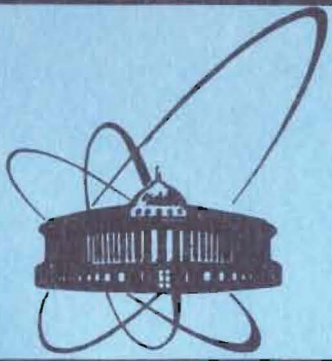

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ENTROPY OF THE SYSTEM FORMED IN HEAVY ION COLLISION

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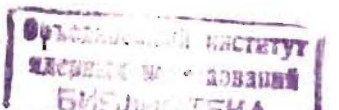
1. Introduction

One of the goals of studying relativistic heavy ion reactions is to investigate nuclear matter under extreme conditions as high temperature and density. However, the information about the properties of the hot and dense matter is obscured by the fact that the intermediate nuclear system remains hot and dense only for a rather short time ($\sim 10^{-22}$ sec.). The interesting quantity which may provide a window into the early stage of the collision process is the produced entropy, because the entropy will grow rapidly when the nucleons make their first collisions and will remain nearly constant when the expansion phase of the nuclear fireball begins. Siemens and Kapusta[1] have suggested that the entropy per baryon (S/A) attained during the collision process may be related to the ratio of the deuteron to the proton yield (R_{dp}) by the following formula:

$$S/A = 5/2 - \ln (R_{dp} / 3\sqrt{2}). \qquad (1)$$

This simple relation is obtained for a dilute gas considered as an ideal mixture consisting of different species being at disassembly in chemical equilibrium. Modifications of (1) arising due to the account of the in-medium corrections have been discussed in refs.[2,3] Provided that thermodynamical equilibrium is established, the application of formula (1) might give us information on the entropy value attained during the collision process if effects connected with the finite size of the system can be disregarded and if the dilute gas approximation is applicable. Concerning the finite size effects, of great interest are the first experimental results of Gutbrod et al.[4] and Doss et al.[5] who measured the cluster abundances in high energy heavy ion collisions by means of a 4π detector (plastic ball). Since they found a saturation of the R_{dp} ratio only for large multiplicities, one is led to the assumption that the cluster production rate is strongly correlated with the volume of the emitting source or, in other words, the cluster abundances become a function of the impact parameter.



In view of this new experimental situation the applicability of the simple formula (1) for extracting the entropy has to be reconsidered. In doing this, one has to start with a theoretical treatment which considers the relevant dynamical aspects of the formation and the disintegration process of the intermediate nuclear system. One of the promising ways to study such dynamical problems is to apply the cascade model[6].

The cascade model permits us to calculate S/A directly without making use of relation (1). Of course, having the specific entropy at our disposal its connection to the cluster production is one of the central questions which will be discussed in a subsequent paper.

The entropy value for the reaction $^{40}Ca + ^{40}Ca$ at 800 MeV/nucl. and zero impact parameter has firstly been calculated by Bertsch and Cugnon [7] using a cascade model. To our knowledge, such types of cascade calculations have not yet been repeated or extended to other projectile and target combinations and different beam energies as well. Therefore, one of the aims of the present paper is to study to what extent the calculated entropy values may be model dependent.

Invoking the cascade model, the time evolution of the entropy production during the collision process can also be investigated. Considering a head-on collision one could expect that at the first stage of the collision the entropy will grow rapidly but as soon as the thermal equilibrium is established the entropy will approach a saturation value. For noncentral collisions the generated hot nuclear matter may be far from thermodynamical equilibrium and, therefore, cascade model simulations can give us reliable information on how such an intermediate system disassembles and how the entropy value per baryon evolves.

In calculating the entropy value we will use two different methods in order to be able to check their relative accuracy. One method to be used is closely connected to that previously employed by Bertsch and Cugnon[7], who based their entropy calculation on a proper subdivision of the six-dimensional phase space.

To get confidence that the entropy value is accurately calculated within the used cascade model approach, we applied a second method which is based on the assumption that a local equilibrium is established for each r-space subvolume of the intermediate system. In doing so the averaging over momentum space is performed. We will show that the second method has to be preferred compared to the first one. This is because the resulting entropy can also be determined adequately in the case of low phase-space densities where statistical fluctuations play a decisive role. This concerns mainly

the latest stage of the collision process when the reaction products go apart and the specific entropy does not change at all. For the practical calculations, the introduction of the cylindrical coordinates yielding an adequate description of the reaction geometry has been proven to be highly advantageous.

The cascade model approach to be used will not be explained in the present paper (for details see ref.[6]). Both the methods to be employed for calculating the entropy are presented in the next section. The time evolution of the specific entropy is studied in section 3. For comparison we consider first an exact soluable model system and then investigate the entropy evolution of the Ca + Ca and Nb + Nb systems for head-on and noncentral collisions as well. At the end of section 3 a comparison is made with the results for specific entropy values inferred from the composite particle yields measured with a 4π detector. Concluding remarks are given in section 4.

