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V.A.Bednyakov, S.G.Kovalenko

ON THE SINGULAR COMPONENT OF THE GLUON DISTRIBUTION FUNCTION

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INTRODUCTION

The notion "vacuum condensate" has now found its place in the modern theory of elementary particles. It has lost its quantum statistical sense and is identified as vacuum mean values of the product of quark (q) and gluon (G) fields (mainly of the form $<q\bar{q}>$ and $<GG>$).

Recent progress of the hadron phenomenology has been stimulated by finding out a close relation of the quark $<q\bar{q}>$ and gluon $<G^2>$ condensates with very important low-energy properties of hadrons (mass, width, etc.)[1].

In this paper we will not deal with the problem of vacuum condensate in that context. We shall consider some consequences of the hypothesis according to which the hadron A has the singular component of the gluon distribution function (DF) of the form $\delta(X)/X$ which can be also called a "condensate" in its original statistical sense[2]. Here X is the fraction of the longitudinal momentum P of the hadron A within the infinite momentum frame $P \to \infty$ (IMF).

If one takes into account that dX/X is a one-particle phase volume within IMF, one usually interprets that component as the gluon bose-condensate in the hadron A, which is natural from the point of view of statistical physics. It is not clear at all how the singular gluon component is related to the vacuum mean $<G^2>$ - the gluon vacuum condensate playing such an important role in low energy physics of hadrons. However, consideration of consequences of the hypothesis on existence of this exotic gluon configuration in hadrons is to our mind of independent interest. The more so, it is the singular function $\delta(X)/X$ that becomes the limit of DF at $Q^2 \to \infty$ in quantum chromodynamics[3].

It should be also stressed that all manifestations of the singular gluon component take plase in the high-energy region, since this is the region where DF and their statistical interpretation have sense.

It is important that the physical consequences of this hypothesis can be checked experimentally. The most specific characteristic consequences are the change of the threshold behaviour of the quark and gluon DF at large Q^2 , the apparent violation of the parton momentum conservation law at its experimental check, appearance of additional power corrections to structure functions.

The statistical parton model and the generating functional formalism are used for the consideration. This approach is shortly described in sec.1.

Sec.2 deals with the DF calculation procedure with allowance for the singular gluon component. In Sec.3 the results obtained are dissensed. Possible variants are considered for Q^2-dependence of the contribution of the singular component of the gluon DF.

1. STATISTICAL PARTON MODEL AND FORMALISM OF GENERATING FUNCTIONALS

We shall briefly present the basic statement of the approach to calculation of one- and multiparticle DF of quarks and gluons in the hadron A, developed by us [4].

Let the limit behaviour of DF be given:

$$\bar{f}_i(X_1, \dots, X_N) = \lim_{X_j \to 0} f_i(X_1, \dots, X_N).$$

One can construct a correct extrapolation procedure $\hat{L}_i$ which takes into account some most characteristics properties of the hadron structure [4], so that the functions

$$f_i(X_1, \dots, X_N) = \hat{L}_i(\bar{f}\,|\,X_1, \dots, X_N)$$

will be well-defined in the whole physical region of variables $\{X_i\}$.

Thus, to find DF one should assign a form of the limit functions $\bar{f}_i(X)$.

Some information on limit functions is provided by the Regge analysis performed for the amplitude of the virtual Compton effect on the nucleon. Hence follow the formulae:

$$\bar{f}_v(X) = \bar{f}_v^{\,j}(X) = a(X)X^{-\alpha}, \quad \bar{f}_s(X) = \bar{f}_s^{\,j}(X) = b(X)X^{-1}/2f, \quad \bar{f}_g(X) = \bar{g}(X)X^{-1}. \qquad (1)$$

Here $j = 1,2,\dots,2f$, f is the number of active quark flavours; $a(X)$, $b(X)$ and $\bar{g}(X)$ are some functions regular at the point $X = 0$ with $a(0) = a > 0$, $b(0) = b > 0$ and $\bar{g}(0) = \bar{g} > 0$.

