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INFRARED ASYMPTOTICS OF PERTURBATIVE QCD. CONTOUR GAUGES

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1. Introduction

This is the first paper of a series devoted to the investigation of the infrared (IR) asymptotics of perturbative QCD and to the application of the results obtained to the analysis of hard hadron-hadron processes for which the problem of a consistent taking into account the IR singularities was not completely solved yet. We tried, however, to organize the presentation in such a way that each paper is maximally independent of both the preceding and subsequent ones.

In this paper, in sections 2-4 we construct a class of the gauge conditions (contour gauges) which are especially suited for the analysis of the IR properties of the gauge theories (these gauges e.g., are free from ghosts and the Gribov ambiguities). In sect. 5 we consider some particular forms of the contour gauges. In subsequent sections the contour gauges are used to study the renormalization properties of the contour (loop) averages and to derive a renormalization group equation for the infrared asymptotics of perturbative QCD.

2. Quantization

Though our analysis will be restricted to the perturbative aspects of QCD, it is desirable to avoid using formalisms that are definitely nonappropriate (or questionable) within the nonperturbative context. Here we discuss the problem of choosing the gauge conditions which are unambigous both in perturbative and nonperturbative approaches to gauge field quantization.

The quantization of the nonabelian gauge fields was first performed in perturbation theory (PT) by Feynman [1], by Faddeev and Popov [2] and by De Witt [3]. The crucial point here was the choice of the gauge condition $\Phi(\hat{A}^{\omega},\omega)=0$. It should obey the following constraints:

a) first, the equation $\Phi(\hat{A}^{\omega},\omega)=0$ should have a unique solution (the problem of one-to-one correspondence);

b) furthermore, the restriction on $\hat{A}_\mu$ together with the original system of nonabelian fields must describe a generalized constrained hamiltonian system (i.e., the two Dirac's quantization conditions/4/ should be fulfilled).

The essentially perturbative nature of the quantization for the most of gauges is a consequence of the fact that normally (e.g., for the most popular Lorentz gauge) the validity of the above constraints can be checked only within PT. However, in 1978 Gribov observed /5/ that the first constraint is not fulfilled for the Lorentz gauge outside the PT applicability region (where both the coupling constant and the gauge potential $\hat{A}_\mu$ are small).

Later it was established, that in some gauges (e.g., in the axial and planar ones/6/) there are no Gribov ambiguities at all.

In what follows we will show that there exists a whole class of gauges free from the above ambiguities.

To construct them explicitly, we incorporate the well-known geometric interpretation of the gauge field/7/ as the connection of the principal fiber bundle $\mathcal{P}(R^4, G)$ with the four-dimensional space-time R^4 being its base and the fiber being the gauge group G. For the gauge field $\hat{A}_\mu(x)$ and each element $g(x)$ of the fiber $G(x)$ we define the field

$$\hat{A}^g_\mu(x) = g^{-1}(x)\,\left(\hat{A}_\mu(x) + \frac{i}{g}\partial_\mu\right) g(x). \tag{1}$$

Then the set $\{\hat{A}^g_\mu(x)\}$ for all $g(x)$ forms the orbit of the gauge equivalent field configurations. For the quantization one shoud choose in a unique way a single element from each class $\{\hat{A}^g_\mu(x)\}$ for all $x \in R^4$, $g \in G$. Note that such a choice can be made in two ways: first, one can write down an analytic equation on $g = g(x, \hat{A})$ (requiring, of course that it should have a unique solution) and second, one can fix "by hand" some element $g(x, \hat{A})$ from each class (without a hope to get an analytic equation for g). The first way corresponds to the standard approach with the general gauge condition $\Phi(\hat{A}^g, g) = 0$. But as it was emphasized earlier, in the general case it can be realized only within the PT framework. The second way can be, in principle, applied without any reference to PT, but normally is not well suited for any practical purposes. In the next section we demonstrate the approach to the gauge fixing problem (and, hence, quantization) which has the positive properties of both the above ways.

