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EFFECTIVE CHIRAL LAGRANGIAN FROM LARGE N QCD

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1. Quantum chromodynamics provides a regular computational scheme for hadron physics in the domain of asymptotic freedom where the perturbation theory is applicable. In a low energy region such a regular scheme at present is absent. At the same time, it has been known for a long time that low energy hadron physics can be described by effective chiral Lagrangian giving a dynamical realization of current algebra [1,2,3].

There is a challenge to give a microscopic derivation of the chiral Lagrangian from QCD and to show in what precise sense it can be considered as a low energy approximation to chromodynamics. If this program is successful it would mean also that we succeeded to sew together the domain of asymptotic freedom and the low energy domain, both being described by the same fundamental QCD Lagrangian.

In this paper we show that the nonlinear chiral Lagrangian including the anomalous Wess-Zumino term [4] and the term responsible for the $\eta'-\pi^0$ mass splitting [5,6,7] can be derived as a low energy approximation to large N QCD if one assumes that the flavour chiral symmetry is spontaneously broken.

Meson fields π^a are described in this approach by the chiral phase of quark field. This phase happens to be the only important dynamical variable in the low energy region. All colour degrees of freedom can be explicitly integrated out resulting in the effective nonlinear chiral Lagrangian for the phases.

Technically it is achieved by the change of variables in the path integral over quark and gluon fields, which separate the degrees of freedom responsible for the spontaneous chiral symmetry breaking and corresponding to the pseudoscalar mesons. The Jacobian of this change produces the Wess-Zumino and Adler-Bell-Jackiw anomalous terms. After this change we perform explicit integration over all colour fields and derive an effective action for meson fields. Under the assumption of spontaneous chiral symmetry breaking the low energy limit of this action is described by the nonlinear chiral Lagrangian including the Wess-Zumino term and $\eta'-\pi^0$ mass splitting.

Our final result coincides therefore with the results of Balachandran et al. [8] and Witten [9]. We would like to stress however that contrary to these authors who constructed effective chiral Lagrangian using symmetry consideration, our paper presents a microscopic derivation of chiral dynamics from QCD. Apart from heuristic interest it allows in principle to express low energy parameters in terms of quark-gluon interaction. It needs however more elaborated technics and will not be discussed here.

Our paper is close in the spirit to the papers /10,11/. However these authors started in their derivation of chiral dynamics from some effective Lagrangian (like Nambu-Iona Lasinio model in /10/ or nonlinear Lagrangian including explicitly meson fields in /11/) assuming that it describes low energy QCD. On the contrary we start directly from the fundamental quark-gluon QCD Lagrangian and transform it identically in the framework of 1/N expansion scheme. The only ad hoc assumption (although a very important one) we made is the hypothesis of spontaneous chiral symmetry breaking.

2. We proceed from the standard Lagrangian of QCD, with SU(N) gauge group of colour symmetry and $U(n)\times U(n)$ of chiral symmetry

$$\mathcal{L}(x) = -\frac{N}{4g^2} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + i\bar\psi(x)\gamma_\mu(\partial_\mu - iA_\mu(x))\psi(x), \tag{1}$$

where the strengths $F^a_{\mu\nu}(x)$ are constructed from the vector gauge fields $A_\mu = A^\alpha_\mu T^\alpha$

Let us denote with t^a the matrices representing the generators of U(n) group with $\mathrm{tr}\, t^a t^b = 2\delta^{ab}$. The composite operators $\bar\psi(x)\gamma^5 t^a\psi(x)$ are related to mesons fields, so it is natural to consider the generating functional

$$Z(\eta) = Z_0^{-1}\int d\mu(A)\, d\bar\psi\, d\psi\, \exp\{i\int[\mathcal{L}(x) + \bar\psi(x)\gamma^5 t^a\psi(x)\eta^a(x)]dx\}, \tag{2}$$

where $d\mu(A)$ is a standard measure for the Yang-Mills field including gauge fixing and ghost terms.