2. Calculation of the entropy

For the entire hot nuclear system formed in the course of a heavy ion collision the calculation of the probability to be in a certain state seems to be hopeless from the point of view of an exact quantum mechanical description, and therefore a rigorous calculation of the time evolution of the density operator and of the entropy seems to be impossible. However, since one deals with a rather hot nuclear ensemble the entropy value might be evaluated by means of the independent particle approximation if one introduces the probability $f \equiv f(\vec{r},\vec{p},t)$ that a certain single-particle state or a certain phase space volume is occupied. In the Fermi gas approximation the entropy can be written as

$$S = -\int d\gamma \left[f \ln f + (1-f)\ln(1-f) \right] , \qquad (2)$$

where $d\gamma = 4\, d^3r\, d^3p/(2\pi\hbar)^3$ represents the phase space volume element (the Boltzmann constant has been dropped). The factor 4 is due to the spin-isospin degrees of freedom. The distribution function is normalized to the particle number A of the system

$$\int f(\vec{r},\vec{p},t)\, d\gamma = A(t) . \qquad (3)$$

In the classical limit one has $f \ll 1$, and the entropy takes the form

$$S/A \simeq 1 - \langle \ln f \rangle \qquad (4)$$

with mean value

$$\langle \ln f \rangle = \int f \ln f \, d\Gamma \Big/ \int f \, d\Gamma . \qquad (5)$$

Providing that thermal equilibrium is reached so that $f = \rho \Lambda^3/4 \exp(-p^2/2MT)$ one has

$$\langle \ln f \rangle = \ln \langle f \rangle - \frac{3}{2}(1-\ln 2) \qquad (6)$$

and $\langle f \rangle = \rho \Lambda^3/4 \cdot 2^{3/2}$ where ρ is the particle number density of the system and $\Lambda = (2\pi\hbar^2/MT)^{1/2}$ denotes the thermal wave length of a nucleon. Combining (6) and (4) one finds the usual entropy expression for the noninteracting Fermi gas in the classical limit

$$S/A = 5/2 - \ln(\rho\Lambda^3/4) = 5/2 - \ln[2^{3/2}\langle f \rangle] . \qquad (7)$$

Notice that the phase space occupation function f introduced above does not contain two-particle correlations and therefore the in-medium corrections and cluster formation effects are not contained a priori from the point of view of a fundamental description. However, in the case of local thermodynamical equilibrium these effects can be accounted for by solving a Bethe-Goldstone-type equation (cf. refs. /3,8/). In the low density limit the phase occupation function f can be related to the ratio of deuterons to protons (R_{dp}) and formula (1) follows from (7).

According to these simplified derivations for the calculation of the entropy within the cascade model two strategies may be applied. In the first one which is similar to that described in ref. /7/ the entire phase space is divided in cells with $\Delta\Gamma_i = 4\,\Delta^3 r_i \Delta^3 p_i/(2\pi\hbar)^3$ and the distribution function is then

$$f_i = \frac{\bar{N}_i}{\Delta\Gamma_i} = \frac{\sum_R N_i(R)}{R\,\Delta\Gamma_i} , \qquad (8)$$

where $N_i(R)$ is the number of nucleons in the i^{th} cell and R gives the number of cascade runs. Introducing f_i in this way one had in principle to consider the limiting case of an infinite number of runs. Since this is not possible for practical reasons, one has to be aware of the statistical fluctuations. Therefore, the method (8) for calculating the distribution function by its average value with respect to one cell i at fixed time t has its own specific features. This is because the division of the phase space in cells has

to be done with some care in order to obtain a rather smooth distribution within the entire phase space. It is clear from (7) that a too small cell division (where fluctuations in the estimation (8) of the distribution function are not negligible) gives a too small entropy, whereas a large size of cells would lead to an overestimation of the entropy value. In dividing the phase space into cells the adequate geometrical description of the cascade processes has to be considered in order to preserve the essential fact that the entropy should not grow if the free motion of all cascade particles sets in. According to the Liouville theorem the entropy remains then constant because the volume of a phase space element does not change. However, the phase space element itself is extremely deformed due to the free motion, and this requires a finer and finer subdivision of the phase space in the cells i .

With (8), the formula for calculating the entropy is

$$S = -\sum_i \left[f_i \ln f_i + (1-f_i)\ln(1-f_i) \right] \Delta\Gamma_i . \qquad (9)$$

The respective results for the entropy per baryon obtained by the latter formula will be discussed after the second method employed has been introduced.

