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U(1) LATTICE GAUGE-HIGGS MODEL

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1. INTRODUCTION

Formulation of gauge theories on the space-time lattice provides a possibility of going beyond the perturbation theory and to study such non-perturbative effects as confinement, phase transitions and others.

In spite of the fact that for the investigation of lattice theories we are forced to use lattices of comparatively small size, the numerical results allow us to get information on the properties of physical systems in the physical (continuous) space-time. In this connection it is of a particular interest to study the phase structure of lattice theories. Indeed, according to the concepts existing at the present time [1] we may expect a continuum limit at a critical point, i.e., at a point of second-order phase transition. The lattice action does not, as a rule, depend on the lattice step explicitly, but via the "bare" couplings ($g_0(a), \lambda_0(a), \ldots$) and "bare" masses ($m_0(a)$). In most of the cases these functions are unknown. At the same time, general considerations show that the correlation length in the lattice system and the lattice step ξ are related in a simple way [1] $a \sim 1/\xi(g_0, \lambda_0, m_0, \ldots)$. At a critical point of the studied lattice system the dimensionless correlation length approaches infinity, and correspondingly, the lattice step tends to zero. More precisely, a continuum limit for (gauge-invariant) vacuum averages exists if the critical behaviour is defined by a fixed point of a renormalization group. At this point the lattice (statistical) system can be compared with a certain field theory with infinite cutoff which is the continuum limit of the lattice theory.

The existence of several or even infinite number of fixed points of the lattice system is allowed, which in fact may imply the existence of several theories in the continuum. Thus the investigation of the phase structure (phase diagrams) of lattice theories and the search for critical points will be an important condition for studying the continuum limit of lattice theories.

The phase diagrams in the Higgs-gauge models have been much studied in recent years [2-5]. In most of the papers a very simplifying assumption was used, i.e., the radial mode of the Higgs field was thought to be frozen $|\Phi_i| = \text{const}$. Thus assumption was based on the considerations following from the renormalization group approach [1] and reading that the continuum limit may exist in the vicinity of the critical point (more exactly, at a fixed point of a renormalization group with an infinite correlation length). Therefore, from the point of view of the continuum limit, the actual size of the scalar field is not essential in the regularized lattice action and the fixation of the radial mode is not a very restrictive condition. However, this treatment has a chance to be correct only when the theory has a single fixed point. If in the theory there are several fixed points, which seems more realistic, the fixation of the radial mode (corresponding to a definite choice of the bare action on the canonical surface) may lead us to a fixed point we do not need.

Therefore, it is important to take into account the fluctuations of the radial mode of the Higgs field.

As far as we know, for the first time this has been done in[3] for the Z_2-symmetric model. Variable radial mode has been used also in[3-5] for the models with groups of symmetries Z_N, U(1), SU(2), SU(3).

The present paper is devoted to the investigation of the U(1) gauge-Higgs model phase structure. The radial mode of the Higgs field is considered to be active. The dependence of the phase structure on the scalar self-interaction constant is investigated by the Monte-Carlo method as well as by approximate calculations using an effective potential of the Coleman-Weinberg-type. A special attention has been paid to the end-points of phase diagrams and the behaviour of the model in the vicinity of such points.

The paper is organized as follows: The second section is devoted to the formulation of the model; in the third section the effective potential of the Coleman-Weinberg-type is constructed and used to analyse the type of phase transitions by one of the order parameters and the dependence of the points of phase transitions on the parameters of the action; the fourth section is devoted to some details of the Monte-Carlo calculations, and the fifth section is a summary of our numerical results.

2. THE MODEL

We choose the action for a gauge field interacting with Higgs field in the fundamental representation of the group U(1) in the form

$$S = \beta \sum_{\square} S_{\square} + \sum_{L} S_L , \tag{2.1}$$

where

$$S_{\square} = 1 - \mathrm{Re}\, U_{ij} U_{jk} U_{k\ell} U_{\ell i} , \tag{2.2}$$

and the gauge field $U_{ij} \equiv U_L$ is defined on the link $L \equiv (i,j)$ outgoing from the site i and ending on the site j. U_ℓ is the element of the group U(1), i.e.,

$$U_L = e^{i\alpha_L} , \quad 0 \le \alpha_L < 2\pi . \tag{2.3}$$

The summation in the first term in (2.1) is performed over all plaquetts.

In the second sum (2.1) in (2.1) summation is over all links L and the quantity S_L has the form

$$S_L = \Phi_i^* \Phi_i - \frac{1}{2} [\Phi_i^* U_{i,i+\mu} \Phi_{i+\mu} + \text{h.c.}] + V(\Phi_i) , \tag{2.4}$$

where

$$V(\Phi_i) = \frac{1}{4} [\frac{m^2}{2} \Phi_i^* \Phi_i + \lambda (\Phi_i^* \Phi_i)^2] , \tag{2.5}$$

and Φ_i is the Higgs field in the lattice side i, transforming according to the fundamental representation U(1).

The radial mode of the Higgs field is active, i.e., $R_i = \sqrt{\Phi_i^* \Phi_i} \ne \mathrm{const}$ and $\Phi_i = R_i \phi_i$, $\phi_i = e^{i\theta_i} \in U(1)$. Everywhere in what follows we assume $m^2 < 0$.