3. <u>Derivation of the Contour Gauges Condition</u>

Let us fix some point $(x_0, g(x_0))$ in the fiber bundle. To uniquely arrive at a neighbouring point, one must define two directions, i.e., the direction in the base and that in the fiber. The direction in the base can be defined by the tangent vector of some curve going through the point x_0. The direction in the fiber can be unambiguously defined as the direction in the tangent subspace corresponding to the parallel transport. As a result, we arrive in a unique way a neighbouring point of the fiber bundle. Moreover, we know the equation of the parallel transport

$$\frac{\partial z^\mu}{\partial s} D_\mu[A]\, g(z(s)) = 0, \tag{2}$$

where $D_\mu[A] = \partial_\mu - ig\hat{A}_\mu$ is the covariant derivative and $z_\mu(s)$ is the parametrization for the curve going through the x_0 point $z^\mu(0) = x^\mu_0$. One can easily find the solution of eq. (2)

$$g(z) = P\exp\left(ig\int_{x_0}^{z} dy^\mu \hat{A}_\mu(y)\right) g(x_0). \tag{3}$$

The fixation of $g(z)$ ensures the unique choice of the element in each orbit of the gauge equivalent fields $\hat{A}^g_\mu(z)$ (eq. (1)). As it follows from eq. (3), the field $\hat{A}^g_\mu$ is determined by the form of the $z_\mu(s)$-contour, and, consequently, the resulting gauge condition on its components also depends on the contour choice. The most important constraints on this choice are:

a) absence of the self-intersection points and

b) existence of some contour connecting x_0 and an arbitrary point x.

The comparatively simple form of eq. (3) enables us to find the explicit form of the corresponding gauge condition. To this end we note the following property of the path-ordered exponentials

$$P\exp\left(ig\int_{x_0}^{x} dz^\mu \hat{A}^g_\mu(z)\right) = g^{-1}(x)\, P\exp\left(ig\int_{x_0}^{x} dz^\mu \hat{A}_\mu(z)\right) g(x_0)$$

which, with account of the explicit form of $g(x)$ (eq. (3)), gives the gauge condition on the components of $\hat{A}_\mu(x)$ in the following general form /8/:

$$P\exp\left(ig\int_{x_0}^{x} dz^\mu \hat{A}_\mu(z)\right) = \mathbb{1} \tag{4}$$

for any x. The gauges of this type will be referred to as the contour gauges.

Another form of the gauge condition, equivalent to eq. (4) can be obtained by substituting eq. (3) into eq. (1):

$$\hat{A}^g_\mu(x) = \int_{x_0}^{x} dz_\nu \frac{\partial z_\rho}{\partial x_\mu} \hat{E}^+(z,x_0)\, \hat{G}_{\nu\rho}(z;\hat{A})\, \hat{E}(z,x_0) \tag{5}$$

$$- \hat{E}^+(z,x_0)\, \hat{A}_\nu(z)\, \hat{E}(z,x_0) \frac{\partial z_\nu}{\partial x_\mu}\Big|_{s=0}^{s=1}$$

$$+ \hat{E}^+(x,x_0)\, \hat{A}_\mu(x)\, \hat{E}(x,x_0),$$

where

$$\hat{E}(z,x_0) = P\exp(ig\int_{x_0}^{z(s)} dy^\mu \hat{A}_\mu(y))$$

and

$$z_\mu(1) = x_\mu .$$

The r.h.s. of eq. (5) depends on the original field $\hat{A}_\mu$, and the starting point x_0 of the contour, in principle, depends on x . As a result of these dependences, even after the gauge condition is imposed, there remains the arbitrariness in the choice of $\hat{A}^g_\mu$ dictated by the residual gauge transformation law

$$\hat{A}^g_\mu(x) \to U^+(x_0)\,(\hat{A}^g_\mu(x) + \frac{i}{g}\,\partial_\mu(x))\; U(x_0). \tag{6}$$

Some important specific examples of eq. (5) will be considered in section 5.

4. Quantization Using the General Contour Gauge Condition

After fixing the gauge condition on $\hat{A}_\mu$ as given by eq. (4), the quantiztion can be performed in the standard way (see, e.g., ref./4/). One should only check whether the three constraints on the gauge choice mentioned in sect. 2 are fulfilled. The first constraint is fulfilled automatically by construction. As for the two other constraints (i.e., that the commutator of the gauge conditions applied to different points can be represented as a linear combination of the gauge conditions and that the commutator of the gauge condition and the "natural" connection of the Lagrangian can be inverted), their fulfillment can be established without any use of PT.