Using (9) the entropy can only be evaluated if the statistical errors in the particle numbers of each cell have been kept small and if at the same time the translational motion of a certain cell in the six-dimensional phase space due to the free motion is considered properly. In practice it might be difficult to fulfill simultaneously both requirements, and another way for calculating the entropy value might be more promising. Let us assume that in the comoving reference frame a local equilibrium is established for each subvolume ΔV_i of the entire system. In this case we characterize the momentum distribution in each subvolume ΔV_i by the mean velocity $\vec{u}_i(t) = \langle \vec{p}_i/M \rangle = \sum_{R,j} \vec{p}_{j,i}(R)/R\bar{N}_i M$ *) and the temperature T_i determined by $\sum_{R,j} [\vec{p}_{j,i}(R) - M\vec{u}_i(t)]^2/2M\bar{N}_i R = 3T_i/2$. The distribution function is

$$f_i(\vec{r},\vec{p},t) = \bar{f}_i \exp\left[-(\vec{p} - M\vec{u}_i(t))^2/2MT_i(t) \right] \qquad (10)$$

with

$$\bar{f}_i = \frac{\bar{N}_i \Lambda_i^3}{4\,\Delta V_i} = \frac{\rho_i \Lambda_i^3}{4} , \qquad (11)$$

*) Actually, in the calculation we have employed everywhere relativistic formulae and used also the relativistic Boltzmann distribution function (see ref. /9/ for details).

where $\bar{N}_i$ is the mean number of particles in the considered space subvolume, see eq.(8), and the factor 4 is due to spin-isospin degeneracy. Having the distribution function (10) at our disposal the momentum integration can be performed in calculating the entropy value. For the limiting case $\rho_i \Lambda_i^3 \ll 1$ the expression for the entropy per particle takes the form

$$S/A = 5/2 - \sum_i \bar{f}_i \ln(\rho_i \Lambda_i^3/4) / \sum_i \bar{f}_i , \qquad (12)$$

which for a uniform distribution goes over into (7).

In the actual calculation we used the general expression following from (2), when the integration over momentum space of the phase space elements is completed. One should expect that after having smoothed over the momentum distribution, the final results for S/A will not be very sensitive to the choice of the subvolume size. In this way (12) can be treated as an additional test for the applicability of (9) to calculate the entropy value attained in the course of a heavy ion reaction. The numerical analysis is represented in the next section. Since local equilibrium was assumed in the latter method one obtains an upper bound for the entropy value. This is because the smoothed distribution function (10) is inferred via the principle of extremal entropy at given mean values of $\bar{f}_i$, $\vec{u}_i$ and T_i.

3. Calculational results

It was already pointed out in ref.[7] that in calculating the entropy the cell configuration has to be determined in an approximate way, whereby it is not practical to divide the six-dimensional phase space into cells without considering the sensitivity of the distribution function to some of the six variables. Several aspects of the cell subdivision have been discussed in ref.[7]. Here, we will not repeat or even copy to some extent this discussion but we will immediately begin with the representation of our results.

First, we consider a model system of 80 nucleons the Boltzmann equation of which is exactly soluable. We are mainly concerned with the problem of the phase space division and investigate in subsection 3.1. the applicability of the two methods to evaluate the entropy during the stage where the free motion of the reaction products sets in.

In the next subsection we consider the collision process of a real system and begin the discussion of the entropy evolution for head-on collisions. Of special interest are the absolute values of the entropy attained in the latest stage of the collision process.

Our approach permits us simultaneously to study the time dependence of both the distribution function f and its various moments such as the particle number density, the local mean velocity, the local temperature, etc. Finally, we investigate the entropy for noncentral collisions. In subsection 3.3. we compare our results with those where the entropy is determined via the cluster abundances.

3.1. Model system calculations

Since the phase space subdivision has to be done properly we investigate initially a model system of a freely expanding nucleon gas characterized by an initial temperature $T_0 = 40$ MeV and an initial Gaussian distribution with mean density $\langle\rho\rangle = \rho_0 = 0.17$ fm^{-3}. Such a model system may describe the disassembly stage of a head-on collision after the violent reaction process has taken place. A total number of 80 nucleons has been chosen so that the results can be related to some extent to those found for the ^{40}Ca + ^{40}Ca central collision at 800 MeV (see also ref.[7]). For this model system the Boltzmann equation can be solved exactly (see Appendix A). The resulting value of the entropy per particle S/A (see(A7)) is constant in time and given in Fig. 1. First, we have calculated the specific entropy as a function of time employing the method of ref.[7]. As is clearly seen in Fig. 1, the corresponding values of S/A are somewhat larger than the exact ones and increase with time. (For the subdivision of

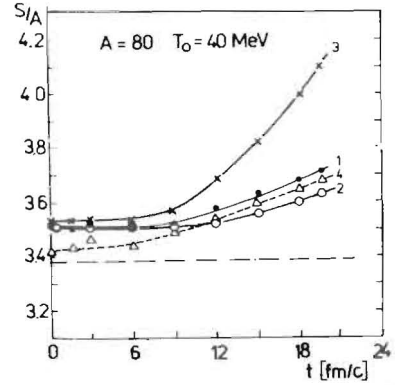


Fig. 1.