To separate explicitly the chiral phase factor of quark fields it is convenient to follow the Faddeev-Popov procedure. For that purpose the integral (2) must be multiplied by "one", where

$$1 = \Delta(\bar\psi,\psi)\int\delta(F^a(\bar\psi^\Omega,\psi^\Omega))\,d\mu(\Omega), \tag{3}$$

$$\psi^\Omega = (\Omega^+ P_R + P_L)\psi, \quad \bar\psi^\Omega = \bar\psi(\Omega P_L + P_R). \tag{4}$$

Here $\Omega(x) = \exp\{i\pi(x)\}$, $\pi(x) = 2\pi^a t^a$, $P_{L,R} = \frac{1\pm\gamma^5}{2}$ and $F^a(\bar\psi,\psi)$ are colour invariant conditions imposed on the quark fields. For simplicity we choose them in the form $F^a = \bar\psi\gamma^5 t^a\psi$.

Introducing variables

$$\psi^\Omega \to \psi \tag{5}$$

we obtain

$$Z(\eta) = Z_0^{-1}\int d\mu(A)\, d\bar\psi\, d\psi\, d\mu(\Omega)\, J\, \Delta(\bar\psi,\psi) \times$$
$$\exp\{i\int[\mathcal{L}(x) + i\bar\psi(x)\gamma_\mu L_\mu(x) P_R\psi(x) + \varphi^a(x)\bar\psi(x)\gamma^5 t^a\psi(x) +$$
$$\bar\psi^{\Omega^{-1}}(x)\gamma^5 t^a\psi^{\Omega^{-1}}(x)\eta^a(x)]d^4x\}. \tag{6}$$

Here we used the exponential representation for the δ -function with the help of Lagrange multiplier φ^a , and J is the Jacobian. The Faddeev-Popov determinant can be rewritten, on the surface $\bar\psi\gamma^5 t^a\psi = 0$, in the form

$$\Delta(\bar\psi,\psi) = \int d\bar c\, dc\, \exp\{\tfrac{i}{2}\int[\bar\psi(x)\bar c^a(x)(\gamma^5[t^a,t^b] + [t^a,t^b]_+)c^b(x)\psi(x)]. \tag{7}$$

Here, as usual, $\bar c$ and c are anticommuting ghost fields.

Making use of the explicit form of the transformation (4) and δ -function, the source term can be represented in the form

$$-\frac{i}{2}\bar\psi[\pi, t^a]_+\psi\,\eta^a + \cdots \tag{8}$$

where ... denotes the terms of higher order of fields π^a, which give no contribution in leading order of the 1/N expansion.

Farther on, we will integrate over all colour fields in (6). For the functional $Z(\eta)$ we will obtain

$$Z(\eta) = \int d\mu(\Omega)\exp\{i\int\mathcal{L}_{eff}(\pi,\eta)d^4x\}, \tag{9}$$

where the effective Lagrangian $\mathcal{L}_{eff}$, which depends on the mesons fields only, will be computed in leading order of 1/N and in low energy limit.

3. To calculate the Jacobian of the transformation (5) we make the following observation. The difference between the Jacobians of transformation $\psi^\Omega \to \psi$, and the transformation $\psi^{\Omega'} \to \psi$, where $\Omega' = \Omega(1+\delta\Omega)$, may be presented in the form

$$\ln J(\Omega') - \ln J(\Omega) = \ln\det\hat{D}(\Omega) - \ln\det\hat{D}(\Omega'), \tag{10}$$

where

$$\hat{\mathcal{D}} = \hat{\partial} + \hat{L}P_R - i\hat{A} - i\varphi^a t^a \gamma^5 + \frac{1}{2}\bar{c}^a(\gamma^5[t^b,t^a] + [t^b,t^a]_+)c^b \tag{11}$$

Making use of the explicit form of the operator $\hat{\mathcal{D}}$ we get

$$\delta \ln J = -\delta \ln\det \hat{\mathcal{D}}(\Omega) = -i\int d^4x\, Tr\, \delta\Omega(x) \mathcal{A}(x) \quad . \tag{12}$$

The right-hand side of the eq. (12) is nothing but the well-known axial anomaly, which has been calculated in /12,13/.