A specific property of the contour gauges is that they all are ghost-free. Indeed, from the gauge transformation law for the path-ordered exponentials given above it follows that on the orbit of the gauge condition eq. (4) the ghost functional does not depend on the gauge field. Hence, the measure of integration over gauge fields

$$\mathcal{D}\hat{A}_\mu \;\; \mathcal{D}g \;\; \delta(P\exp(ig\int_{x_0}^{x} dz^\mu \hat{A}^g_\mu(z)) - 1)$$

does not change under the potential-dependent gauge transformations. Performing the gauge group shift in this expression one can factorize out the (nonphysical) volume of the gauge group. The remaining expression

$$\mathcal{D}\hat{A}_\mu \;\; \delta(P\exp(ig\int_{x_0}^{x} dz^\mu \hat{A}_\mu(z)) - 1)$$

defines the integration measure for the generating functional.

5. Specific Examples of the Contour Gauges

Choosing the contours with the fixed end point x_0 (i.e., not depending on x) one can get some particular examples of the general gauge condition eq. (5). Using the definition of the $\hat{A}^g_\mu$ -field one can obtain from eq. (5) (which gives the relation between the components of the original and transformed fields) the equation for the components of the field $\hat{A}^g_\mu$ itself

$$\hat{A}^g_\mu(x) = \int_{x_0}^{x} dz_\nu \frac{\partial z_\rho}{\partial x_\mu} G_{\nu\rho}(z;\hat{A}^g). \tag{7}$$

Thus, within the class of the contour gauges with the fixed end point of the contour the gluon potential $\hat{A}_\mu$ is a linear functional of the gluon field $\hat{G}_{\mu\nu}$. Now we show how choosing the integration contour in some particular ways one can obtain three well-known gauges.

a) Fock-Schwinger gauge

Let the integration contour be the straight line connecting the points x_0 and x . Then it immediately follows from eq. (7) that

$$\hat{A}_\mu(x) = (x-x_0)_\nu \int_0^1 ds\, s\, \hat{G}_{\nu\mu}(x_0 + s(x-x_0))$$

or

$$(x-x_0)_\mu\, \hat{A}^\mu(x) = 0 .$$

b) Hamilton gauge

Choose the contour be given by the straight line parallel to the time axis and going through the points $x=(x^0,\vec{x})$ and $y=(t^0,\vec{x})$, where t^0 is an arbitrary parameter. The dependence of y on x and the arbitrariness of the t^0 - choice induce, according to eq. (6), the residual gauge transformation with $U(y) \equiv U(t^0,\vec{x})$. From eq. (5) we get

$$\hat{A}^g_\mu(x) = g_{\mu\alpha}\left(\hat{A}_\alpha(t^0,\vec{x}) + \int_0^1 ds\,(x^0 - t^0)\,\hat{G}_{\alpha 0}(z;\hat{A}^g)\right),$$

where

$$z^0(s) = t^0 + s(x^0 - t^0)\,, \quad z_\alpha(s) = x_\alpha,\quad \alpha = 1,2,3.$$

c) <u>Axial gauge</u>

To get the axial gauge, we should take an infinitely long path:

$$z_\mu(s) = x_\mu + (1 - \exp(-\epsilon s))\,\frac{n_\mu}{\epsilon},$$

where ϵ is an infinitesimal parameter and n_μ fixes the direction of the path. Substituting the explicit form of $z_\mu(s)$ into eq. (5) we obtain the relations between the transformed and original fields

$$\hat{A}^g_\mu(x) = n_\nu \int_0^\infty ds\,\hat{E}^+(x+ns,\infty)\,\hat{G}_{\nu\mu}(x+ns;\hat{A})\,\hat{E}(x+ns,\infty)\,e^{-\epsilon s} \tag{8}$$

and between the components of $\hat{A}^g_\mu(x)$:

$$\hat{A}^g_\mu(x) = n_\nu \int_0^\infty ds\,\hat{G}_{\nu\mu}(x+ns;\hat{A}^g)\,e^{-\epsilon s} \tag{9a}$$

$$= \int \frac{d^4k}{(2\pi)^4}\,e^{ikx}\,\frac{i n_\nu}{(kn)+i\epsilon}\,\hat{G}_{\nu\mu}(k;\hat{A}^g), \tag{9b}$$

where

$$\hat{E}(x,\infty) = P\exp\left(ig\int_0^\infty ds\,n_\mu\,\hat{A}_\mu(x+ns)\,e^{-\epsilon s}\right). \tag{10a}$$

Hence, in axial gauges one can also express the potential $\hat{A}_\mu$ in terms of the field $\hat{G}_{\mu\nu}$.