The evolution of the specific entropy for the model system calculated by using method which is based on a subdivision of the 6-dimensional phase (cf. ref.[7]). The exact result is shown by a dot-dashed line. The curves are calculated by using different subdivisions (spherical reference frame): $\Delta r \times N_{\theta_{pr}} \times N_p \times N_{\theta_p}$ = 2 x 4 x x 5 x 2 (1), 2 x 4 x 5 x 4 (2), 2 x 4 x 5 x 2 (3), 1 x 4 x 5 x 2 (4). Here Δr is the step along the radius (in fm), $N_{\theta_{pr}}$, N_p, N_{θ_p} are in turn the number of steps in coordinate angle θ_{pr} (measured relatively to $\vec{p}$ direction), momentum p and its polar angle θ_p. The subdivision in θ_{pr} is the same as in ref.[6] besides for curve (3) where it is uniform in cos θ_{pr}. The subdivision in cos θ_p is uniform everywhere.

the phase space see figure caption). As discussed above, the increase in the entropy with time is solely due to the fact that an initially fixed subdivision begins to fail when the system has considerably expanded. The average phase space density becomes smaller and consequently the entropy increases. This effect can be obscured by choosing relatively large cells because then one can get indeed a saturation behaviour for a longer time interval but simultaneously the absolute value for the "quasiconstant" specific entropy is larger than the exact one. This is illustrated in part by curve 3 in Fig. 1.

From the results represented in Fig. 1 it is seen that the precise value of the entropy can only be reached with a relatively small cell mesh size with respect to the radial extension. For $\Delta r = 1$ fm about 500 runs turned out to be quite sufficient in getting reliable results. For even smaller intervals the number of cascade runs has to be increased.

The results represented in Fig. 2 are obtained by employing the second method of section 2 which is based on a smoothing over the momentum space and where a local temperature for each cell in the co-ordinate space has been introduced. In doing so, the collective motion of the single phase space cell which does not contribute to the temperature or the entropy can very easily be extracted. Using a reasonable cell subdivision, the exact results for the specific entropy, the mean temperature and the mean density are reproduced for a large time interval.

If statistical fluctuations become relatively large, the calculated entropy deviates from the exact one. The curves labeled by 3 and 4 in Fig. 2 demonstrate the importance of having enough runs. Curve 4 is calculated by using $\mathcal{R}=10$ and demonstrates that due to

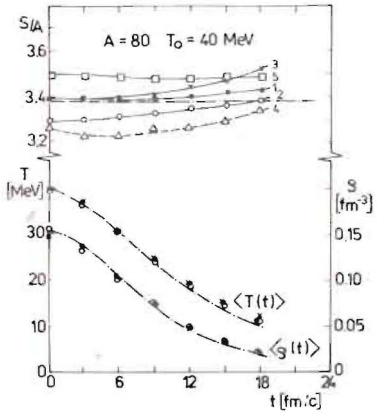


Fig. 2.

The evolution of the specific entropy, average temperature and density of the model system calculated by means of the momentum averaging method. The exact result is given by a dashed line. The subdivisions for the various curves are:

$\Delta r \times N_{\theta_r}$ = 2 x 8 (1), 1 x 8 (2), 2 x 4 (3), 2 x 4 (4), 3 x 8 (5).

The subdivision in the polar angle θ_r is chosen as uniform everywhere. The number of runs is R = 500 except for case (4) where R =10.

the improper account of statistical fluctuations the calculated value of the entropy may be smaller than the exact one.

Curve 5 illustrates that a saturation-like behaviour can be achieved by using rather large cell sizes. The lower part of Fig. 2 gives the temperature and particle number density as a function of the time. All quantities considered are in excellent agreement with the exact ones for times smaller than 15 fm/c.

The conclusion from these model calculations is that the second method based on a smooting over the momentum space has to be preferred with respect to the first one. At the beginning of the collision process where the size of the entire system is still small there seems to be no difference in applying both methods. However, when the collision process approaches the disassembly stage and the free expansion of the intermediate system begins, the second method seems to be more appropriate to calculate the entropy value because then the population of the entire phase space can better be controlled and, especially, the relevant thermal motion within a cell can be separated from the cell's collective motion.

3.2. Entropy evolutions for real systems

After having discussed these model system calculations let us turn to a real case and consider the reaction Ar + Ar at 0.8 GeV at zero impact parameter. The results obtained for the specific entropy by using the two methods described in section 2 are shown in Fig. 3 (see also the figure caption). The phase space subdivision as in ref. /6/ yields no pronounced saturation pattern for S/A when the collisions have ceased and the minimum value is about 0.6 units larger than obtained by the momentum smoothing procedure. In this respect it should be noted that for a non-Gaussian distribution the momentum smoothing procedure gives an upper limit for the entropy. As can be seen in the lower part of Fig. 3 the momentum averaging procedure gives a saturation behaviour in a wider range of time.