Finally, the N-particle DF to be found are described by the formula

$$f^A_{m,k,\ell}(X_1^v, \dots, X_m^v, \dots, X_1^g, \dots, X_\ell^g) = \bar{f}^A_{m,k,\ell}(X_1^v, \dots, X_m^v, \dots, X_1^g, \dots, X_\ell^g)$$

$$\frac{1}{W_A[\bar{f}\,|\,1]} \cdot \frac{\delta^N W_A[\bar{f}\,|\,1]}{\delta\bar{f}_v(X_1^v)\dots\delta\bar{f}_v(X_m^v)\dots\delta\bar{f}_g(X_1^g)\dots\delta\bar{f}_g(X_\ell^g)}, \qquad (2)$$

where

$$\bar{f}^A_{m,k,\ell}(X_1^v, \dots, X_m^v, X_1^s, \dots, X_k^s, X_1^g, \dots, X_\ell^g) =$$

$$= \frac{1}{m!}\prod_{i=1}^{m}\bar{f}_v(X_i^v)\,\frac{1}{k!}\prod_{i=1}^{k}\bar{f}_s(X_i^s)\,\frac{1}{\ell!}\prod_{i=1}^{\ell}\bar{f}_g(X_i^g)$$

are the limit multiparticle distributions; m, k, ℓ are the number of valence, see quarks and gluons in the N-particle DF ($N = m + k + \ell$).

Further we shall confine ourselves to consideration of only one-particle DF. Generalization to multiparticle DF is not difficult.

The regularised generating functional W_A has the form [4]:

$$W_A[\bar{f}\,|\,a] = \int_{-\infty}^{+\infty} d\xi\, e^{i\xi a}\, D_v^n(\xi)(\xi - io)^{-b-g}$$

$$\exp\{D_g(\xi) + D_s(\xi)\}\,\theta(a), \qquad (3)$$

where n is the number of valence quarks in the hadron A (for the nucleon $n = 3$).

$$D_v(\xi) = \int_0^\infty dt e^{-i(\xi - io)}\,\bar{f}_v(t),$$

$$D_s(\xi) = \int_0^\infty dt e^{-i(\xi - io)t}\left(\bar{f}_s(t) - \frac{b}{2ft}\right), \qquad (4)$$

$$D_g(\xi) = \int_0^\infty dt e^{-i(\xi - io)t}\left(\bar{f}_g(t) - \frac{g}{t}\right)$$

are the regularised Laplace transform of the limit functions $\bar{f}_i(X)$.

Further we shall assume that the limit gluon function $\bar{f}_g(X)$ contains the singular δ-like contribution of the condensate type:

$$\bar{f}^k_g(X) = \bar{f}_g(X) + K\frac{\delta(X)}{X} \qquad (5)$$

and, obtain the DF of quarks and gluons in the hadron A.

The quantity K has a simple physical sence: this is the part of the hadron A longitudinal momentum which is due to the singular gluon component, therefor $0 \le K \le 1$.

2. GENERATING FUNCTIONAL AND SINGULAR GLUON COMPONENT

In this section we shall find the DF $f_i^K(X)$ taking into account the singular gluon component (the gluon bose-condensate). For this purpose we shall generalise the formalism described in Sec.1 to the case of singular limit functions $\bar{f}_g^K(X)$ (5).

Indeed, for $K \neq 0$ uncertain quantities like $\int dX \frac{\delta(X)}{X} \bar{f}_i(X)$ appear in functional (3) owing to the addend of (5). To eliminate those uncertainties we shall employ the following standard method. We shall regularise the δ-function by one of the known methods. The exponential regularisation

$$\delta(X) = \lim_{\eta \to \infty} \eta e^{-\eta X} \quad (6)$$

is most suitable.

We shall take the DF of quarks and gluons in the hadron in the following limit sense:

$$f_{v,s}^K(X) = \bar{f}_{v,s}(X) \lim_{\eta \to \infty} \frac{\delta \ln W_\eta[\bar{f}|1]}{\delta \bar{f}_{v,s}(X)},$$

$$f_g^K(X) = \bar{f}_g^K(X) \lim_{\eta \to \infty} \frac{\delta \ln W_\eta[\bar{f}|1]}{\delta \bar{f}_g(X)}. \quad (7)$$