Integrating the eq. (12) according to /4,14/ one obtains for the topologically nontrivial part

$$\ln J = i Z_{W-Z} + i\frac{\sqrt{2n}}{8\pi^2} tr \int F_{\mu\nu}(x) F_{\beta\sigma}(x) \varepsilon^{\mu\nu\beta\sigma} \pi^0(x) d^4x \quad , \tag{13}$$

where Z_{W-Z} is Wess-Zumino term

$$Z_{W-Z} = \frac{N}{48\pi^2}\int_0^1 d\tau \int d^4x\, \varepsilon^{\mu\nu\beta\sigma} Tr L_5(\tau) L_\mu(\tau) L_\nu(\tau) L_\beta(\tau) L_\sigma(\tau) . \tag{14}$$

$$L_\mu(x,\tau) = \Omega^{-1}(x,\tau)\partial_\mu \Omega(x,\tau) \, , \; L_5(x,\tau) = \Omega^{-1}(x,\tau)\partial_\tau \Omega(x,\tau) . \tag{15}$$

$$\Omega(x,\tau) = exp\{ i\tau \pi(x)\} .$$

The second term in eq. (13) is due to the Abelian anomaly, π^0 is the component of the multiplet π^a, corresponding to the generator $t^0 = 1\sqrt{\frac{2}{n}}$.

4. Let us consider again the representation (6) for the functional $Z[\eta]$. We remind the reader that our goal is to integrate out all the colour variables and obtain the effective action in terms of the fields π^a. For this purpose we integrate firstly over gluon fields. The gluons interact with the other fields via the terms $\bar\psi T^a \gamma_\mu \psi A^a_\mu$ and $\frac{\sqrt{2n}}{8\pi^2}\pi^0 tr F_{\mu\nu} F_{\beta\sigma} \varepsilon^{\mu\nu\beta\sigma}$. Therefore the integration over the gluon fields produces the factor

$$exp\{ i S_0(\bar\psi T^a \gamma^\mu \psi, \pi^0)\} \tag{16}$$

which is nothing but the generating functional for the Green functions of the fields $A^a_\mu(x)$, $q(x)$

$$q(x) = \frac{\sqrt{2n}}{8\pi^2} tr F_{\mu\nu}(x) F_{\beta\sigma}(x) \varepsilon^{\mu\nu\beta\sigma} \tag{17}$$

in pure gluodynamics.

The invariance properties of S_0 with respect to the colour rotations give us the information which allows to perform explicitly the integration over quark fields $\bar\psi$, ψ and to develop for the remaining integral the 1/N expansion resulting the leading order in the nonlinear chiral Lagrangian for the meson fields π^a.

The important property of the functional S_0 is that it depends in fact only on colour singlet bilocal combination of fields $\bar\psi^i_\varkappa(x)\psi^j_\varkappa(y)$, where $\varkappa$ is a colour index and i, j are flavour indices.

To establish this fact it is convenient to introduce a diagram technique indicating explicitly the flow of colour indices /15/. Because a gluon carries two colour indices $A^{\varkappa\varkappa'}_\mu(x) = A^a_\mu(x) T^a$, the gluon propagator is represented by two lines with different direction and an analogous representation holds for the ghost field propagator as well.

To see where the indices go it is enough to follow the arrows. The index lines flow continuously and they either close to make a loop, or beginning from some $\bar\psi_\varkappa(x)$ they end in some $\psi_{\varkappa'}(y)$. To every such line corresponds $\delta_{\varkappa\varkappa'}$. Summing over the colour indices will give N, for every index loop, and colour singlet combination of fermion fields $\bar\psi_\varkappa(x)\psi_\varkappa(y)$ in the other case. So that the $\bar\psi_\varkappa(x)$ and $\psi_{\varkappa'}(y)$ dependence of S_0 is only through singlet combinations.