The naive continuum limit of action (2.1)-(2.5) can easily be obtained by substituting

$$\Phi_i \to a\Phi(x) , \quad m^2 \to a^2 m^2 , \quad U_{i,i+\mu} \to e^{iagA_\mu(x)} ,$$

where a is the lattice step. Then, in the limit $a \to 0$, g, λ, m^2 fixed we get for the action S:

$$S = \frac{1}{4} \int d^4x \, (F_{\mu\nu}(x))^2 + \frac{1}{2} \int d^4x \, |(\partial_\mu - igA_\mu(x)) \Phi(x)|^2 + \int d^4x [\frac{m^2}{2} \Phi^* \Phi + \lambda (\Phi^* \Phi)^2] .$$

Now we use the Higgs polar variables (R_i, ϕ_i) to determine the partition function $Z \equiv \int \prod d\mu(R_i) d\phi_i \prod dU_L \exp\{-S\}$, where $d\mu(R_i) \equiv R_i dR_i$ is the radial measure of the Higgs field, and $d\phi_i = d\theta_i / 2\pi$ is the Haar measure on group U(1). The order parameters used in our paper are determined as follows:

$$\langle R^2 \rangle \equiv Z^{-1} \int \prod_i d\mu(R_i) d\phi_i \prod_L dU_L \Phi_i^* \Phi_i e^{-S} ,$$

$$\langle 1 - \square \rangle \equiv Z^{-1} \int \prod_i d\mu(R_i) d\phi_i \prod_L dU_L (1 - \mathrm{Re}\, U_{\square}) e^{-S} ,$$

$$\langle \mathrm{Re}\,\phi^* U \phi \rangle \equiv Z^{-1} \int \prod_i d\mu(R_i)\, d\phi_i \prod_L dU_L\, \mathrm{Re}(\phi_i^* U_{i,i+\mu}\, \phi_{i+\mu})\, \ell^{-S}. \qquad (2.6)$$

If $m^2 \to \infty$ or $\lambda \to \infty$, then, obviously, $\langle R^2 \rangle \to 0$ and we are left with a pure U(1) gauge U(1) theory depending on one parameter β and having a second-order phase transition in the order parameter $\langle 1 - \square \rangle$ at $\beta \approx 1$ (ref. [6]).

3. THE EFFECTIVE POTENTIAL METHOD

One approach to investigating possible phase transitions is to study the behaviour of the order parameter $\langle R \rangle$ by means of an effective potential of the Coleman-Weinberg-type [7]. Let's use the representation

$$Z = \int \prod_i d\mu(R_i)\, \tilde Z\{R_i\}, \qquad (3.1)$$

where

$$\tilde Z\{R_i\} \equiv \ell^{-\tilde S\{R_i\}} = \int \prod_\ell dU_\ell \prod_i \frac{d\alpha_i}{2\pi}\, \ell^{-S[\{R_i\},\{\alpha_i\},\{U_\ell\}]} \qquad (3.2)$$

and the action S is defined by (2.1). The "radial free energy" $\tilde S$ can be expanded in powers of β:

$$\tilde S[\{R_i\},\beta] = \tilde S_0 + \beta \tilde S_1 + \beta^2 \tilde S_2 + \dots , \qquad (3.3)$$

where the expansion coefficients $\tilde S_0$, $\tilde S_1$, $\tilde S_2$, ... are related to the "radial partition function" $\tilde Z$ by

$$\tilde S_0 = -\ln \tilde Z\big|_{\beta=0}, \quad \tilde S_1 = -\tilde Z^{-1} \frac{d}{d\beta} \tilde Z\big|_{\beta=0}, \quad \tilde S_2 = \frac12 (\tilde S_1^2 - \tilde Z^{-1} \frac{d^2}{d\beta^2} \tilde Z)\big|_{\beta=0}. \qquad (3.4)$$

One can easily calculate the first several terms in (3.3):

$$\tilde S_0\{R_i\} = \sum_i \sum_\mu [(1 + \frac{m^2}{8}) R_i^2 + \frac{\lambda}{4} R_i^4 - \ln I_0(R_i R_{i+\mu}) - \frac18 \ln R_i^2],$$

$$\tilde S_1\{R_i\} = N_\square - \sum_\square \prod_{\ell\in\square} \frac{I_1(R_i R_{i+\mu})}{I_0(R_i R_{i+\mu})}, \qquad \ell \equiv (i,\mu),$$

$$\tilde S_2\{R_i\} = -\frac14 N_\square - \frac14 \sum_\square \prod_{\ell\in\square} \frac{I_2(R_i R_{i+\mu})}{I_0(R_i R_{i+\mu})} - 5 \sum_{\substack{\square,\square' \\ \square\cap\square' = L}} [1 + \frac{I_2(R_i R_{i+\mu})}{I_0(R_i R_{i+\mu})}] \cdot \qquad (3.5)$$

$$\cdot \prod_{\ell\in\square\cup\square'/L} \frac{I_1(R_j R_{j+\mu})}{I_0(R_j R_{j+\mu})} + \sum_{\substack{\square,\square' \\ \square\cap\square' \neq \emptyset}} \prod_{\ell\in\square} \frac{I_1(R_i R_{i+\mu})}{I_0(R_i R_{i+\mu})} \prod_{\ell\in\square'} \frac{I_1(R_j R_{j+\mu})}{I_0(R_j R_{j+\mu})},$$

where I_n is the modified Bessel function.