The regularised generating functional W_η has the form

$$W_\eta[\bar{f}|a] = \int_{-\infty}^{+\infty} d\xi e^{i\xi a} D_v^n(\xi)(\xi - io)^{-b-g}$$

$$\exp\{D_g(\xi) + D_s(\xi)\}(\xi - i\eta)^{-K\eta} \theta(a). \quad (8)$$

Taking into consideration the relation $\lim_{\eta \to \infty} (\frac{\xi}{i\eta} - 1)^{-K\eta} = e^{-iK\xi}$ and the property of functionals D_i:

$$\frac{\delta D_i(\xi)}{\delta \bar{f}_j(X)} = \delta_{ij} e^{i\xi X} \quad (9)$$

we obtain general relations for DF

$$f_v^K(X) = n\bar{f}_v(X) \frac{H_{n-1}(X+K)}{H_n(K)} \theta(1 - X - K)$$

$$\begin{Bmatrix} f_s^K(X) \\ f_g^K(X) \end{Bmatrix} = \begin{Bmatrix} \bar{f}_s(X) \\ \bar{f}_g^K(X) \end{Bmatrix} \frac{H_n(X+K)}{H_n(K)} \theta(1 - X - K). \quad (10)$$

Here

$$H_n(a) = \int_0^1 dt\, t^{g+b-1} \mathcal{P}_n(a+t)\theta(1-a-t),$$

$$\mathcal{P}_n(\beta) = \int_{-\infty}^{+\infty} d\xi e^{i\xi(1-\beta)} D_v^n(\xi) \exp\{D_s(\xi) + D_g(\xi)\}. \quad (11)$$

Noteworthy is an important property of functions (10). They become zero at $X = 1 - K < 1$, and not at $X = 1$. Consequences of this are discussed in Sec.3.

Let, for example,

$$\bar{f}_v(X) = X^{-\alpha}, \quad \bar{f}_4(X) = \frac{b}{X}, \quad \bar{f}_g^K(X) = (ge^{-\beta X} + K\delta(X))/X. \quad (12)$$

We obtain then:

$$D_s(\xi) = 0, \quad D_v(\xi) = \Gamma(1+\alpha)(i\xi)^{\alpha-1},$$

$$D_g(\xi) = g\int_0^\infty \frac{dt}{t} e^{-i(\xi - io)t}(e^{-\beta t} - 1) = g \ln \frac{\xi - io}{\xi - i\beta},$$

$$\mathcal{P}_n(y) = \int_{-\infty}^{+\infty} d\xi e^{i\xi(1-y)} \left[\frac{\Gamma(1-\alpha)}{(i\xi)^{1-\alpha}}\right]^n \cdot \left(\frac{\xi - io}{\xi - i\beta}\right)^g, \quad (13)$$

$$H_n(y) = (\Gamma(1-\alpha))^n \int_{-\infty}^{+\infty} d\xi e^{i\xi(1-y)} \frac{(i\xi + \beta)^g}{(i\xi)^{n(1-\alpha)+b}} =$$

$$= 2\pi \frac{(\Gamma(1-\alpha))^{n}(1-y)^{n(1-\alpha)+b+g-1}}{\Gamma(n(1-\alpha)+b+g)} \phi(g, n(1-\alpha)+b+g; -\beta(1-y)),$$

where ϕ (a,b,z) is the degenerated hypergeometric function.
Substituting $H_n(y)$ into (10) we obtain the final result:

$$f_v^K(X) = X^{-\alpha} \frac{(1-X-K)^{r}\phi(g, r+1, -\beta(1-X-K))\,\theta(1-K-X)}{B(1-\alpha, r+1)(1-K)^{r+1-\alpha}\;\phi(g, r+2-\alpha; -\beta(1-K))},$$

$$f_s^K(X) = \frac{b}{X}\frac{(1-X-K)^{r+1-\alpha}}{(1-K)^{r+1-\alpha}} \frac{\phi(g, r+2-\alpha; -\beta(1-K-X))\theta(1-K-X)}{\phi(g, r+2-\alpha; -\beta(1-K))}, \tag{14}$$

$$f_g^K(X) = \left(\frac{g}{X}e^{-\beta X} + \frac{K}{X}\delta(X)\right)\frac{(1-K-X)^{r+1-\alpha}}{(1-K)^{r+1-\alpha}},$$

$$\frac{\phi(g, r+2-\alpha; -\beta(1-K-X))}{\phi(g, r+2-\alpha; -\beta(1-K))}\;\theta(1-K-X),$$

where $f_v^K(X)$ is normalised to 1, and $r = (n-1)(1-\alpha)+b+g-1$.
The free parameters r, β, g, K can be fixed by comparison with the experimental data. The parameter $\alpha = \alpha(0)$ is the intercept of the leading non-singlet trajectory (usually $\alpha(0) = 1/2$). If one takes $\beta = 0$, expressions (14) coincide in the form with the known Buras-Gaemers parametrisations[5], modified with the condensate type gluon configuration.
Here are the expressions for the moments of the DF $f_i^K(X)$:

$$<f_v^K>_n \equiv \int_0^1 dX \cdot X^{n-1} f_v^K(X) = <f_v>_n^{(K)} (1-K)^{n-1},$$

$$<f_s^K>_n = <f_s>_n^{(K)} (1-K)^{n-1}, \tag{15}$$

$$<f_g^K>_n = <f_g>_n^{(K)} (1-K)^{n-1} + K(\delta_{n2} + \delta_{n1} \int_0^1 \frac{dX\,\delta(X)}{X}).$$

Here

$$<f_v>_n^{(K)} = \frac{B(r+1, n-\alpha)}{B(r+1, 1-\alpha)}\cdot\frac{\phi(g, r+1-\alpha+n; -\beta(1-K))}{\phi(g, r+2-\alpha; -\beta(1-K))}, \tag{16}$$

$$<f_s>_n^{(K)} = \frac{r-g}{8} B(r+2-\alpha, n-1)\frac{\phi(g, r+1-\alpha+n; -\beta(1-K))}{\phi(g, r+2-\alpha; -\beta(1-K))},$$

$$<f_g>_n^{(K)} = gB(r+2-\alpha, n-1)\frac{\phi(g+n-1, r+1-\alpha+n; -\beta(1-K))}{\phi(g, r+2-\alpha; -\beta(1-K))}. \tag{16}$$

It is evident that for $K = 0$ expressions (14) and (16) lead to usual DF $f_i(X)$ and their moments, obtained by us earlier[6].

3. ON INTERPRETATION OF THE SINGULAR GLUON COMPONENT

As is shown in ref.[6], the logarithmic Q^2-dependence of DF due to the QCD perturbation theory[7] can be introduced in the formulae like (14) and (16) by means of parameters r, β and g (e.g., in the form $r = r^{(0)} + r^{(1)}\cdot S$, etc., where $S = \ln\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}$ - is the standard evolution variable). In this case the parameter K may remain constant or be in power dependence on Q^2.

Here we consider two possible variants:
a) $K \to 0$ at $Q^2 \to \infty$, b) $K \to K_0 \neq 0$ at $Q^2 \to \infty$.
In limit case a) we parametrise the Q^2 dependence of K in the simplest way:

$$K(Q^2) = K_0/Q^2, \tag{17}$$

where K_0 is the constant.
Expanding the binomial $(1-K)^{n-1}$ in (16) in a power series of K, taking into account (17) and the parton links of distribution functions (14) with the structure functions

$$<F_2^K>_n = \int_0^\infty dX X^{n-2} F_2^K(X, Q^2) = <f_v^K>_n + \frac{20}{9}<f_s^K>_n \tag{18}$$

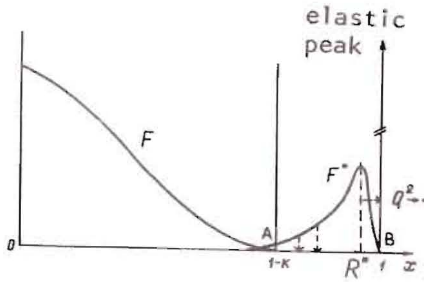
we obtain the expansion in inverse power series of Q^2 for the moments of the structure function F_2:

$$<F_2^K>_n = <F_2>_n^{(0)} + \frac{K_0}{Q^2}(n-1)<F_2>_n^{(0)} + \frac{K_0^2}{Q^4}\frac{(n-1)(n-2)}{2}<F_2>_n^{(0)} + \dots \tag{19}$$

So the singular component of the gluon DF can be a source of additional power corrections ($1/Q^2$ - corrections) to the structure functions.

We also note that the first power correction $-\frac{K_0(n-1)}{Q^2}\langle F_2\rangle_n^{(0)}$ - exactly corresponds to the result obtained within QCD in the soft gluon approximation[8]. This coincidence is probably due to the fact that the gluon component, being singular for $X = 0$, corresponds to the limit case of the soft gluon approximation when each gluon has a zero share of the hadron A momentum.

In the limit case b), when $K \to K_0$ at $Q^2 \to \infty$, there is a peculiar phenomenon - a reduction of the physical region for DF determination. Usually ($K = 0$) DF are determined in the interval $0 \le X \le 1$; introduction of the singular component makes it as narrow as $0 < X \le 1 - K < 1$.