In contrast to ref. /7/, in the Dubna version of the cascade model /6/ nucleons are considered as bounded initially in a finite - range average potential of the Saxon-Woods type. Therefore, the results of both approaches cannot immediately be compared. To make a direct comparison to the results of ref. /7/, we have replaced the initial nucleon density distribution by a sharp-edged one, neglected the binding as well and repeated the cascade calculations for Ca + Ca at 0.8 GeV/nucl. In this case the entropy value obtained by our momentum smoothing procedure turns out to be S/A=3.2, i.e. it is about the unit smaller than that given by Bertsch and Cugnon /7/ who considered additionally the interaction among the participants.

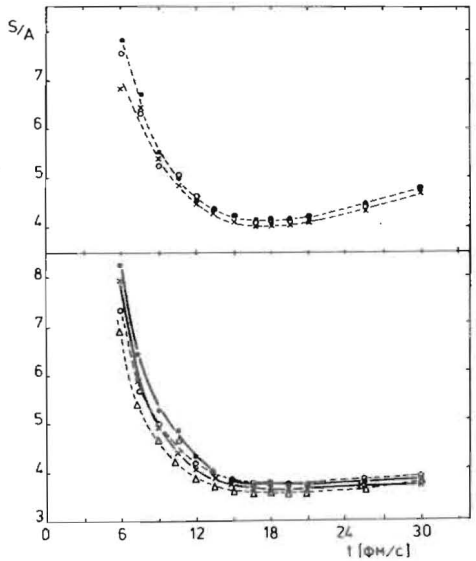


Fig. 3.

The evolution of the specific entropy for central Ar + Ar (0.8 GeV/A) collision. Upper part: Bertsch-Cugnon method as in ref./7/ (see also Fig. 1) with $\Delta r \times N_{\theta_{pr}} \times N_p \times N_{\theta_p}$ = 2 x 4 x 5 x 2(•), 2 x 4 x 5 x 4 (x), 2.5 x 4 x 5 x 5 (o). Lower part: Method of the present work (see also Fig. 2) with $\Delta r \times N_{\theta_r}$ = 2.5 x 8 (o), 1.0 x 8 (▲) for spherical symmetry and $\Delta r \times \Delta z$ = 2 x 2 (•), 1.5 x 1.5 (x) for cylindrical symmetry.

We have estimated the participant -participant interaction and found that it does not contribute very much in this energy range. The difference of about one unit can mainly be attributed to the more correct treatment of the collective motion of each volume element.

Taking into consideration the diffusivity of the system one would obtain an additional increase of the specific entropy about 0.6 units.

Having a reliable method for calculating the specific entropy at our disposal it is interesting to study the time evolution of quantities characterizing the system. We show this in Fig. 4, where in the upper part the relative position of the projectile and target has been sketched for four different time situations. From Fig. 4 it can be seen that the specific entropy reaches the saturation value when the two nuclei maximally overlap. (Note that we represent the specific entropy S(t)/A(t) where A(t) is the number of struck particles. Since the number of struck particles is rather small at the beginning of the collision, the specific entropy decreases with time). In contrast, other mean values as the mean number of struck nucleons saturate at the disassembly stage. In this way, the entropy measured indirectly by relating the cross sections of composite particles (see refs./4,5,11,12/) gives us at least valuable information about the system when it has bypassed this stage of the evolution. The stability of the numerical calculations is demonstrated by the

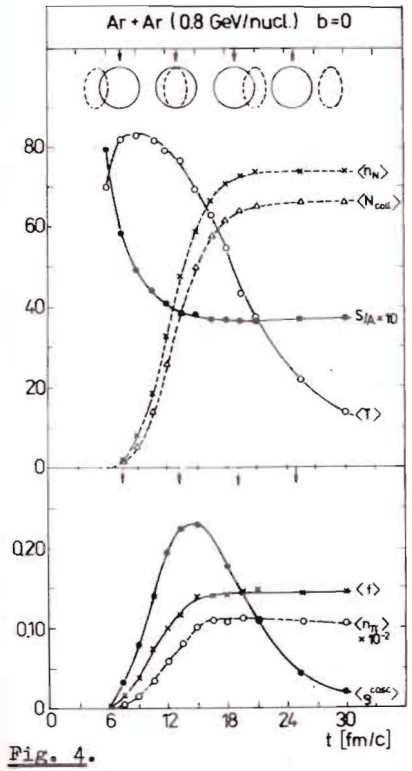


Fig. 4.

The evolution of different average characteristics for central Ar+Ar (0.8 GeV/A) collisions. On the upper part of the figure it is shown how the colliding nuclei are situated at the time moment marked by the arrows.

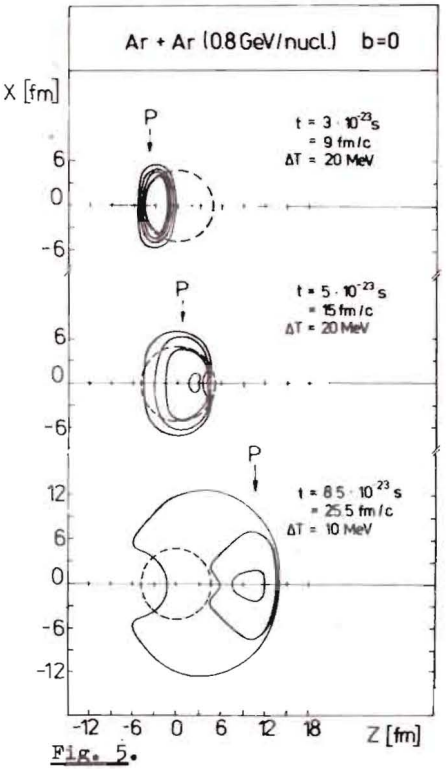


Fig. 5.