Now, let us consider $\tilde S$ as a new action depending only on the radial variables $\{R_i\}$ which can be obtained in the standard way as an effective potential of the Coleman-Weinberg-type. For our purpose it is sufficient to use only the lowest order approximation for the effective potential $V_{eff}(\bar R)$. It is determined as

$$V_{eff}(\bar R) \equiv \frac{1}{N_L} \tilde S\{R_i\}\big|_{R_i = \bar R} = \sum_{k\ge 0} \beta^k V_{eff}^{(k)}(\bar R), \qquad (3.6)$$

where

$$V_{eff}^{(0)}(\bar R) = (1 + \frac{m^2}{8}) \bar R^2 + \frac{\lambda}{4} \bar R^4 - \ln I_0(\bar R^2) - \frac18 \ln \bar R^2, \qquad (3.7a)$$

$$V_{eff}^{(1)}(\bar R) = \frac32 \{1 - [\frac{I_1(\bar R^2)}{I_0(\bar R^2)}]^4\}, \qquad (3.7b)$$

$$V_{eff}^{(2)}(\bar R) = -\frac32 \{\frac14 [1 + (\frac{I_2(\bar R^2)}{I_0(\bar R^2)})^4] - 5 (\frac{I_1(\bar R^2)}{I_0(\bar R^2)})^6 [1 + \frac{I_2(\bar R^2)}{I_0(\bar R^2)}] + \frac{21}{2} [\frac{I_1(\bar R^2)}{I_0(\bar R^2)}]^8\} \qquad (3.7c)$$

and so on*.

In what follows we shall use only the first three terms in the expansion (3.6) written explicitly in (3.7).

As we shall see the effective potential (3.6)-(3.7) turns out to be a highly useful tool for studying the nature of phase transitions in the order parameter $\langle R^2 \rangle$ or $\langle R \rangle$. At $\beta = 0$ this expression for the effective potential has been used by us in [4,5] for describing phase transitions in the Higgs-gauge models with symmetry groups Z_N and SU(2) and it led to a good agreement with the results of the Monte-Carlo calculations. Note, that $V_{eff}^{(1)}$ and $V_{eff}^{(2)}$ vanish for $\bar R \to \infty$, i.e., it may be expected that the phase with large values of the radial mode of the Higgs field can be well described by the first term of the expansion (3.6). At small values of $\bar R$ the ratio $V^{(2)}/V^{(1)}$ is approximately 1/4, which means that the natural region of application of (3.6)-(3.7) is $|\beta| \le 1$. A comparison of the results obtained from (3.6)-(3.7) with the Monte-Carlo calculations shows that this estimate is reasonable.

The shape of the effective potential for different values of $|m^2|$, β and λ helps us to understand the nature of the phase transitions with the change of these parameters. Figure 1 shows

*Really the effective potential must be apparently a convex one (see, for instance [8]). But the definition of V_{eff} we have used [7] is absolutely noncontradictory for the lowest approximation and is more convenient for calculations.

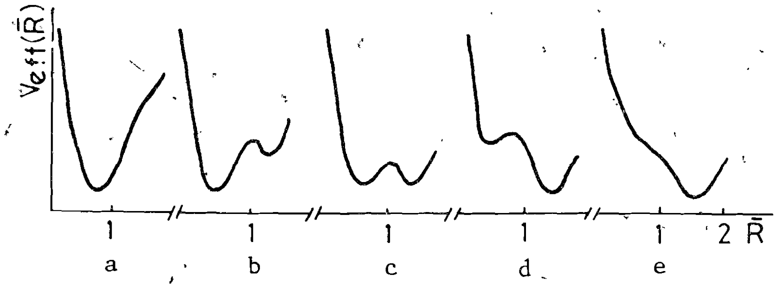


Fig.1. Behaviour of the effective potential as a function of $\bar{R}$ at fixed λ and β around the point of the first-order phase transition. Curves a–e correspond to growing values of $|m^2|$.

the typical behaviour of the effective potential for different values of m^2 and fixed β and λ. For small values of $|m^2|$ the effective potential has only one minimum at comparatively small values of $\bar{R}$ (see fig.1a). With increasing $|m^2|$ ($m^2 < 0$), V_{eff} acquires a second minimum (see fig.1b) lying above the first, and with increasing $|m^2|$ the values of V_{eff} in both minima become equal (fig.1c). With further increasing $|m^2|$ the value of the effective potential in the right minimum becomes less than in the left (1d), and eventually the left minimum vanishes at all (fig.1e). It is obvious that the left minimum of V_{eff} in fig.1b corresponds to a stable phase whereas the right-to a metastable one and the situation is reversed in fig.1d. At $m^2 = m_c^2$, where the minima become equal (fig.1c), there occurs a phase transition of the first order. With the help of V_{eff} one can recover the β dependence of m_c^2 for not very large values of β. A typical form of this dependence is shown on fig.2. The falling tails at the ends are due to inapplicability of our simple approximation in this region, where evidently, the higher-order expansion terms V_{eff} in β become important. Figure 3 demonstrates the dependence of m_c^2 on β at different values of λ. The solid lines are obtained by means of formulae (3.6)-(3.7).