Here we note that the experimentally measured structure functions are presented as a sum of F and F*. F is related to calculated DF (14) through the parton model formulae, and F* corresponds to the contribution of various power and resonance effects. Therefore the reduction of the DF determination region may manifest itself in the experiment only at sufficiently large energies when the contribution from the resonance component of structure functions is suppressed. The situation may be like that in the figure. In the interval AB one can see the contribution of the resonance R* which goes to the boundary of the physical region ($X = 1$) and vanishes at $Q^2 \to \infty$. As a result, the interval AB turns into an analogue of the energy gap: $F = 0$ in it.

A more indirect manifestation of the singular gluon component is as follows.

When one determines independently the total momentum of quarks $\langle X_q\rangle$ and - separately - of gluons $\langle X_g\rangle$, one may "find" violation of the parton momentum conservation law: $\langle X_q\rangle + \langle X_g\rangle < 1$. This could be an indication of a singular component which cannot be directly observed. It would carry the missing hadron momentum $K = 1 - \langle X_q\rangle - \langle X_g\rangle$. We note that in this approach one can try to find manifestation of the singular gluon component at finite values of Q^2, i.e. both in mode a) and b).

At present, however, the momentum share of all gluons $\langle X_g\rangle$ is determined just on the basis of the momentum conservation law; this does not allow one to detect the indicated "effect". A possibility to independently measure gluon distributions will be probably provided by a detailed study of heavy quark and massive lepton pair production in lepton-hadron interactions.

An indication of an extremally strong growth of the gluon DF in the $X \to 0$ region[9] can be considered as possible signal from the condensate type gluon configuration. From the experimental point of view this growth agrees with presence of δ-like configuration of the gluon spectrum at $X = 0$.

CONCLUSION

Some consequences of the hypothesis on existence of the singular $\sim K\delta(X)/X$ component of the gluon distribution function are considered.

Distribution functions of quarks and gluons in the hadron for the fixed value $Q^2 = Q_0^2$ have been found within the framework of the statistical parton model with allowance for the adopted hypothesis. At $K = 0$ those functions turn into usual normalised distribution functions that we obtained and used earlier[5].

Two limit cases with a physical interpretation have been studied under the assumption of the Q^2-dependence of K.

1. ($K \to 0$ at $Q^2 \to \infty$). An inverse power series of Q^2 (when $K = K_0/Q^2$) has been obtained for moments of distribution functions (structure functions).

2. ($K \to K_0 \neq 0$; 1 at $Q^2 \to \infty$). Here the determination region for structure functions is reduced and an analogue of the energy gap appears ($1 - K_0 < X < 1$). This situation can in principle be detected experimentally.

At finite values of Q^2 the contribution of the singular component can be registered through a "apparent" violation of the parton momentum conservation law $\langle X_q\rangle + \langle X_g\rangle = 1 - K < 1$, where $\langle X_q\rangle$ and $\langle X_g\rangle$ are total momenta of quarks and gluons extracted independently from the data.

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Бедняков В.А., Коваленко С.Г. E2-85-485
О сингулярной компоненте глюонной функции распределения

На основе статистической партонной модели получены функции распределения кварков и глюонов в адроне с учетом гипотетической сингулярной компоненты глюонной функции распределения. Наличие такой компоненты, интерпретируемой как статистический глюонный бозе-конденсат, может быть, в принципе, экспериментально обнаружено. Наиболее характерные проявления сингулярной компоненты следующие: изменение порогового поведения функций распределения кварков и глюонов в области больших переданных импульсов, кажущееся нарушение закона сохранения импульса партонов при его экспериментальной проверке, возникновение дополнительных степенных поправок в структурных функциях.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Bednyakov V.A., Kovalenko S.G. E2-85-485
On the Singular Component of the Gluon Distribution Function

Distribution functions of quarks and gluons have been found within the framework of the statistical parton model with allowance for the existence of the singular component of the gluon distribution function into hadron. The existence of such component, which interprets as statistical gluon bose-condensate, can be checked experimentally. The most characteristic consequences of the singular gluon component are the change of the threshold behaviour of the quark and gluon distribution functions at large Q^2, the apparent violation of the parton momentum conservation law at its experimental check, appearance of additional power corrections to structure functions.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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