The temperature field for cascade nucleons from central Ar+Ar (0.8 GeV/A) collisions. The contours for the temperature field are defined by k x T (k= 1,2,...). At given time moment, the dashed circles indicate the undisturbed target nucleus ($R_{1/2}$ = 1.07 $A^{1/3}$), and the arrows show the position of the projectile nucleus.

fact that the specific entropy remains fairly constant, although the entire system is enormously cooled down.

Since we have calculated the entropy via a smoothing over the momentum space it might be instructive to show the contour plot of the temperature field. In Fig. 5 the dashed circles give the position of the initial target nucleus with $R_{1/2}$ = 1.07 $A^{1/3}$, whereas the full lines denote the different contours. (Note the different temperature increment ΔT). The formation of the well defined temperature field shows clearly that a smoothing over the momentum space

is very favourable for considering the later stage of the collision when the free expansion sets in.

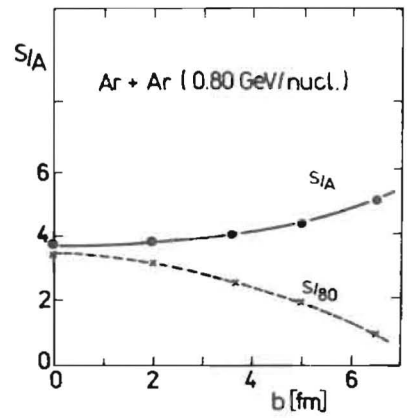


Fig. 6.
Impact parameter dependence of the specific entropy for the Ar + Ar (0.8 GeV/A) collision. The values of S/A are shown for the breakup density $\langle \rho(t) \rangle = \rho_0/3$.

Fig. 6 shows the dependence of the entropy on the impact parameter. It is clearly seen that the specific entropy increases steadily as the collision process becomes more peripheral. Such a tendency can also be obtained from the analysis of the 4π data of refs.[4,5]. This is because for the peripheral collision the struck particles become rather spare in phase space, whereby the associated temperature exceeds that of central collisions. For comparison we have also shown in Fig. 6 the entropy normalized to the total nucleon number. As expected, that quantity approaches zero for large impact parameter values.

The phase space occupancy function $\langle f \rangle$ for noncentral collisions has been shown in ref.[13,14] for Ca + Ca at 0.8 GeV/nucleon and is in reasonable agreement with our findings.

3.3. Comparison to entropy values inferred from composite particle production of relativistic heavy ion collisions

Finally, let us discuss our results in connection with the other models relating the production of composites to the produced entropy in the system. Concerning the 4π data an extensive discussion of the situation is performed in ref.[5] where entropy values are extracted by using the method of Kapusta[11] and Stöcker[12], respectively. Both models are based on quantum statistical approaches for infinite nuclear matter and include the finite size effects of the clusters. Kapusta's model [11] predicts the number of real deuterons and deuteron-like pairs contained in the heavier clusters, but does not specify these clusters. The model of Stöcker et al.[12] includes the formation of heavy clusters up to mass number 20 and the decay of all unbound resonances of these species. Furthermore, it also includes the contribution to the entropy originating from the pion and delta production.

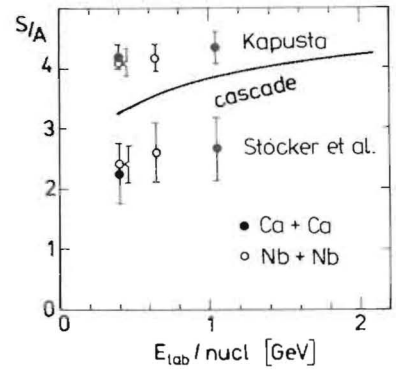


Fig. 7.
Projectile energy dependence of the specific entropy for central Ar+Ar collisions. "Experimental" points from /5/ are obtained by means of two different procedures /11,12/.

The results of the analysis are shown in Fig. 7. It is rather striking that the two models give rather distinct results. The appearing difference of about two units cannot be explained by the contribution of the heavy fragments at lower energies and/or by the contribution of the pion production at high beam energies. From the fig.7 it is seen that the cascade results for the specific entropy are in between the predictions of the other two models and seem not to favour one of the methods for beam energy up to 1 GeV/nucleon. For higher energies the cascade results seem to cope with those obtained by Kapusta's method (see also ref.[14]).