We can see that our effective potential predicts a shift with increasing λ of lines of phase transitions to the right and upward. The circles indicate that the lines of the first order phase transitions have end points (at least for not very small λ). Near the end point $(m_c^2(\lambda); \beta_c(\lambda))$ the minima of the effective potential move nearer to each other and the local maximum between them disappears. Figure 4 exemplifies the behavious of the effective potential near an end point ($\beta = \beta_c$) for different values of $m^2 \sim m_c^2$. At some values of $|m^2| < |m_c^2|$ near the end point the effective potential has, in addition to the minimum at some relatively small value of R, also an inflection point to the right of the minimum (see fid.4a). With

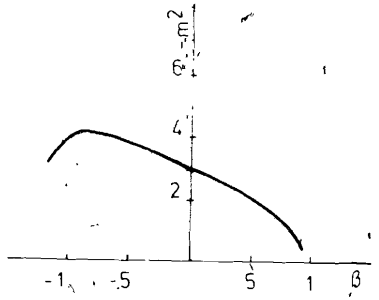


Fig.2. The point of phase transition m_c^2 as a function of β at $\lambda = 0,1$. The curve is found by formula (3.7).

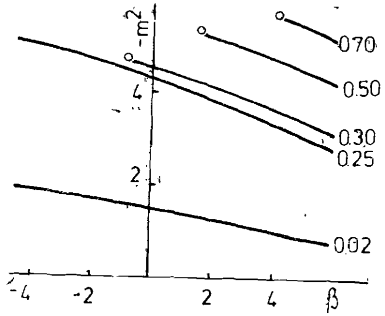


Fig.3. Lines of the first-order phase transition for various values of λ (λ = 0.02; 0.25; 0.30; 0.50; 0.70), calculated by formula (3.7).

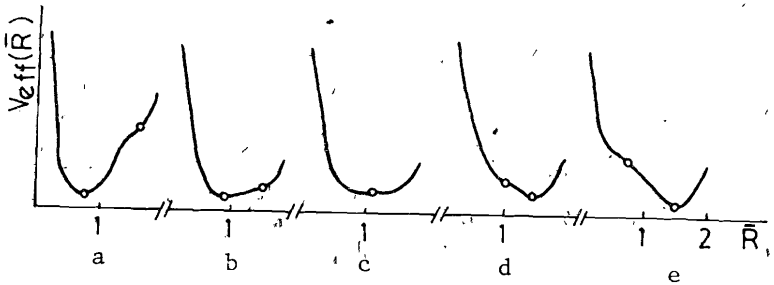


Fig.4. Dependence of the effective potential on $\bar{R}$ at different values of m^2 and fixed λ and $\beta = \beta_c$. Circles represent the minimum and point of inflexion of V_{eff}.

increasing $|m^2|$ the minimum and the inflection point approach each other, (fig.4b) and at some value of $m^2 = m_c^2$ "coincide" (fig.4c). With further increasing $|m^2|$, the inflection point shifts to the left of the minimum (fig.4d) and with further increasing $|m^2|$ the minimum shifts to the right (fig.4e). It is obvious that at $m^2 = m_c^2$ the derivative $(d/dm^2)\bar{R}(m^2)$ has a singularity. Indeed, let $\bar{R}_c$ be the solution of the equation

$$V'_{eff} = \frac{d}{d\bar{R}} V_{eff}\,(\bar{R}_c\,, m^2) = 0\,, \qquad (3.8)$$

i.e., $\bar{R}_c$ is the value of the order parameter $\langle R \rangle$ at a given value of m^2. By differentiating eq.(3.8) with respect to m^2, we find

$$0 = \frac{d}{dm^2} V''_{eff} [\bar{R}_c(m^2), m^2] = \frac{\partial}{\partial m^2} V'_{eff} [\bar{R}_c(m^2), m^2] + V''_{eff} \frac{d}{dm^2} \bar{R}_c$$

and hence, using the relation (3.7) we get

$$\frac{d}{dm^2} \bar{R}_c(m^2) = - \frac{\bar{R}_c(m^2)}{4 V''_{eff} [\bar{R}_c(m^2), m^2]} . \qquad (3.9)$$

Since $V''_{eff} \to 0$, when $m^2 \to m_c^2$, it follows that $(d/dm^2)\bar{R}_c(m^2) \to \infty$ (see fig.5).

The singularity of the derivative of the order parameter $\langle R\rangle = \bar{R}$ with respect to m^2 at the end point allows us to assert that it is a point of second order phase transition. To the left of this point (i.e., at $\beta < \beta_c$) the derivative of the order parameter $\langle R\rangle$ is regular and we observe a change of regime (crossover), represented by the dashed line on fig.5.

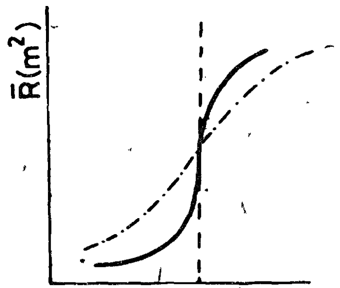


Fig.5. Behaviour of $\bar{R}$ *at* $\beta = \beta_c$ *(solid curve) and at* $\beta < \beta_c$ *(dotted line).*

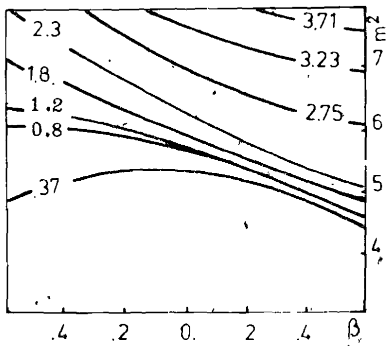


Fig.6. Lines of the level of function $\bar{R}(\beta ; m^2)$.