For the sake of convenience we introduce a colour singlet bilocal collective coordinates /16,17/ by the relation

$$exp\{ i S_0\} = \int d\xi\, \delta(N\xi^{ij}(x,y) - \bar\psi^i_\varkappa(x)\psi^j_\varkappa(y)) exp\{ i V(\xi,\pi^0)\} \tag{18}$$

Substituting this expression into (6) and using again the exponential representation for the δ-function

$$\delta(N\xi^{ij}(x,y) - \bar\psi^i_\varkappa(x)\psi^j_\varkappa(y)) = \tag{19}$$

$$\int d\lambda\, exp\{ i\int d^4x\, d^4y\, \lambda^{ij}(x,y)(N\xi^{ij}(x,y) - \bar\psi^i_\varkappa(x)\psi^j_\varkappa(y))\} ,$$

we can integrate over the fermion fields $\bar\psi$, ψ. The result is

$$Z(\eta)=Z_0^{-1}\int d\mu(\Omega)\,d\xi\,d\lambda\,d\bar c\,dc\,d\varphi\,\exp\{i[V(\xi,\pi^0)+ N\,\mathrm{Tr}\,\lambda\xi+Z_{W\text{-}Z}+(-i)N\,\mathrm{Tr}\ln(i\gamma^\mu\partial_\mu\delta^{ij}+R^{ij})]\}, \tag{20}$$

where

$$R^{ij}=i\hat L_{ij}P_R+\varphi^a\gamma^5 t^a_{ij}-\lambda_{ij}+\frac{1}{2}\bar c^a(\gamma^5[t^b,t^a]+[t^b,t^a]_+)_{ij}\,c^b. \tag{21}$$

The functional $V(\xi,\pi^0)$ can be decomposed into two parts

$$V(\xi,\pi^0)=V(\xi,0)+V'(\xi,\pi^0), \tag{22}$$

where $V'(\xi,\pi^0)$ includes at least one source π^0.

We can draw up Feynman diagrams for any term in $V(\xi,\pi^0)$ just as in QCD. The only difference is that we have to associate with every fermion line $\delta_{xx'}\xi$, instead of the free fermion propagator.

As a result of this, to determine the N-dependence of $V(\xi,\pi^0)$ we have to follow the well-know considerations in QCD /15/. The leading order in N corresponds to pure gluonic planar diagrams and every quark loop introduces a factor N^{-1}. Therefore, the large N behaviour of $V'(\xi,\pi^0)$ is determined by the pure Yang-Mills theory and does not depend of ξ, $V'(\xi,\pi^0)=V_0'(\pi^0)+O(1/N)$.

The diagrams, corresponding to the potential $V(\xi,0)$, contain at least one fermion loop, so that the leading ones are of order N.

By this reason $V(\xi,0)$ may be expanded in term of N^{-1}

$$V(\xi,0)=N[V_0(\xi)+\frac{1}{N}V_1(\xi)+\dots]. \tag{23}$$

To integrate over the fields ξ, λ, $\bar c$, c and φ in the leading order of N, one must expand the action in exponent (20) around the stationary point. The last one is defined by the equations

$$\xi^{ij}_{st}(x-y)=-i\left(\frac{1}{i\hat\partial-\lambda_{st}}\right)^{ij}(x-y), \tag{24}$$

$$\lambda^{ij}_{st}(x-y)=\left(\frac{\delta V_0}{\delta \xi_{st}}\right)^{ij}(x-y), \tag{25}$$

$$\bar c^a=c^a=\varphi^a=\pi^a=0. \tag{26}$$

Since we have assumed that U(n) x U(n) symmetry is broken down to the diagonal U(n), the stationary solutions λ^{ij}_{st}, ξ^{ij}_{st} have a structure

$$\xi^{ij}_{st}=\delta^{ij}\xi_{st}\ ;\ \lambda^{ij}_{st}=\delta^{ij}\lambda_{st}. \tag{27}$$

We are not able to obtain explicit solution of the eq. (24)-(25) because we do not know the potential $V_0(\xi)$. However, for our purposes it is sufficient to assume that the Fourier transform of the function λ_{st}, $\tilde\lambda_{st}(P)$ at small momenta P is a constant different from zero $\tilde\lambda_{st}(P)\xrightarrow[P\to 0]{} C\neq 0$.