To investigate the origin of the large differences between the entropy values inferred by using either Kapusta's or Stöcker's method, the cascade model predictions for the cluster formation have also to be analyzed. For that aim we have developed following the line of a previous paper[8] a dynamical coalescence model which enables us to describe the composite formation as a function of the temperature and density of the nuclear medium and relative momentum of the composites. The specific entropy values represented in Fig. 7 are not much changed when allowing for clusterization but one obtains information how the composite production is related to the specific entropy value. Details of the calculations will be published elsewhere.

4. Conclusions

We have presented a cascade model calculation of the specific entropy. The used method is based on a smoothing of the distribution function in the momentum space by introducing a temperature field. Compared to the method which is based on a straightforward subdivision of the six-dimensional phase space[6] the method presented here allows us to calculate the entropy in the saturation region more accurately. This is because the thermal motion can well be separated form the collective motion of the cell. The numerical results have

shown that the specific entropy for the head-on collision of $^{40}Ca+^{40}Ca$ at 0.8 GeV is S/A =3.2 and about one unit smaller than that given in ref.[7]. This is mainly due to the too rough phase space subdivision and the partial inclusion of collective motion effects in the calculated entropy. Furthermore it has been found that the account of the diffusivity of the medium increases the specific entropy by about 0.6 units for $^{40}Ca + ^{40}Ca$.

It is also worthwhile to note that the cascade results for the specific entropy agree well with the predictions of the hydrodynamic calculations including viscosity effects[15]. This concerns the absolute values of the entropy and their dependence on the beam energy as well.

Compared to a recent analysis of 4π data of ref.[5] the entropy values do not favour the predictions of Kapusta[11] or Stöcker[12] et al. but are clearly smaller than those obtained by using Kapusta's method. It seems to be dubious whether or not the relation (1) can be used to derive the entropy from the deuteron or higher composite formation, because this relation is valid only in the low density limit where in-medium effects on the cluster formation are negligible. We suppose that cascade model calculations give a reasonable and independent approach to evaluate the entropy. On the other hand, the formation of composites should be performed within a coalescence model which takes into account in-medium corrections. In this way a consistent description of cluster abundances as well as entropy values may be found.

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Appendix A

Neglecting the collision term the Boltzmann equation takes the form

$$\frac{\partial f(\vec{r},\vec{p},t)}{\partial t} + \frac{\vec{p}}{M}\vec{\nabla} f(\vec{r},\vec{p},t) = 0 \qquad (A1)$$

The initial condition at time $t=0$ is written as .

$$f(\vec{r}_0,\vec{p},t=0) = \rho(\vec{r}_0)\, g(\vec{p})$$

where

$$\rho(\vec{r}_0) = \frac{A}{(2\pi\Delta^2)^{3/2}}\, e^{-\frac{r_0^2}{2\Delta^2}}$$

and

$$g(\vec{p}) = \frac{(2\pi\hbar)^3}{4(2\pi M T_0)}\, e^{-\frac{p^2}{2MT_0}} = \frac{1}{4^3}\, e^{-\frac{p^2}{2MT_0}}$$

are the distribution functions for the density and momentum, respectively. The normalization conditions are

$$4\pi \int \rho(r) r^2 dr = A$$

$$4\pi \int g(p) p^2 \frac{4dp}{(2\pi\hbar)^3} = 1 .$$

Since for $t>0$ the system expands as $\vec{r} = \vec{r}_0 + \frac{\vec{p}}{M}t$ and at the same time the momentum distribution remains unchanged, the solution of (A1) can be written in the form

$$f(\vec{r},\vec{p},t) = \rho(\vec{r} - \frac{\vec{p}}{M}t)\; g(\vec{p}) \qquad (A2)$$

whereby the local density, velocity and temperature are given by

$$\rho(\vec{r},t) \equiv \int \frac{d^3p}{(2\pi\hbar)^3} f(\vec{r},\vec{p},t) \Big/ \int d\Gamma f = \frac{1}{(2\pi\Delta^2\beta(t))^{3/2}}\, e^{-\frac{r^2}{2\Delta^2\beta(t)}} \qquad (A3)$$

,

$$\vec{u}(\vec{r},t) \equiv \int \frac{d^3p}{(2\pi\hbar)^3} \frac{\vec{p}}{M} f(\vec{r},\vec{p},t) \Big/ \int d\Gamma f = \frac{\vec{r}\,t}{M\Delta^2}\, T_0/\beta(t) \qquad (A4)$$

.

$$T(\vec{r},t) \equiv \frac{2}{3}\int \frac{d^3p}{(2\pi\hbar)^3} \frac{(\vec{p}-M\vec{u})^2}{2M} f(\vec{r},\vec{p},t) \Big/ \int d\Gamma f = T_0/\beta(t) \qquad (A5)$$

with $\beta(t) = 1 + T_0 t^2/M\Delta^2$. With the help of expressions (A3)-(A5) the complete solution of the Boltzmann equation (A1) reads[10]

$$f(\vec{r},\vec{p},t) = \left(\frac{\beta(t)}{2\pi\, T_0 M}\right)^{3/2} \left(\frac{1}{2\pi\Delta^2\beta(t)}\right)^{3/2} e^{-\frac{[\vec{p}-M\vec{u}(\vec{r},t)]^2}{2MT_0/\beta(t)}}\, e^{-\frac{r^2}{2\Delta^2\beta(t)}} \qquad (A6)$$

So, the time evolution of the density distribution function is characterized by a broadening of the effective width $\Delta^2 \to \Delta^2\beta(t)$ and a lowering of the associated temperature $T_0 \to T_0/\beta(t)$.

