ti}|^2\, \mathrm{Re}\left[V(m_t^2) - V(0)\right] \qquad (A\ 12)$$

where $V(m_t^2)$ is the function as defined in eq. (3.3) of /17/ for $q^2 = -M_Z^2$ and s_i has the same meaning as in eq. (24).

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Received by Publishing Department
on August 15, 1985

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Ахундов А.А., Бардин Д.Ю., Риманн Т. E2-85-617

Электрослабые однопетлевые поправки к распаду нейтрального векторного бозона

В стандартной теории вычислены электрослабые радиационные поправки к парциальным ширинам распада Z бозона на пару лептонов или кварков: $Z \to \bar{\ell}\ell$, $\bar{\nu}\nu$, $\bar{q}q$. Результаты представлены в терминах двух электрослабых формфакторов ρ и k с простым физическим смыслом. Вследствие используемой схемы вычислений поправки не превышают нескольких процентов. Они оказываются достаточно стабильными при варьировании массы бозона Хиггса M_H. В случае тяжелого t-кварка эффективный параметр смешивания $k \cdot \sin^2\theta_W$ может отличаться от $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$ на величину порядка 10%.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Перевод автора

Akhundov A.A., Bardin D.Yu., Riemann T. E2-85-617

Electroweak One-Loop Corrections to the Decay of the Neutral Vector Boson

The electroweak radiative corrections to the partial decay widths of the Z-boson into a pair of leptons or quarks have been calculated in the standard theory: $Z \to \bar{\ell}\ell$, $\bar{\nu}\nu$, $\bar{q}q$. Results are presented in terms of two electroweak form factors ρ and k with simple physical interpretation. As a result of the calculational scheme used, the corrections are at most of the order of some percents. They are rather stable against a variation of the Higgs boson mass M_H. In case of a heavy t-quark, the effective mixing parameter $k \cdot \sin^2\theta_W$ may deviate from $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$ by about 10%.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985