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DISCUSSION OF THE SOLAR NEUTRINO PROBLEM IN THE LIGHT OF REACTOR NEUTRINO OSCILLATION EXPERIMENTS

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1. Oscillations of solar and reactor neutrinos: introduction

1. The possible significance of neutrino oscillations in solar neutrino astronomy has been recognized even before the first solar neutrino experiments had been performed[1]. In the absence of any information on neutrino mixing, one would expect that the flux of detectable solar neutrinos might decrease through the presence of oscillations by a factor at most equal to n , where n is the number of neutrinos of definite (and different) masses ("$\frac{1}{n}$ -rule") [2,3]. As a matter of fact R.Davis et al.[4] have found an intensity of solar neutrinos detected through the $^{37}Cl - ^{37}A$ method[5] smaller than the one expected from the standard solar model[6]. This discrepancy may mean that neutrinos are massive and oscillate, but the conclusion, as is well known, is not certain (because it is difficult to assess the error affecting the expected $^{37}Cl - ^{37}A$ "no-oscillation" value of the solar neutrino flux).

At present different and complementary methods of investigating solar neutrinos are being considered. We have in mind especially the Ga-Ge method[7], the corresponding "no-oscillation" neutrino flux being practically model independent.

Presently, artificial neutrinos from accelerators and reactors are also being used on a wide scale program with the aim to search for oscillations. In this note we shall consider only solar and reactor investigations because the results of the corresponding experiments are directly comparable, the Sun and the reactor being sources of electron-type neutrinos (ν_e , $\bar{\nu}_e$) respectively.

In a recent paper[8] we started to consider the problem as to how the information on the presence of oscillations possibly revealed in reactor experiments would influence the discussion and the interpretation of present and future solar neutrino experiments. There we limited ourselves to the case of small amplitude oscillations with only one oscillation length. We showed that the presence of such oscillations possibly revealed in reactor experiments would result in a maximum decrease of the detectable solar neutrino flux by a factor equal to $(n-1)$ instead of the factor n corresponding to the "$\frac{1}{n}$ -rule". In this paper we present a more general discussion of the problem.

2. Basic assumptions and results

As is well known the "current" neutrino fields ν_ℓ $(\ell=e,\mu,\tau,..)$ may be different from the fields ν_i $(i = 1,2,3,...)$ of neutrinos with definite masses m_i (either Dirac or Majorana). In the general case[9]

$$\nu_{\ell L} = \sum_{i=1}^{n} U_{\ell i}\, \nu_{iL} . \qquad (1)$$

Here the index L denotes left-handed components, U is a unitary mixing matrix. Let us note that the number of neutrinos with definite masses n may either coincide with the number of charged leptons or be twice larger than such a number[10]. In the last case together with $n/2$ "active" (current) neutrinos $\nu_e, \nu_\mu, \nu_\tau, \ldots$ there must exist also $n/2$ "sterile" particles ν_{eL} , $\nu_{\mu L}$, $\nu_{\tau L}$, ... which do not take part in the standard weak interaction.

In this paper we will limit ourselves to the simplest case when in reactor experiments oscillations with only one oscillation length are observed; in other words the case in which only one difference of squared neutrino masses enters the expressinn of the reactor antineutrino oscillation amplitude.

The indication in favour of finite neutrino masses, if any, is as follows:

i) In the ITEF group investigation of the β -spectrum of ^3H it is claimed that the mass of electron antineutrinos is about 30 eV [11]. Incidentably, such a value is very desirable from a cosmological point of view[12].

ii) The solar neutrino flux (I_ν =1,8 ± 0,3 SNU) measured[4] with the help of the $^{37}Cl - ^{37}A$ method is smaller than that expected theoretically[6] on the assumption that no oscillations take place (I_ν^0 = 7,9 ± 1,5 SNU); if the difference is due to oscillations,the result implies[13] that $|m_i^2 - m_k^2| \gtrsim 10^{-12}$ eV^2 (m_i and m_k being the masses of the corresponding neutrino mass eigenstates).

iii) In a recent [14] high statistics investigation performed at a power reactor it is claimed that some indication is found in favour of oscillations (with oscillation length corresponding to a square neutrino mass difference around 0,2 eV^2 and an amplitude around 0,2) . Below we take points i), ii) and iii) at their face value and try to reconcile them from a phenomenological point of view, although we are of the opinion that points i), ii) and iii) are in no way final.