The specific entropy at $t=0$ is given by

$$S/A \simeq 1 - (\langle \ln\rho \rangle + \langle \ln g \rangle)$$

$$= 5/2 - \ln\frac{\langle\rho\rangle\Lambda^3}{4} - \frac{3}{2}(\ln 2 - 1) \qquad (A7)$$

Inserting into (A7) the time-dependent solution (A6), one sees that the specific entropy is conserved for any time instant. The quantity $\frac{3}{2}(\ln 2-1) \approx 0.46$ appearing in (A7) additionally compared to the expression for the entropy given in the text (eq. (7)) is due to the fact that in (A3) the Gaussian distribution function is used.

References

1. Siemens P.J., Kapusta J., Phys.Rev.Lett., 1976, 43, p. 1486.
2. Knoll J., Münchow L., Röpke G., Schulz H., Phys.Lett., 1982, 112, p. 13.
3. Röpke G., Münchow L., Schulz H., Nucl.Phys., 1982, A379, p. 563; Phys.Lett., 1982, 110B, p. 21; Röpke G., Münchow L., Schmidt M., Schulz H., Nucl.Phys., 1983, A399, p. 587.
4. Gutbrod H., Löhner H., Poskauzer A.M., Renner T., Riedesel H., Ritter H.G., Warwick A., Weik F., Wieman H., Phys.Lett., 1983, 127B, p. 317.
5. Doss K.G.R., Gustafson H.A., Gutbrod H.H., Kolb B., Löhner H., Ludewigt B., Poskanzer A.M., Renner T., Riedesel H., Ritter H.G., Warwick A., Wieman H., GSI-Preprint, GSI-85-14, 1985.
6. Gudima K.K., Toneev V.D. Yad.Fys., 1978, 27, p. 658; Nucl.Phys., 1983, A400, p. 173c.
7. Bertsch G., Cugnon J., Phys.Rev., 1981, C24, p. 2514.
8. Schulz H., Röpke G., Toneev V.D., Gudima K.K., Phys.Lett., 1983, 124B, p. 458.
9. Gudima K.K., Toneev V.D., Yad. Fys., 1980, 31, 1455.
10. Aichelin J., Nucl.Phys., 1983, A411, 474.
11. Kapusta J., Phys.Rev., 1984, C29, p. 1735.
12. Stöcker H., Buchwald G., Graebner G., Subramanian P., Maruhn J.A., Greiner W., Jacak B.V., Westfall G.D., Nucl.Phys., 1983, A400, p. 63c; Stöcker H., Jacak B.V., Westfall G.D., Phys.Rev., 1984, C29, p. 1744.
13. Bertsch G., Nucl.Phys., 1983, A400, p. 221c.
14. Cugnon J., L'Hote D., In: Proc. of the 7-th High Energy Heavy Ion Study, GSI-85-10, Darmstadt, 1985, p. 253.
15. Stöcker H., J.Phys. G: Nucl.Phys., 1984, 10, p. L11.

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Энтропия системы, образованной в столкновении тяжелых ионов

В рамках каскадной модели исследуется эволюция энтропии системы, образованной при столкновении тяжелых ионов. Метод вычисления энтропии основан на сглаживании функции распределения в импульсном пространстве путем введения поля температуры. Показано, что результирующая энтропия, приходящаяся на один нуклон, весьма чувствительна к процедуре разбиения фазового пространства на ячейки на стадии свободного разлета продуктов реакции. Из сравнения с последними экспериментальными результатами для удельной энтропии, извлеченной из 4π-измерений выхода составных частиц, найдено, что каскадные вычисления не позволяют отдать предпочтение какому-то одному методу модельной оценки энтропии и предсказывают значения энтропии меньше тех, которые следуют из рассмотрения на основе равновесной статистики.

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Entropy of the System Formed in Heavy Ion Collision

The entropy evolution in heavy ion collisions is investigated by means of a cascade model study. The method for calculating the entropy is based on a smoothing of the distribution function over the momentum space by introducing a local temperature field. It is shown that the resulting specific entropy is rather sensitive to the proper choice of the phase space subdivision at the disassembly stage of the collision. Compared to recent results for specific entropy values inferred from the composite particle yield of 4π measurements, it is found that the cascade calculations do not favour other model treatments but as a trend we obtain smaller entropy values than following from conventional considerations within equilibrium statistics.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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