The expressions (8)-(10) play a very important role in our analysis of the IR structure of perturbative QCD (for a brief outline of our approach see ref. /8/). So, let us discuss their structure in more detail. By definition of the P-ordered exponential, we have

$$\hat{E}(x,\infty) = \sum_{k=0}^\infty (ig)^k \int_0^\infty ds_1 \ldots \int_0^\infty ds_k\,\exp\left(-\epsilon\sum_{i=1}^k s_i\right)\,\theta(s_1 < \ldots < s_k)$$
$$(n\hat{A}(x+ns_1))\ldots(n\hat{A}(x+ns_k)) \tag{10b}$$

or, in momentum representation,

$$\hat{E}(k) = \int d^4x\,e^{ikx}\,\hat{E}(x,\infty) = \sum_{\ell=0}^\infty (-g)^\ell \int \prod_{i=1}^\ell \frac{d^4k_i}{(2\pi)^4}\,\delta(k - \sum_j k_j) \tag{10c}$$
$$\frac{(n\hat{A}(k_\ell))\ldots(n\hat{A}(k_1))}{(n(k_1+\ldots+k_\ell)+i\epsilon)\ldots(nk_1+i\epsilon)}.$$

We emphasize the very important property of eq. (10c): the sign of the $i\epsilon$ term dictating the position of the corresponding poles is an unambiguously fixed by the equation for the contour defining the gauge.

Also useful is the momentum representation for eq. (9)

$$\hat{A}^g_\mu(k) = -i\,\lambda^a n_\nu \int \frac{d^4p}{(2\pi)^4}\,\frac{G^b_{\nu\mu}(p)}{(pn)-i\epsilon}\,\tilde{E}_{ab}(k-p), \tag{11}$$

where $\tilde{E}$ is given by eq. (10) with $\hat{A}_\mu \to \tilde{A}_\mu \equiv A^a_\mu \sigma_a$, σ_a being the matrices of the adjoint representation of the $SU(3)$ group.

One of the basic results of the present series of papers (partially formulated in ref. /8/) is that the IR asymptotics of the QCD Green functions are determined by the vacuum averages of the contour integrals, viz., by path-ordered exponential along open paths for the gauge-dependent Green functions and by Wilson loops for gauge-invariant quantities. The specific form of the integration path in both the cases is determined by the momenta of external lines. Our goal in the subsequent sections is to derive a renormalization group equation for the IR behaviour of the on-shell quark form factor in QCD using the multiplicative renormalizability properties of the path-ordered exponentials established in refs. /9-11/.

6. Renormalization of the Contour Averages

First we summarize some results obtained in the above-mentioned papers /9-11/. It was established that the renormalization properties of the contour averages are completely determined by the smoothness properties of the integration contour. In the case of a smooth closed contour the effect of the ultraviolet divergences that appear after one expands the Wilson loop $W[C]$

$$W[C] = \frac{1}{N_c}\,Tr\,\langle 0|\,P\exp\left(ig\oint_C dz^\mu\,\hat{A}_\mu(z)\right)|0\rangle$$

into the PT series reduces to renormalization of the coupling constant, while $W[C]$ remains to be a renorminvariant quantity [9].

The situation changes if the contour has singular points of the following two types:

a) endpoint of an open smooth contour

b) cusps (angles, see fig. 1) and self-intersections.

For the first case it was proved [9,10] that the additional ultraviolet singularities result in a multiplicative renormalization of the contour average $\hat{E}(C,g,\mu) = P\exp(ig\int_C dz^\mu \hat{A}_\mu(z))$

$$\hat{E}(C, g(\mu),\mu) = Z_C\left(\frac{\mu}{\mu'}, g(\mu')\right) \hat{E}(C, g(\mu'),\mu'). \tag{12}$$

For the type b) singularities Polyakov[9] found the additional contribution to $W[C]$

$$W^{(2)}[C] = -\frac{\alpha_s}{\pi} C_F (\gamma\,\mathrm{ctg}\,\gamma - 1)\ \ln\frac{1}{a} \tag{13}$$

(where a is the ultraviolet cut-off parameter) and proposed that in higher orders of PT these singularities are grouped into a multiplicative factor Z_{cusp}. This statement was then proved in ref. [11].