Having in mind the possible large decrease of the measured solar neutrino flux with respect to the expected one, let us assume that there are $n > 2$ neutrino masses. If the reactor experiments reveal only one oscillation length one would guess that some neutrino masses are so close that the corresponding oscillation lengths are much larger than the distance reactor-detector. Thus, we are naturally led to the idea that there may exist a group of close neutrino masses $m_1 \simeq m_2 \simeq \ldots$ the difference of which is relevant in solar but not in reactor oscillation experiments. The origin of the reactor oscillation would be then due to the existence of one mass m_n significantly different from the masses of the neutrinos belonging to the above-mentioned group. The amplitude of the transition $\nu_l \to \nu_{l'}$ ($l, l' = e, \mu, \tau, \ldots$) may be written in the form

$$\mathcal{A}_{\nu_{l'};\nu_l}(t) = e^{-iE_j t}\Big[\sum_{i=1}^{n} U_{l'i}\big(e^{-i(E_i-E_j)t}-1\big)U_{li}^{*} + \delta_{l'l}\Big], \qquad (2)$$

where t is the time, $E_i = \sqrt{p^2+m_i^2}$, p is the neutrino momentum. In expression (2) the index j is fixed, and m_j is the mass of any of the neutrinos belonging to the group. Our starting point that only one oscillation length is revealed in reactor experiments is taken to indicate that

$$|E_n - E_j|t \simeq \frac{|m_n^2 - m_j^2|}{2p} R \gtrsim 1,$$
$$|E_i - E_j|t \simeq \frac{|m_i^2 - m_j^2|}{2p} R \ll 1, \quad i \neq n, \qquad (3)$$

where R is the distance between the reactor and the detector ($p \gg m_i$, $i = 1, 2, \ldots n$). For the probability of transition $\nu_l \to \nu_{l'}$ ($\bar\nu_l \to \bar\nu_{l'}$) from (2) and (3) we get [*].

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[*] In the presence of sterile neutrino in addition to (1) we must add the expression /10/

$$\nu_{lL}^{c} = (\nu_{lR})^{c} = \sum_{i=1}^{n} U_{li}^{*}\,\nu_{iL},$$

where $\nu^c = C\bar\nu^T$ is the charge conjugated field. The probability $\nu_l \to \bar\nu_{l'L}$ can be obtained from (4) by substitution $|U_{l'i}|^2 \to |U_{l'i}^{*}|^2$.

$$P_{\nu_{l'};\nu_l}(R/p) = P_{\bar\nu_{l'};\bar\nu_l}(R/p) = 2|U_{l'n}|^2|U_{ln}|^2\Big(1-\cos\frac{1}{2}\Delta\frac{R}{p}\Big), \qquad (4)$$

where

$$\Delta = |m_n^2 - m_j^2|.$$

From (4) one finds [*]

$$P_{\nu_l;\nu_l}(R/p) = P_{\bar\nu_l;\bar\nu_l} = 1 - \sum_{l' \neq l} P_{\nu_{l'};\nu_l}(R/p) = 1 - \frac{1}{2}A\Big(1-\cos\frac{1}{2}\Delta\frac{R}{p}\Big). \qquad (5)$$

Here

$$A = 4|U_{ln}|^2\big(1-|U_{ln}|^2\big) \qquad (6)$$

is the oscillation amplitude. It is clear that

$$0 \leq A \leq 1$$

and the maximum of the amplitude corresponds to $|U_{ln}|^2 = \frac{1}{2}$.

For the intensity of electron antineutrinos $I_{\bar\nu_e}(R,p)$ with momentum p at a distance R from the reactor we have

$$I_{\bar\nu_e}(R,p) = P_{\bar\nu_e;\bar\nu_e}(R/p)\, I_{\bar\nu_e}^{0}(R,p), \qquad (7)$$

where $I^0(R,p)$ is the intensity of $\bar\nu_e$ expected in the absence of oscillations and the probability $P_{\bar\nu_e;\bar\nu_e}(R/p)$ is given by the expression (5).

Let us discuss now solar neutrino experiments. Since in such experiments $R/p \simeq 10^{11}\,\frac{m}{MeV/c}$, we shall assume that for all $i \neq j$

$$|m_i^2 - m_j^2|\frac{R}{p} \gg 1,$$

i.e., every oscillation length is much smaller than Sun-Earth distance. The properly averaged detected neutrino flux is

$$I_{\nu_e} = \delta\, I_{\nu_e}^{0}, \qquad (8)$$

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[*] In presence of transitions to sterile states the sum in (5) must include also such states.

where $I^{0}_{\nu_e}$ is the flux which should be expected under the assumption that oscillations are absent and δ is

$$\delta = \sum_{i=1}^{n} |U_{ei}|^{4} . \tag{9}$$

Suppose that there is no any apriori information on oscillations. Usually the maximum decrease of the dectable solar neutrino flux due to the presence of oscillations is then considered. As is well known/2,3/ such a maximum decrease corresponds to

$$\delta_{min} = \frac{1}{n} , \tag{10}$$

which is obtained when $|U_{ei}|^{4} = \frac{1}{n}$ $(i=1,2,\ldots n)$.