This assumption is the precise form of the hypothesis of spontaneous chiral symmetry breaking we use. As one can show, $\tilde\xi_{st}(P)$ is equal to the quark propagator to the leading order in N. Therefore, $\tilde\lambda_{st}(P)\xrightarrow[P\to 0]{} C\neq 0$ means that quarks acquire dynamical masses violating chiral symmetry.

We start with the calculation of quadratic in the currents L_μ terms as the most important in the low energy region.

In the vicinity of the stationary point the exponent in eq. (20) has the form

$$Z_{W\text{-}Z}+V_0'(\pi^0)+N\,\mathrm{Tr}\{\tfrac{1}{2}i\hat L P_R\Phi i\hat L P_R+\tfrac{1}{2}\varphi\gamma^5\Phi\gamma^5\varphi+\tfrac{1}{2}\lambda\Phi\lambda-\tfrac{1}{2}i\hat L P_R\Phi\lambda-\tfrac{1}{2}\lambda\Phi i\hat L P_R+\tfrac{1}{2}i\hat L P_R\Phi\gamma^5\varphi+\tfrac{1}{2}\varphi\gamma^5\Phi i\hat L P_R-\tfrac{1}{2}\lambda\Phi\gamma^5\varphi-\tfrac{1}{2}\varphi\gamma^5\Phi\lambda+\lambda\xi+\tfrac{1}{2}\xi K\xi\}+\dots, \tag{28}$$

where the coefficient functions of quadratic terms in the expansion of trln (...) and $V_0(\xi)$ are denoted as Φ and K respectively. We wrote explicitly only terms of the second order in variables φ, π, ξ, because all higher order terms, after integration over the fields φ, π, ξ will produce corrections of order O(1/N) and can be omitted in the limit $N \to \infty$. By the same reasons the source term is reduced to $-N\pi^a(x)\tau^a(x)\xi_{sr}(0)$. Up to normalization factor it is simply the source for the field π^a. It is important to mention, that, the only term in quadratic form with ghost fields $\bar{C}$ and C is the term of the form $\bar{C}$YC. Therefore, after integration over the fields $\bar{C}$ and C this gives no contribution in the leading order of N and we drop it.

Integrating over the fields ξ we obtain for the π - field quadratic form

$$-G^{-1} = [\Phi - K^{-1}].$$

Straightforward computations lead to the large-N Bethe- Salpeter equation for G. For simplicity reasons we will represent it only graphically

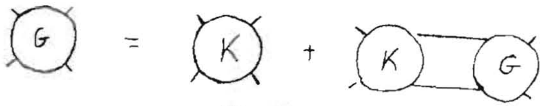

Fig.1

To every fermion line ———— corresponds $\xi_{sr}(x-y)$ (the quark propagator to the leading order in N).

The poles in the solution G(x,x';y,y') determine the fermion antifermion boundstates, so it is clear that the bilocal fields describe composite particles.

As the chiral symmetry is spontaneously broken, there exist massless boundstate associated with the pseudoscalar Goldstone boson. We assume that it is the only massless state in the theory and all singularities at small momenta are due to the exchange by the Goldstone bosons.

Performing the integration over the field π, we obtain for the new Lagrangian density the expression

$$\begin{aligned} &\mathcal{L}_{W-Z}(L) + V_0'(\pi^a) + \frac{N}{2}\mathrm{Tr}\{i\hat{L}P_R(\Phi + \Phi G\Phi)i\hat{L}P_R + \\ &\varphi\gamma^5(\Phi + \Phi G\Phi)\gamma^5\varphi + i\hat{L}P_R(\Phi + \Phi G\Phi)\gamma^5\varphi + \\ &\varphi\gamma^5(\Phi + \Phi G\Phi)i\hat{L}P_R\} + \cdots \end{aligned} \tag{29}$$

Let us introduce the designations

$$\begin{aligned} &\mathrm{Tr}\,\gamma^5 t^a(\Phi + \Phi G\Phi)\gamma^5 t^b = \Gamma^{ab}(x-y), \\ &\mathrm{Tr}\,i\gamma_\mu P_R t^a(\Phi + \Phi G\Phi)\gamma^5 t^b = \Gamma_\mu^{ab}(x-y), \\ &\mathrm{Tr}\,i\gamma_\mu P_R t^a(\Phi + \Phi G\Phi)i\gamma_\nu P_R t^b = \Gamma_{\mu\nu}^{ab}(x-y). \end{aligned} \tag{30}$$