Hence, for an arbitrary contour

$$\hat{E}(C,g(\mu),\mu) = Z_C\left(\frac{\mu}{\mu'}, g(\mu')\right) Z_{cusp}\left(\{\gamma_i\}, \frac{\mu}{\mu'}, g(\mu')\right) \hat{E}(C, g(\mu'),\mu'). \tag{14}$$

In fact, using the contour gauge technique one can derive the multiplicativity of the cusp singularities from the multiplicativity of the endpoint ones, i.e., from eq. (12).

Note first, that $\hat{E}(C,g,\mu)$ is nothing else but the gauge transformation from an arbitrary gauge to the C -contour one. This means that eq. (12) itself states just the fact that the use of the C -contour gauge does not affect the multiplicative renormalizability of the theory. In this sense the contour gauges are on the same footing with the ordinary ones (assumed in refs. [9-11]).

Observe now that an arbitrary contour C having a cusp or self-intersection at the point x can be represented as a sum of two contours C_1 and C_2 with x being their common point (see fig. 1). Imposing the contour gauge

$$P\exp(ig\int_{C_1} dz^\mu \hat{A}_\mu(z)) = 1$$

and incorporating the gauge-invariance of $W[C]$ we get the contour average

$$W[C] = \frac{1}{N_c} Tr \langle 0|\, P\exp(ig\int_{C_2} dz^\mu \hat{A}_\mu(z))\, |0\rangle \Big|_{C_1-\text{contour gauge}} \tag{15}$$

defined on the smooth contour C_2. According to eq. (12), $W[C]$ in this case is multiplicatively renormalizable, with the Z -factor determined by the endpoint singularities of the smooth open contour C_2. Thus, the cusp singularities of the contour C are equivalent to the endpoint singularities of the smooth contour C_2 calculated in the $C_1 = C/C_2$ contour gauge

$$Z_{cusp}(C) = Z_{C_2}\Big|_{C/C_2-\text{contour gauge}}. \tag{16}$$

In view of the arbitrariness in the splitting of C into C_1 and C_2, the gauge invariance of the r.h.s. of eq. (16) should be maintained by the corresponding changes in Z_{C_2}. From eq. (15) it follows also that in the C_1 -contour-gauge the path-ordered exponential for C_2 is renorminvariant if C_1 and C_2 constitute a smooth closed contour. In the next section we shall demonstrate the validity of eq. (16) for the simplest contour shown in fig. 2. The relevant specific form of the contour gauge in this case is the axial gauge. Using now eq. (14) we obtain the renormalization group equation for the contour averages

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_C(g) + \gamma_{cusp}(g,\{\gamma_i\})\right) \hat{E}(C,g,\mu) = 0, \tag{17}$$

where, by definition

$$\gamma_C(g) = -\frac{\partial \ln Z_C}{\partial \ln\mu}\Big|_{\mu=\mu'} \tag{18}$$

is the endpoint anomalous dimension and

$$\gamma_{cusp}(g,\{\gamma_i\}) = -\frac{\partial \ln Z_{cusp}}{\partial \ln\mu}\Big|_{\mu=\mu'} \tag{19}$$

is the gauge-invariant cusp anomalous dimension. In the Euclidean space γ_{cusp}, as follows from eq. (13), is given by

$$\gamma_{cusp}(g,\gamma) = \frac{\alpha_s}{\pi} C_F (\gamma\,\mathrm{ctg}\,\gamma - 1).$$

In the next section we will obtain its analog in the Minkowski space.

7. Analysis of the Infrared Asymptotics

As an example of the application of our approach to the IR properties of QCD we consider the IR asymptotics of the simplest quark-photon vertex function in the on-shell regime for quarks. Its IR-singular part is given by *)

$$\mathcal{M}_{IR} = \frac{1}{N_c} \mathrm{Tr} \langle 0 | \hat{E}^{+}_{-q}(0,\infty)\, \hat{E}_{p}(0,\infty) | 0 \rangle , \tag{20}$$

where p and q are the quark momenta, $p^2 = q^2 = m^2$ and

$$\hat{E}_p(0,\infty) = P\exp\left(ig\int_0^\infty ds\, p_\mu \hat{A}_\mu(ps) e^{-\epsilon s}\right) \tag{21}$$

is the path-ordered exponential generating the gauge transformation into the axial gauge specified by the vector p_μ. The r.h.s. of eq. (20) is gauge-invariant, and one can actually treat it as the path-ordered exponential corresponding to the contour shown in fig.2. Hence, it can be analyzed using the technique developed in the preceding section.