Figure 6 shows the lines of level of order parameter $\langle R\rangle$ in the plane (β, m^2) at $\lambda = 0.5$. In the place of confluence of two level lines there occurs a jump, the first-order phase transition. At the end point (β_c, m_c^2) the level lines get separated (in the figure these are the lines with order parameter 0.8 and 1.2) though being quite close to each other. This proximity of two different level lines testify to a sharp change of the regime. It may be said that the crossover is a trail of the critical point. Analogous situation may probably occur in the phase plane (β, β_A) in the pure gauge SU(2)-symmetric theory with a mixed action[9].

4. DESCRIPTION OF THE MONTE-CARLO PROCEDURE

The model (2.1)-(2.5) has been numerically investigated by the Monte-Carlo method. We have used the Metropolis algorithm[10] which is more convenient in the case of noncompact manifold of fields.

When implementing the Metropolis algorithm the number of updates of the field variables in each site and on each link of the lattice has been optimized, as was the maximal deviation of the new value of the field variable from the old one, in order to obtain an acceptance rate within 50-70%. Usually this required four-five attempts for each degree of freedom. One Monte-Carlo iteration means a systematic upgrading of all links and sites of the lattice. We have employed the Monte-Carlo method to calculate the following order parameters: $\langle 1-\Box\rangle$, $\langle R^2\rangle$ and $\langle \mathrm{Re}\, \phi^* U \phi\rangle$. The behaviour of the order parameters near a phase transition point was studied by two different methods:

i. Thermal cycles

One of the parameters (β or m^2) was being slowly changed in a given interval, first in one direction and then back. On the average, 8-12 iterations were made at each step, and the order parameter was averaged over the last 4-5 iterations. The typical step in β was equal to 0.05; and in m^2 , to 0.1. In the thermal cycles the final configuration at the previous value of $\beta(m^2)$ was used as an initial configuration at the current value of $\beta(m^2)$. At the beginning of the thermal cycle 100-200 iterations were made to equilibrate the system. The appearance of a hysteresis loop (see, for instance, fig.7) may indicate a possible phase transition.

ii. Different starts

At a point of first order phase transition the value of the order parameter determined by the Monte-Carlo method can depend on the choice of the initial configuration (start). Mainly, we used two types of starts : "hot", when all the values of field variables are chosen at random, and "cold", when all the gauge degrees of freedom are fixed by the condition $U = 1$ and the initial values of the Higgs fields are set equal to $R_i = 0,1$, $\phi_i = 1$.

The existence of two phases can reveal itself in the hisrogram of R_i^2 after a large number of iterations. In terms of the effective potential, a peak in such a histogram indicates a minimum of the potential. When the peaks corresponding to different initial configurations do not coincide, this points to a two-minimum structure of the effective potential, i.e., the two-phase picture.

As an approximation of the group U(1), its discrete subgroup Z_{300} was used. All the calculations were made on lattices 4^4 and 6^4 with periodic boundary conditions. The results of the calculations on these two lattices practically coincide.

5. RESULTS OF THE MONTE-CARLO CALCULATIONS

As it has been pointed out, we calculate three order parameters $\langle R^2 \rangle$, $\langle 1-\Box \rangle$ and $\langle \mathrm{Re}\phi^* U \phi \rangle$. When performing thermal cycles in m^2 at fixed β and λ, the phase transition points were determined from the behaviour of the three order parameters. The results of calculations show that, for comparatively small λ, there exists a first order phase transition with changing m^2. Figure 7 demonstrates a typical hysteresis loop at $\lambda = 0.3$ and $\beta = 0.6$. An increase of the number of iterations per point does not change noticeably the form of the hysteresis loop, which testifies to a first order phase transition. This conclusion is confirmed by the dependence of the order parameter $\langle R^2 \rangle$ on the number of iterations for two different starts.

Figure 8 shows the dependence of the current value of the average $\langle R^2 \rangle$ as a function of the number of iterations for $m^2 = -3.1$; -2.9 and -2.5 at $\beta = 0.6$, $\lambda = 0.3$. The dots in fig.8 correspond to the "ordered" start, whereas the crosses correspond to the "random" one. It is obvious that with changing m^2 from $m^2=-3.1$ to $m^2=-2.5$, the order parameter $\langle R^2 \rangle$ changes sharply. In Figure 9 we compare our Monte-Carlo results (denoted by x) with calculations by formula (3.7) at $\lambda = 0.3$ and $\beta = 0.6$ The upper solid line corresponds to the position of the right minimum of V_{eff}, whereas the lower curve to the left minimum. The vertical line corresponds to the point of phase transition defined by formula (3.7). At $m^2 = -2.5$ the state corresponding to the "ordered" start is stable while at $m^2 = -3.1$ stable is the state corresponding to the "disordered" start. At $m^2 = -2.9$ we observe two "long-living" states, one being stable and the other metastable. The existence of two "long-living" states (stable and metastable) can be illustrated by the histogram of distribution of R_i^2 at the lattice nodes. At $m^2 = -2.9$ the histograms have pinnacled form (see fig.10), the position of peaks being different for two various starts.

Considering an analytic continuation in β to the region of negative values at fixed (not very small) value of λ, we notice that the hysteresis loop begins to shrink and finally disappears completely. This is illustrated in fig.11. We see that at $\lambda = 0.25$ the line of first order phase transitions has an end-point at $\beta_c \approx -0.75$ (fig.11a). At this point the behaviour of the order parameter shows a sharp change of regime. At $\beta < \beta_c$ the order parameter $\langle R^2 \rangle$ changes more smoothly (fig.11b).