Let us assume now that some information on the value of $|U_{en}|^{2}$ has been obtained from reactor experiments. In this case instead of relation (10) we get /8/

$$\delta'_{min} = \frac{(1-|U_{en}|^{2})^{2}}{n-1} + |U_{en}|^{4} \tag{11}$$

Such a minimum corresponds to

$$|U_{ei}|^{2} = \frac{1-|U_{en}|^{2}}{n-1} , \; i=1,2,\ldots n-1. \tag{12}$$

Let us notice that expression (11) was obtained under the assumption $n \geq 3$. Clearly if $n=2$ the knowledge of $|U_{e2}|^{2}$ permits to predict the parameter δ unambiguously $(\delta = |U_{e2}|^{4} + (1-|U_{e2}|^{2})^{2})$.

Let us discuss now which information on $|U_{en}|^{2}$ may be obtained from reactor neutrino experiments. If in such experiments oscillations with one oscillation length had been observed, this would yield the value of $|m_n^2 - m_d^2|$ and the oscillation amplitude A. From (6) it is obvious that to a given value A there correspond two values $|U_{en}|^{2}$:

1. $$|U_{en}|_1^{2} = \frac{1-\sqrt{1-A}}{2} , \tag{13}$$
2. $$|U_{en}|_2^{2} = \frac{1+\sqrt{1-A}}{2} . \tag{14}$$

Thus, we are confronted with two possible minima of the parameter δ:

$$(\delta'_{min})_1 = 1 - \frac{1}{2}A - \alpha_+^{2}\left(1 - \frac{1}{n-1}\right) , \tag{15}$$

$$(\delta'_{min})_2 = 1 - \frac{1}{2}A - \alpha_-^{2}\left(1 - \frac{1}{n-1}\right) , \tag{16}$$

where

$$\alpha_{\pm} = \frac{1 \pm \sqrt{1-A}}{2} . \tag{17}$$

Since $\alpha_+ \geq \alpha_-$ the deepest minimum is $(\delta'_{min})_1$.

It can be shown that information about the presence of one oscillation length with the amplitude A in reactor experiment permits us also to yield the following upper limit

$$\delta < 1 - \frac{1}{2}A . \tag{18}$$

As a matter of fact

$$\delta = |U_{en}|^{4} + \sum_{i \neq n} |U_{ei}|^{4} . \tag{19}$$

Taking into account the unitarity of the mixing matrix U we have

$$\sum_{i \neq n} |U_{ei}|^{4} < (1-|U_{en}|^{2})^{2} . \tag{20}$$

The inequality (18) follows from (6), (19) and (20).

Let us notice that the meaning of this inequality is quite clear from a physical point of view. The parameter δ is the averaged probability that a solar (electron) neutrino is detected as such at the Earth. Clearly, such a probability is smaller than the analogous probability, given by the right-hand side of equation (18), that an antineutrino is detected as such in reactor experiments.

Thus, the observation in reactor neutrino experiments of oscillations characterized by one oscillation length would permit one to draw the following conclusion: the parameter δ characterizing the decrease of the flux of detectable solar neutrino is bounded from above (see (18)) as well as from below (see (15) and (16)). The upper limit of δ depends only on oscillation amplitude A. The lower limit depends*) upon A as well as the number n of neutrinos with definite masses.

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*) Let us stress that such a conclusion holds under the assumption that $n-1$ neutrino masses are grouped closely and one mass is significantly different from the masses of the group.

3. Small oscillation amplitude

Here we consider the case when reactor experiments indicate the presence of one oscillation length with small amplitude. With an accuracy up to linear terms in A from expression (15), one has

$$(\delta'_{min})_1 \simeq \frac{1}{(n-1)}\left(1-\frac{1}{2}A\right). \qquad (21)$$

This result was already discussed in paper/8/. As far as the second solution is concerned we have

$$(\delta'_{min})_2 \simeq 1-\frac{1}{2}A. \qquad (22)$$

By comparing (18) and (22) we conclude that in linear with respect to A approximation the parameter δ for the second solution is obtained unambiguously from the measurement of the amplitude A. In other words for this case in solar neutrino oscillations there is no new physics with respect to the one already seen in reactor experiments.