First we identify all possible sources of the ultraviolet singularities.

a) The contour is open, and hence one may expect the end-point singularities. However, due to the cut-off factor $\exp(-\epsilon L)$ (where L is the running length of the contour) these singularities are absent in our case. Hence, the end-point anomalous dimension vanishes:

$$\gamma_c(g) = 0 . \tag{22}$$

b) The contour shown in fig. 2 has no self-intersections, but it has a cusp at the point 0. For the kinematics

$$Q^2 = -(p-q)^2 \neq 0$$

we are interested in the cusp never disappears and, hence, the renormalization factor of eq. (20) is determined just by the cusp singularities.

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*) Eq.(20) may be understood either in terms of the eikonal approximation or on the basis of the fact that the one-particle irreducible quark-photon vertex function has no IR singularities both in p - and q -axial gauges. A more detailed discussion and generalization to more complicated situations will be given in a subsequent paper.

Apart from the UV singularities, eq. (20) contains also the IR ones that are due to the long-range gluon exchanges. Their study is just the problem of our main concern. As is clear from fig.2, the relevant scale is determined by the length L of the contour which is infinite in our case. Thus, to regularize the IR singularities we cut off the length of the contour by $L<\infty$. For the UV singularities we will use two regularization schemes: dimensional regularization and the cut-off used by Polyakov in ref.[9] that amounts to adding the small parameter a^2 to the denominator of the gluon propagator written down in the configuration space representation.

Expanding eq.(20) in the formal (unregularized) PT series gives expressions of the type

$$\int_0^\infty \prod_i ds_i \exp(-\epsilon s_i)\; D\left(\left\{\frac{p_j^\mu}{\sqrt{p_j^2}} s_j\right\}, \left\{\frac{(p_k p_\ell)}{\sqrt{p_k^2 p_\ell^2}}\right\}\right) , \quad p_i = (p,q)$$

which are scale invariant with respect to transformations $\{s_i\} \to \lambda\{s_i\}$. After regularization of all singularities this transformation must be substituted by

$$\{s_i\} \to \lambda\{s_i\}, \quad L \to \lambda L, \quad \mu \to \frac{1}{\lambda}\mu \quad (a \to \lambda a).$$

As a result, the final expressions for the PT expansion of eq. (20) contain only the dimensionless combinations invariant under the above scale transformation, viz. (μL) in the dimensional regularization scheme and (L/a) in the UV cut-off scheme. This means that the IR and UV singularities of eq.(20) have the same structure, and one can apply the renormalization group to study the IR singularities.

To order α_s eq.(20) can be written as

$$\begin{aligned} \mathcal{M}_{IR}\left(\frac{Q^2}{m^2}, \mu L\right) &= 1 - \frac{1}{2} g^2 C_F \oint\oint dx_\mu dy_\nu D_{\mu\nu}(x-y) \\ &= 1 - \frac{\alpha_s}{2\pi} C_F \int_0^\infty ds\, e^{-\epsilon s} \int_0^\infty dt\, e^{-\epsilon t} \left[\frac{p^2}{p^2(s-t)^2} + \frac{q^2}{q^2(s-t)^2} + \frac{2(pq)}{(pt+qs)^2}\right] , \end{aligned} \tag{23}$$

where the last contribution contains the leading contribution of the cusp singularity for $s,t \to 0$. Calculating it in the two above-mentioned regularization schemes gives

$$\int_0^\infty ds\, e^{-\epsilon s} \int_0^\infty dt\, e^{-\epsilon t} \frac{2(pq)}{(pt+qs)^2 + a^2} = \frac{1+r^2}{r} \ln\frac{1+r}{1-r} \ln\frac{L}{a} + (\text{finite terms}) \tag{24a}$$

and

$$\int_0^\infty ds\, e^{-\epsilon s}\int_0^\infty dt\, e^{-\epsilon t}\,\mu^{2\delta}\,\frac{2(pq)}{(pt+qs)^{2-2\delta}} = \frac{1+r^2}{r}\ln\frac{1+r}{1-r}\left(\frac{1}{2\delta}+\ln(L\mu)\right), \qquad (24b)$$

where $r = (1+4m^2/Q^2)^{-\frac{1}{2}}$.