Using the iterative solution of the Bethe-Salpeter equation, one can see that Γ^{ab} (x-y) is a sum of diagramms

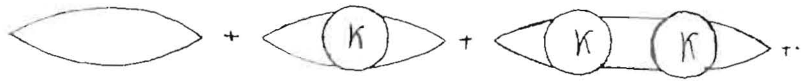

Fig.2

It is evident, that

$$N\Gamma^{ab}(x-y) = i\langle T\bar{\psi}(x)\gamma^5 t^a\psi(x)\;\bar{\psi}(y)\gamma^5 t^b\psi(y)\rangle_{(0)}, \tag{31}$$

where $\langle\dots\rangle_{(0)}$ denotes the leading order of 1/N.

In analogy, we obtain for the other functions

$$\begin{aligned} &N\Gamma_\mu^{ab}(x-y) = i\langle T\bar{\psi}(x)i\gamma_\mu P_R t^a\psi(x)\;\bar{\psi}(y)\gamma^5 t^b\psi(y)\rangle_{(0)} \\ &N\Gamma_{\mu\nu}^{ab}(x-y) = i\langle T\bar{\psi}(x)i\gamma_\mu P_R t^a\psi(x)\;\bar{\psi}(y)i\gamma_\nu P_R t^b\psi(y)\rangle_{(0)}, \end{aligned} \tag{32}$$

Due to the existence of the Goldstone pole in the Bethe-Salpeter amplitudes the functions Γ also have pole singularities. Therefore, the small momenta behaviour of the Γ functions has the form

$$\begin{aligned} &\int d^4x\, e^{ipx}\Gamma_{\mu\nu}^{ab}(x) = \frac{f^2}{4}\delta^{ab}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) + O(p) \\ &\int d^4x\, e^{ipx}\Gamma_\mu^{ab}(x) = i\frac{Bf}{2}\delta^{ab}\frac{p_\mu}{p^2} \\ &\int d^4x\, e^{ipx}\Gamma^{ab}(x) = -B^2\delta^{ab}\frac{1}{p^2} + O(p). \end{aligned} \tag{33}$$

To receive the final expression for the effective Lagrangian it remains to integrate over the fields φ. The result is

$$\mathcal{L}'_{eff} = \mathcal{L}_{W-Z}(L) + V'_0(\pi^0) + \frac{N}{2} L^\mu_a \Gamma^{ab}_{\mu\nu} L^\nu_b - \frac{N}{2} L^a_\mu \Gamma^\mu_{ac} (\Gamma^{-1})_{cc'} \Gamma^\nu_{c'b} L^b_\nu . \tag{34}$$

From the small momenta behaviour of Γ functions (33) we obtain

$$\mathcal{L}_{eff} = -\frac{F_\pi^2}{16} Tr L_\mu L_\mu + V'_0(\pi^0) - \frac{N}{48\pi^2} \int_0^1 d\tau \, \varepsilon^{\mu\nu\rho\sigma} Tr L_5(\tau) L_\mu(\tau) L_\nu(\tau) L_\rho(\tau) L_\sigma(\tau) , \tag{35}$$

where $F_\pi^2 = N f^2$, F_π - is a meson decay constant (F_π = 190 MeV).

We must mention, that because of the dynamically generated quark mass, the coefficient functions in the expansion of the functional trln (...) have a smooth properties near the zero momenta. Therefore, there is no possibility to obtain a term of Wess-Zumino type from this functional. The only possible terms are those of higher order of the momenta. But they are droped in low energy approximation.

Let us make few comments concerning the eq. (35). We kept in this formula only the terms quadratic in L because they are leading at small momenta. However, it is true only if the coefficient functions in the expansion of the effective action with respect to L are nonsingular at zero momenta. This is not obvious because the Goldstone bosons do generate singularities at small momenta. Above we showed explicitly that in the coefficient function of the quartic in L term these poles cancel. We emphasize that this cancelation arises after the integration over the field φ, i.e., is due to the explicit account of the constraint on the fields (see eq. (3) and eq. (6)).