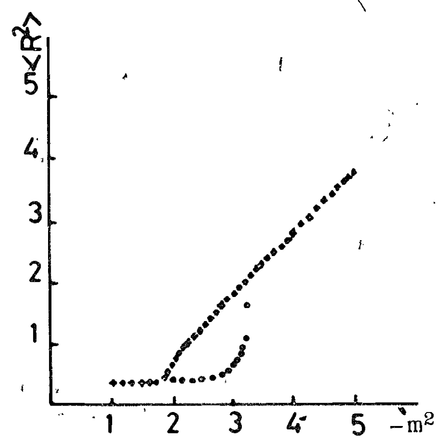
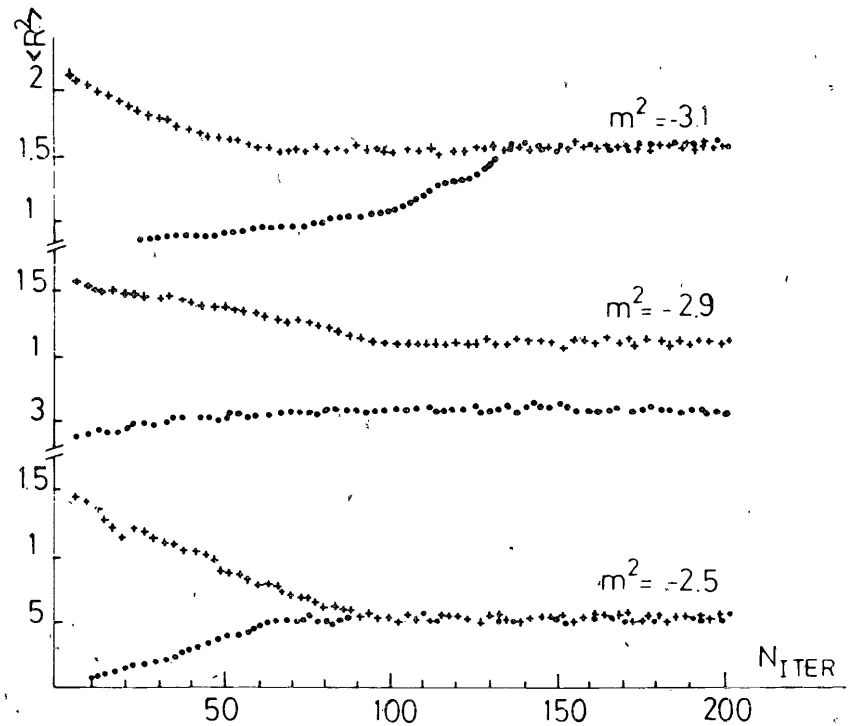


Fig.7. Thermal cycle in parameter m^2 at $\lambda = 0.3$ and $\beta = 0.6$.

Fig.8. Dependence of the order parameter $\langle R^2 \rangle$ on the number of iterations for various starts. Crosses represent the disordered start, whereas dots the ordered one.



Our analysis of the effective potential shows that a second order phase transition is what happens at the end-point. With changing β towards large positive values, we find that the hysteresis loop also becomes narrower at sufficiently large β. At $\beta = \infty$ there is no hysteresis (see fig.12), however, the type of a phase transition, if it exists at all, is yet an open question.

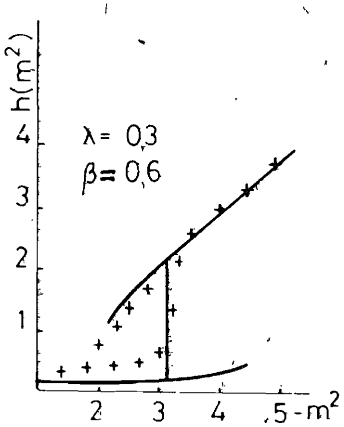


Fig.9. Parameter $\bar{R}^2$ at minima of V_{eff} for $\lambda = 0.3$ and $\beta = 0.6$. Crosses are some Monte-Carlo points of the thermal cycle given in Fig.6.

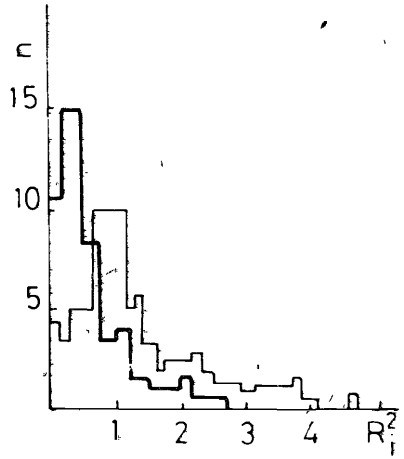


Fig.10. Distribution R_i^2 after 200 iterations for $\lambda = 0.3$, $m^2 = -2.9$ and $\beta = 0.6$. The left peak corresponds to the ordered start, and the right one to the disordered start.