The first two terms in eq.(23) correspond to vacuum averaging for the P-exponential related to the straight-line (infinite) contours calculated in Feynman gauge. The only UV singularity in this case is the end-point singularity *) logarithmically dependent on the regularization parameter. In particular,

$$\int_0^\infty ds\, e^{-\epsilon s}\int_0^\infty dt\, e^{-\epsilon t}\,\frac{n^2}{n^2(s-t)^2+a^2} = \pi\frac{L}{a} - \ln\frac{L}{a}, \qquad (25a)$$

$$\int_0^\infty ds\, e^{-\epsilon s}\int_0^\infty dt\, e^{-\epsilon t}\,\mu^{2\delta}\,\frac{n^2}{\left[n^2(s-t)^2\right]^{1-\delta}} = -\frac{1}{2\delta} - \ln(L\mu). \qquad (25b)$$

The linear divergence in eq.(25a) may be interpreted as a renormalization of the mass of a test particle /9/. Hence, it may be extracted from eq.(20) into the factor $\exp(-c_0\frac{L}{a})$. The cusp singularity was related in ref. /9/ to the gluon bremsstrahlung at the bending point of the contour shown in fig. 2.

Summing the contributions of eqs. (24), (25) gives

$$m_{IR}\left(\frac{Q^2}{m^2},\frac{L}{a}\right) = 1 - \frac{\alpha_s}{\pi}C_F\left[\frac{\pi}{2a}(L+L) + \ln\frac{L}{a}\left(\frac{1+r^2}{2r}\ln\frac{1+r}{1-r}-1\right)\right]. \qquad (26a)$$

$$m_{IR}\left(\frac{Q^2}{m^2},L\mu\right) = 1 - \frac{\alpha_s}{\pi}C_F\left[\frac{1}{2\delta} + \ln(L\mu)\right]\left(\frac{1+r^2}{2r}\ln\frac{1+r}{1-r}-1\right). \qquad (26b)$$

In the first relation we stressed the additive nature of the linear singularities.

Substituting eq.(26) into the definition of the cusp anomalous dimension (eq.(19)) we get

*) In the UV cut-off scheme there appears also the linear singularity $\sim L/a$ /9/ resulting from the contraction of all points of a connected subgraph.

$$\Gamma_{cusp}(g,\gamma) = \frac{\alpha_s}{\pi}C_F\left(\frac{1+r^2}{2r}\ln\frac{1+r}{1-r}-1\right) + O(\alpha_s^2 N_c) \qquad (27)$$

$$= \frac{\alpha_s}{\pi}C_F(\gamma\,\mathrm{cth}\,\gamma - 1) + O(\alpha_s^2 N_c),$$

where the Minkowskian cusp angle γ is defined by

$$\mathrm{ch}^2\gamma = \frac{(pq)^2}{p^2q^2}.$$

Thus, the anomalous dimension Γ_{cusp} depends on the momenta p and q.

Combining eqs. (17), (26), (27) gives the renormalization-group equation for the IR singularities of the on-shell quark form factor

$$\left(L\frac{\partial}{\partial L} + \beta(g)\frac{\partial}{\partial g} + \Gamma_{cusp}(g,\gamma)\right)m_{IR}\left(\frac{Q^2}{m^2},\mu L\right) = 0. \qquad (28)$$

Note that eqs.(27), (28) are in fact universal since they reflect just the properties of the contour averages. In the case where more complicated Green functions are introduced, the only essential change is in the number of the cusp singularities followed by the trivial additive change in the anomalous dimension in eq.(28). It should also be mentioned here that the validity of eq.(28) in the leading logarithm approximation was checked in ref. /12/ up to the three-loop level, with the gluon mass $\lambda\sim 1/L$ serving as the IR regularization parameter.