Now we shall give the arguments showing that analogous cancelation occurs also in the higher order terms.

If we are interested in the higher order terms, we cannot use any more the eq. (28) and should take into account next terms in the expansion near stationary points (24-27). In particular the terms depending on ξ have a form

$$N\{\tfrac{1}{2}\xi K \xi + \sum_{n\geq 3} K_n \xi^n + \lambda \xi\} . \tag{36}$$

Integrating over ξ in the limit N $\to \infty$ we get

$$\exp\{i\mathcal{L}_{eff}\} = \int d\lambda\, d\varphi \exp\{i\int[\mathcal{L}_{W-Z}(L) + V'_0(\pi^0) + N\, Tr\{-\tfrac{1}{2}\lambda G^{-1}\lambda + \sum_{n\geq 3}(-1)^n K_n (K^{-1}\lambda)^n + \tfrac{1}{2}\varphi\gamma^5 \Phi \gamma^5 \varphi - \tfrac{1}{2}\lambda\Phi\gamma^5\varphi - \tfrac{1}{2}\varphi\gamma^5\Phi\lambda + \mathcal{F}(\lambda+\varphi, L)]]\} \tag{37}$$

In this formula we wrote explicitly the terms quadratic in λ and φ in the expansion of the trln (...) , and the remaining terms were denoted by $\mathcal{F}(\varphi+\lambda, L)$. The propagator of the field λ is the Bethe-Salpeter amplitude G, which has a pole corresponding to the pseudoscalar Goldstone boson. Therefore we can separate the term responsible for this pole and replace the integration over λ (x,y) by the integration over the local fields λ^a(x) with propagator $\delta^{ab}\, 1/k^2$, and the bilocal field $\tilde{\lambda}$ (x,y) with the propagator regular at zero momenta. This corresponds to the following change:

$$-\tfrac{1}{2}\lambda(x,y)\, G^{-1}(x,y;x',y')\lambda(x',y') \to -\tfrac{1}{2}\lambda^a(x)\Box\lambda^a(x) - \tfrac{1}{2}\tilde{\lambda}(x,y)\tilde{G}^{-1}(x,y;x',y')\tilde{\lambda}(x',y') \tag{38}$$

and

$$\lambda^a(x,y) \to \delta(x-y)\gamma^5\lambda^a(x) + \tilde{\lambda}^a(x,y) \tag{39}$$

in the remaining terms

Now we can perform a change of variables

$$\varphi^a(x) \to \varphi^a(x) - \lambda^a(x) \tag{40}$$

eliminating λ^a dependence of $\mathcal{F}$.

The result looks as follows

$$\exp\{i\mathcal{L}_{eff}\} = \int d\lambda^a d\tilde{\lambda} d\varphi \exp\{i\int[\mathcal{L}_{W-Z}(L) + V'_0(\pi^0) + N[-\tfrac{1}{2}\lambda^a(\Box\delta^{ab} + Tr\, t^a\gamma^5\Phi t^b\gamma^5)\lambda^b - \tag{41}$$

$$-\mathrm{Tr}\Big[\tfrac{1}{2}\tilde{\lambda}\tilde{G}^{-1}\tilde{\lambda}+\sum_{n\geq 3}(-1)^n K_n\left[K^{-1}(\lambda^a\gamma^5 t^a+\tilde{\lambda})\right]^n + \qquad (41)$$

$$\varphi\gamma^5\Phi\gamma^5\varphi\Big]+\mathcal{F}(\varphi,L)\Big)\Big]\Big\}.$$

This formula shows that the integral over λ^a, which produced before the pole singularities at zero momenta, after the change (40) does not produce singularities anymore, because

$$\mathrm{Tr}\,\gamma^5 t^a\Phi\gamma^5 t^b\big|_{p^2=0}\sim\delta^{ab}B\neq 0 \qquad (42)$$

where the constant B was defined in the eq's (30,33).

Therefore, we see that one-particle singularities at zero momenta disappear from the effective action (41).