Let us notice that the first solution implies a small value of $|U_{en}|^2$ $(|U_{en}|_1^2 \simeq \frac{1}{4}A)$. In such a case the unitarity of the mixing matrix U allows large amounts of mixing of the remaining elements of the e- row, so that the decrease of the detectable solar neutrino flux may be considerable $\left((\delta'_{min})_1 \simeq \frac{1}{n-1}\right)$. In the second solution $|U_{en}|^2$ is near one $(|U_{en}|_2^2 \simeq 1-\frac{1}{4}A)$. Then, the unitarity of the matrix U requires the parameter $(\delta'_{min})_2$ to be close to one independently of the value of n. Physically this means that there is no mixing with the exception of that already shown in reactor experiments, more exactly there is no large amounts of mixing of the remaining elements of the e- row of matrix U.

Suppose now that the parameter is found to be definitely less than one in the case considered in this paragraph $(A \ll 1)$. This would obviously mean that the second solution for $|U_{en}|^2$ is to be excluded, that is, there may be noticeable mixing. If $\delta \leq \frac{1}{2}(1-\frac{1}{2}A)$ from solar neutrino data it is possible to obtain lower bound on n. As a matter of fact from (21) we get in this case

$$n \geqslant 1+\frac{1-\frac{1}{2}A}{\delta} \qquad (23)$$

Let us underline here that the time is not far-distant when some direct information on the number n' of neutrinos emitted in Z^0-decay will be obtained/15/. The question arises as to whether the number n' coincides with the number n of neutrinos with definite masses, the lower limit of which n_{min} can be obtained from solar neutrino experiments (we have in mind, for example, expressions such as $\delta_{min} = \frac{1}{n}$ or $\delta'_{min} \simeq \frac{1}{n-1}$). The question may be answered if it will turn out that

$$n_{min} > n'. \qquad (24)$$

The inequality (24) would mean that there exists some sort of neutrinos which are not emitted in Z^0-decay. This would be first an indication in favour of the evidence of sterile neutrinos and second, a proof that Z^0-bosons emit only "current" neutrinos, as it is usually assumed. Incidentally such an assumption is quite natural in the frame of all we know about the electroweak interaction but nevertheless in our opinion it should be tested. As a matter of fact an analysis of known data about nonconservation of parity in neutrino neutral current interactions allows one to make the following conclusion: the exotic possibility that Z^0-boson emits sterile neutrinos in addition to current neutrinos does not contradict the known facts. The question of the Z^0 decay into two neutrinos was discussed in paper/16/.

4. Large oscillation amplitude

Let us consider now the case whereby reactor experiments show the existence of oscillations with one oscillation length, the amplitude being close to the maximum value ($A=1$). In this case the two solutions (15) and (16) for δ'_{min} coincide:

$$\delta'_{min} = \frac{1}{4}+\frac{1}{4}\frac{1}{(n-1)} \qquad (25)$$

It is seen from (18) and (25) that

$$\frac{1}{4} < \delta < \frac{1}{2}. \qquad (26)$$

Thus, if at $A=1$ turns out to be $< \frac{1}{4}$, such a decrease of a detectable flux of solar neutrinos could not be explained in terms of oscillations. Of course the conclusion for the time being is rather academic; as a matter of fact the accuracies of the experiments and the accuracy of the predicted value of a solar neutrino flux in the absence of oscillations will not be sufficient for a long time.

Let us notice that the value of δ'_{min}, as is seen from (25) slightly depends upon n (in contrast with the case $A \ll 1$ and of course with the case $\delta_{min} = \frac{1}{n}$, corresponding to the absence of any information on mixing).

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Биленький С.М., Понтекорво Б. E2-85-589
Проблема солнечных нейтрино
и реакторные нейтринные эксперименты

Обсуждается связь между экспериментами по регистрации солнечных нейтрино и опытами с реакторными антинейтрино. В общем случае n массивных нейтрино получено соотношение между максимальным и минимальным значениями коэффициента /обусловленного осцилляциями/ уменьшения потока детектируемых солнечных нейтрино и амплитудой осцилляций реакторных антинейтрино. Подробно рассмотрены случаи малых и больших амплитуд осцилляций.

Работа выполнена в Лаборатории теоретической физики и Лаборатории ядерных проблем ОИЯИ.

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Bilenky S.M., Pontecorvo B. E2-85-589
Discussion of the Solar Neutrino Problem
in the Light of Reactor Neutrino Oscillation Experiments

A connection between results of solar neutrino experiments and reactor neutrino oscillation experiments is discussed. In the general case of n massive neutrinos a relation between the maximum and minimum values of the decrease coefficient of a solar neutrino flux and the amplitude of reactor antineutrino oscillations is obtained. The cases of small and large amplitudes are considered in detail.

The investigation has been performed at the Laboratory of Theoretical Physics and Laboratory of Nuclear Problems, JINR.

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