In the limit $Q^2 \gg m^2$ the expression for $\Gamma_{cusp}(g,\gamma)$ can be simplified:

$$\Gamma_{cusp}(g,\gamma) = \frac{\alpha_s}{\pi}C_F\ln\frac{Q^2}{m^2} + O(\alpha_s^2 N_c) \qquad (29)$$

and the solution of eq.(28) reproduces the well-known double logarithmic $\sim\left(\alpha_s\ln\frac{Q^2}{m^2}\ln\frac{Q^2}{\lambda^2}\right)^N$ asymptotics of the on-shell vertex function. Within the contour space approach this asymptotics is the consequence of the cusp singularity for the contour relevant to eq.(20). However, if one uses in eq.(20) the axial gauge with p (or q) being the gauge fixing vector, then, on the one hand, the final result for m_{IR} is unchanged since it is gauge-invariant, but, on the other hand, the corresponding anomalous dimension will be generated by the end-point singularity of the P-exponential for the straight-line going from 0 to infinity along q (or p, respectively) and calculated in the p-(or q-) axial gauge. As a consequence, the double logarithmic asymptotics in these axial gauges will be generated by the radiation corrections to the quark propagator rather than by those to the (one-particle) irreducible vertex function.

8. Conclusions

Here we formulate the main results of the present paper.

1. A new class of the gauge fixing conditions - contour gauges - was introduced. In the case of the simplest straight line contours the contour gauges reproduce the well-known ghost-free gauges.

2. Using the contour gauge technique enables us to find a relation between the end-point and cusp UV singularities of the contour averages.

3. Analysing the particular contour average absorbing all the IR singularities of the on-shell quark form factor we have demonstrated that its IR singularities are in one-to-one correspondence with its UV singularities, and consequently, it has been established that in this case one can use the renormalization group methods for studying the IR asymptotics of the QCD amplitudes. The relevant anomalous dimension $\gamma_{cusp}(g,\gamma)$ was calculated to order α_s.

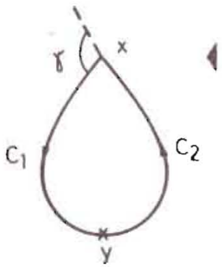

Fig.1.
Example of a contour having a cusp singularity at the point x.

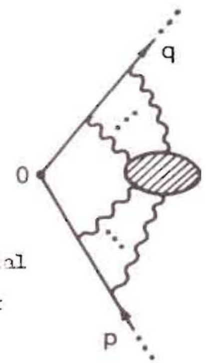

Fig.2.
Integration contour for the path-ordered exponential determining the infrared asymptotics of the vertex function.

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Иванов С.В., Корчемский Г.П., Радюшкин А.В. E2-85-595

Инфракрасная асимптотика пертурбативной КХД. Контурные калибровки

Введен новый класс калибровочных условий - контурные калибровки. Показано, что в случае простейших прямолинейных контуров они воспроизводят общеизвестные бездуховые калибровки. С помощью контурных калибровок найдена связь между концевыми и угловыми ультрафиолетовыми особенностями контурных средних. На основе анализа контурного среднего, поглощающего все инфракрасные особенности кваркового формфактора вблизи массовой поверхности, продемонстрирована взаимно-однозначная связь этих особенностей с ультрафиолетовыми и тем самым установлено, что инфракрасная асимптотика КХД амплитуд может исследоваться с помощью метода ренормгруппы. Найдена соответствующая аномальная размерность в порядке α_s. Метод может быть применен к анализу инфракрасной асимптотики различных процессов в квантовой хромодинамике.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Ivanov S.V., Korchemsky G.P., Radyushkin A.V. E2-85-595

Infrared Asymptotics of Perturbative QCD. Contour Gauges

A new class of the gauge fixing conditions - contour gauges - is introduced. It is shown that in the case of the simplest straight line contours the contour gauges reproduce the well-known ghost-free gauges. Using the contour gauge technique enables us to find a relation between the end-point and cusp singularities of the contour averages. On the basis of an analysis of the particular contour average absorbing all the infrared singularities of the on-shell quark form factor it is demonstrated that its IR singularities are in a one-to-one correspondence with its UV-singularities, and consequently, it has been established that one can use the renormalization group methods for studying the IR asymptotics of the QCD amplitudes. The relevant anomalous dimension was calculated to order α_s. The method can be applied to study the IR asymptotics of various QCD processes.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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