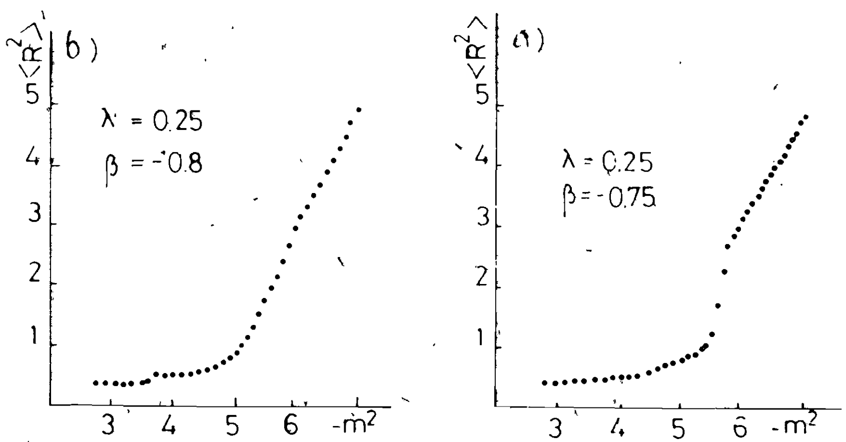
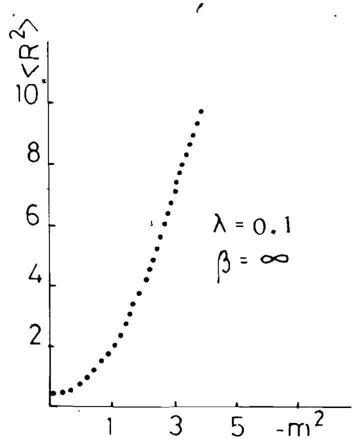
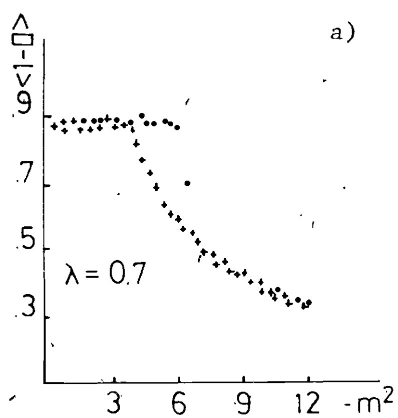


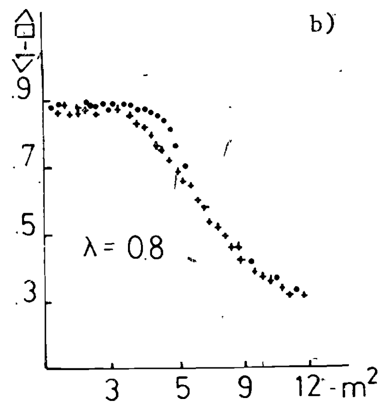
Fig.11. Behaviour of the order parameter $\langle R^2 \rangle (m^2)$ at $\lambda = 0.25$; $\beta = -0.75$(a) and $\beta = -0.80$ (b).

Thus, at fixed λ we observe a "horizontal" line of first order phase transitions in the phase plane $(\beta ; m^2)$ having an end point at which the phase transition is of the second order. Is there any phase transition at $\beta = \infty$ and what is the order of it, we cannot determine at present.

Fig.12. Dependence of $\langle R^2 \rangle$ on m^2 for $\lambda = 0.1$ and $\beta = \infty$.







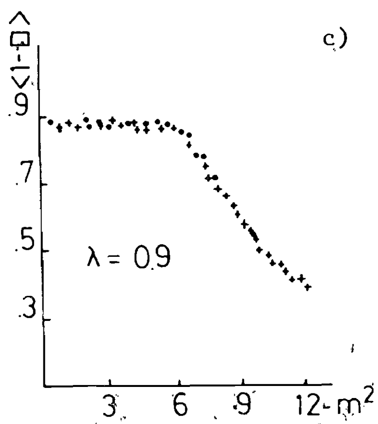


Fig.13. Thermal cycles in m^2 at $\beta = 0.3$ and various λ.

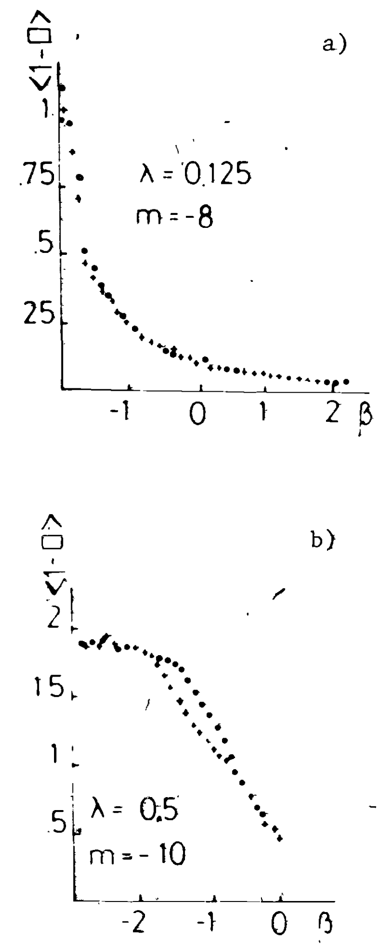


Fig.14. Thermal cycles in β: a) λ = 0.125, $m^2 = -8$ and b) λ = 0.5; $m^2 = -10$.