That means that the coefficient functions may be expanded near the origin and the higher order terms are small in comparison with the quadratic term.

5. Let us consider the functional $V'(\pi^0)$. It is generating functional for connected Green's functions

$$\langle T\, q(x_1)q(x_2)\cdots q(x_n)\rangle_{(c)}$$

in leading order of 1/N . Its expansion in power of π^0 has the form

$$V_0'(\pi^0)=\frac{1}{2}\int\pi^0(x)\langle T q(x)q(y)\rangle_{(c)}\pi^0(y)\,d^4x\,d^4y+\cdots \qquad (43)$$

If we assume, that

$$\int d^4x\,\langle T q(x)q(0)\rangle_{(c)}=a\neq 0,$$

then, from (43) follows, that π^0 particle (η' -meson) receives mass

$$m_{\eta'}^2=\frac{1}{4F_\pi^2}a=\frac{4n}{F_\pi^2}\left(\frac{1}{16\pi^2}\right)^2\int d^4x\,\langle \mathrm{tr}\,\tilde{F}(x)F(x)\;\mathrm{tr}\,\tilde{F}(0)F(0)\rangle_{(c)} \qquad (44)$$

which coincides with Witten's expression [5](see also /11/).

The constant of π meson decay is proportional to $N^{1/2}$ and $\langle \mathrm{TrF}^*\mathrm{F}\ \ \mathrm{TrF}^*\mathrm{F}\rangle_{(c)}\sim N^0$, so $m_{\eta'}^2\sim 1/N$. Hence, the η' -meson mass generation (the restoration of U(1) symmetry) is not in contradiction with the assumption of $U(n)\times U(n)$ symmetry breaking in leading order of N.

6. So we succed to show that the low energy chiral Lagrangian can be obtain from the fundamental QCD Lagrangian by the direct integration of colour degrees of freedom in the limit $N\to\infty$ provided the chiral symmetry is spontaneously broken.

We consider here only quadratic term in the effective action which defines the low energy π -dynamics. One may be interested in the calculation of next order terms, in particular the Skyrme term $\mathrm{tr}\,[L,L]^2$. This term present a special interest because the chiral Lagrangian including the Skyrme term admits stable solutions, which may be interpreted as baryons /8,9/ . In our approach such terms will arise automatically as next order terms generated by the effective action (41). In principle the microscopic derivation proposed here allows to calculate any term in the effective Lagrangian and to express phenomenological parameters like F_π , the Skyrme constant, etc., in terms of fundamental quark-gluon parameters.

However to do it in practice one must know the functional $V_0(\xi)$. Precise determination of this functional requires the exact summation of planar diagrams in gluodynamics, and is at the moment beyond our possibilities.

In the case of the leading quadratic term we succeded to avoid these problems by approximation of the exact Bethe-Salpeter amplitude by the pole term. It allowed us to construct the effective Lagrangian in terms of low energy parameters like F_π .

It is worthwhile to emphasize that to calculate the Skyrme term one needs to know not only singular terms in the Bethe-Salpeter amplitude, but also regular ones and the pole approximation does not work in this case. At present to make some definite predictions concerning this term one needs some additional model assumption.

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Карчев Н.И., Славнов А.А. E2-85-553

Эффективный киральный лагранжиан из $SU(\infty)$ КХД

Нелинейный киральный лагранжиан, включающий аномальное взаимодействие Весса - Зумино и массовый член, описывающий расщепление синглетного и несинглетного мезонов получен как низкоэнергетическое КХД при больших N. С этой целью делается замена переменных в континуальном интеграле, отделяющая степени свободы, ответственные за нарушение киральной симметрии и соответствующие псевдоскалярным мезонам.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Karchev N.I., Slavnov A.A. E2-85-553

Effective Chiral Lagrangian from Large N QCD

The nonlinear chiral Lagrangian with the anomalous Wess-Zumino interaction and mass term that describes the splitting of the singlet meson from the non-singlets is derived in large N approximation of QCD. For that purpose a change of variables in functional integral is made separating the degrees of freedom responsible for a breakdown of chiral symmetry and corresponding to the pseudoscalar mesons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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