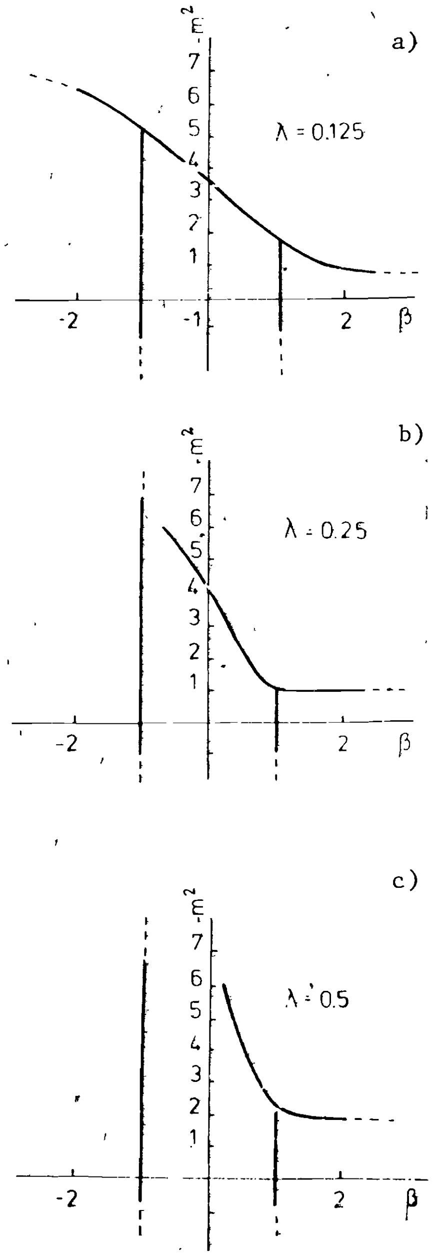


Fig.15. Phase diagrams for different values of λ.

With increasing λ this line shifts in the phase plane to the right and upwards, as it was predicted by our calculations in the framework of the effective potential (see Sec.3). The dependence of the hysteresis loops for the order parameter $\langle 1-\square \rangle$ at $\beta = 0.3$ on m^2 is shown in fig.13. It is seen that with changing λ from λ = 0.7 to λ = 0.9 the hysteresis loops become narrower and then vanish completely. It is not yet clear how far would the left end-point be shifted with increasing λ.

When performing thermal cycles in β at fixed m^2, we observe hysteresis loops only with respect to the order parameters $\langle 1-\square \rangle$ and $\langle \mathrm{Re}\,\phi^* U \phi \rangle$. As it has been pointed out, in the limit $m^2 \to \infty$ we are led to a purely gauge theory with U(1)-symmetry. In this model a second order phase transition was observed at $\beta \approx 1$ [6]. In the case of our Higgs-gauge model we also observe the hysteresis loops with respect to the order parameters $\langle 1-\square \rangle$ and $\langle \mathrm{Re}\,\phi^* U \phi \rangle$ at $\beta \approx 1$ and $\beta \approx -1$. In this case, at sufficiently small λ, at which the left end point discussed above is to the left of $\beta = -1$, the hysteresis loops occur only for values of m^2 which are below the "horizontal" line of the first order phase transitions. With increasing λ the end point shifts towards $\beta > -1$, the hysteresis loops occur at even larger values of m^2. This situation is represented in fig.14. At $\beta \approx 1$ for not very large λ we observe hysteresis loops only for those values of m^2 which are below the "horizontal" line of the first order phase transitions. We can sum up our results in the following series of phase diagrams (see fig.15).

6. CONCLUSION

Thus, we have analysed the phase structure of the U(1)-symmetric lattice gauge-Higgs theory with defrost radial mode. Phase diagrams were constructed by the numerical Monte-Carlo method with the use of an effective potential of the Coleman-Weinberg type. We have shown that for a fixed value of the constant of scalar self-interaction λ in phase plane (β, m^2) there exists an end point which is a critical point like in the phase diagram of the type "gas-liquid-ice". The "trail" of this end point on the phase diagram is the line of crossovers. As follows from our results, at not very small values of λ, the "Higgs" phase and the "confinement" phase are not separated by a continuous line of phase transitions, rather there exists a region of analyticity in the phase plane which permits a continuous transition from one phase to another without jump of the order parameter or its derivative. This is an argument in favour of the so-called complementary principle [11]. However, at present we cannot make an unambiguous conclusion on the nature of phase transition when $\beta = \infty$ (if it exists at all).

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Гердт В.П., Илчев А.С., Митрюшкин В.К. E2-85-59
U(1)-хиггс-калибровочная теория на решетке

В работе методом Монте-Карло, а также с помощью приближенного вычисления эффективного потенциала исследуется фазовая структура U(1)-симметричной хиггс-калибровочной теории. Хиггсовские поля рассматриваются в фундаментальном представлении, и радиальная мода хиггсовских полей разморожена. Построены фазовые диаграммы теории для различных значений константы скалярного самодействия. Показано, что линии фазовых переходов 1-го рода имеют концевые точки, которые являются критическими точками, аналогично тому, как это имеет место на фазовых диаграммах типа "газ-жидкость-лед".

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Gerdt V.P., Ilchev A.S., Mitrjushkin V.K. E2-85-59
U(1) Lattice Gauge-Higgs Model

The phase structure of the U(1)-symmetric Higgs-gauge theory is investigated by the Monte-Carlo method and by an approximate calculation of an effective potential. The Higgs fields are treated in the fundamental representation, and their radial mode is not frozen. Phase diagrams of the theory are determined for various values of the constant of scalar self-interaction. Lines of first-order phase transitions are shown to possess end points which are critical points like in the diagrams of the type "gas-liquid-ice".

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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