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THE CALCULATION OF THE K°-$\bar{K}^\circ$ MATRIX ELEMENT IN THE QUARK LOOPS MODEL (THE QL-MODEL)

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1. Introduction

At the last time there are many theoretical evalutions of the $K^0-\bar{K}^0$ matrix element $\langle\bar{K}^0|\hat{O}_{|\Delta S|=2}|K^0\rangle$ where

$$\hat{O}_{|\Delta S|=2}=[\bar{s}\gamma^{\mu}(1-\gamma^5)d][\bar{s}\gamma_{\mu}(1-\gamma^5)d] \tag{1}$$

is the four-quark operator changing the strangeness by two units[1,2]. The point is that the knowledge of the $K^0-\bar{K}^0$ matrix element value allows in particular to get the information on the t-quark mass value from comparison of experimental data and theoretical evalutions of the K_L-K_S mass difference obtained in the framework of the Kobayashi-Maskawa variant[3] of the Glashow-Weinberg-Salam electroweak model[4]. The K_L-K_S mass difference is proportional to $\langle\bar{K}^0|\hat{O}_{|\Delta S|=2}|K^0\rangle$ but the coefficient of the proportionality is the known function of the t-quark mass m_t [5]. The m_t numerical value essentially depends on the $K^0-\bar{K}^0$ matrix element value[6].

For the calculation of $\langle\bar{K}^0|\hat{O}_{|\Delta S|=2}|K^0\rangle$ it is necessary to know the dynamics of low-energy strong interaction. At present time it is possible only the model investigation of the $K^0-\bar{K}^0$ matrix element. The numerical values of $\langle\bar{K}^0|\hat{O}_{|\Delta S|=2}|K^0\rangle$ obtained in different models are contradictory so far[1]. Obviously the ambiguity of the computation can be explained by the imperfection of the model calculation approaches.

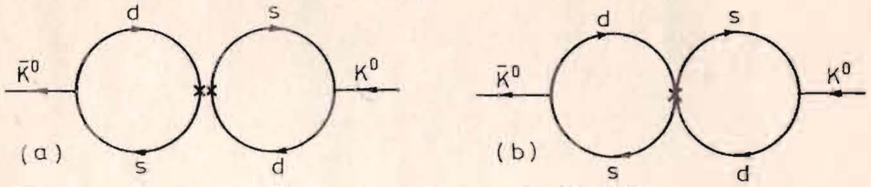
In the present note the $K^0-\bar{K}^0$ matrix element is calculated in the quark loops model (the QL-model). The QL-model unites two quark models, viz. the superconductor quark model[7] and the model of the dominance of quark loop anomalies (the DQLA-model)[8]. Both models supplement each other and describe the dynamics of hadronic low-energy interaction pretty well. Thus the QL-model result of the $K^0-\bar{K}^0$ matrix element computation does seem quite reliable.

The note is organized in the following way. In Sec.2 the matrix element is calculated in the QL-model. In Sec.3 we briefly discuss the result obtained.

2. The $K^0-\bar{K}^0$ Matrix Element

Feynman diagrams defining the $K^0-\bar{K}^0$ matrix element are represented in the Figure. Write down the result of the computation

$$\langle \bar{K}^0 | \hat{O}_{|\Delta S|=2} | K^0 \rangle = (F_\pi^2 m_K / 4\pi^3) \cdot$$
$$\cdot \left\{ [N_1 + (\lambda-1)(N_1 - N_2)]^2 + \frac{1}{3}[N_1 + (\lambda-1)N_2]^2 \right\}. \qquad (2)$$



F i g. Feynman diagrams defining the $K^0 - \bar{K}^0$ matrix element in the QL-model.

Here the first and the second terms define the contributions of the diagrams in the Figure respectively; F_π = 93 MeV is the π -meson decay constant, m_K = 498 MeV is the K^0-meson mass; λ is the unitary symmetry breakdown parameter which equals the ratio of s and d -quark masses ($\lambda = m_s/m_d$). The values N_1 and N_2 are defined by the following formulas

$$N_1 = 4g^2 I(m_s^2, m_d^2) + \left(\frac{3m_K^2}{4\pi^2 F_\pi^2}\right)\left[\frac{\lambda^2+1}{2(\lambda^2-1)^2} - \frac{\lambda^2 \ln\lambda^2}{(\lambda^2-1)^3}\right],$$
$$N_2 = 2g^2 I(m_s^2, m_d^2) + \left(\frac{3m_K^2}{4\pi^2 F_\pi^2}\right)\left[\frac{2\lambda^4+5\lambda^2-1}{6(\lambda^2-1)^3} - \frac{\lambda^4 \ln\lambda^2}{(\lambda^2-1)^4}\right], \qquad (3)$$

where $g = m_d/F_\pi$ is the quark-meson coupling constant, $I(m_s^2, m_d^2)$ is the logarithmic divergent integral

$$I(m_s^2, m_d^2) = -\frac{3i}{(2\pi)^4}\int d^4k \frac{1}{(m_s^2-k^2)(m_d^2-k^2)}. \qquad (4)$$

To remove the logarithmic divergent integral it is sufficient to carry out the $K^0(\bar{K}^0)$ wave function renormalization with the renormalization constant $Z_K^{-1} = 4g^2 I(m_s^2, m_d^2)$ /7/. Then let us write down $(N_1)_R$ and $(N_2)_R$ in terms of renormalized values (the index R is omitted)

$$N_1 = 1 + \left(\frac{3m_K^2}{4\pi^2 F_\pi^2}\right)\left[\frac{\lambda^2+1}{2(\lambda^2-1)^2} - \frac{\lambda^2 \ln\lambda^2}{(\lambda^2-1)^3}\right],$$
$$N_2 = \frac{1}{2} + \left(\frac{3m_K^2}{4\pi^2 F_\pi^2}\right)\left[\frac{2\lambda^4+5\lambda^2-1}{6(\lambda^2-1)^3} - \frac{\lambda^4 \ln\lambda^2}{(\lambda^2-1)^4}\right]. \qquad (5)$$

The numerical value of the $K^0 - \bar{K}^0$ matrix element is usually discussed relative to the ratio to the vacuum insertion value

$$\langle \bar{K}^0 | \hat{O}_{|\Delta S|=2} | K^0 \rangle_{vac} = \frac{8}{3}\langle \bar{K}^0 | \bar{s}\gamma^\mu(1-\gamma^5)d|0\rangle\langle 0|\bar{s}\gamma_\mu(1-\gamma^5)d|K^0\rangle, \qquad (6)$$

where 8/3 is the combinatorial multiplier. Following /1,2/ let us analyse the ratio

$$B = \frac{\langle \bar{K}^0 | \hat{O}_{|\Delta S|=2} | K^0 \rangle}{\langle \bar{K}^0 | \hat{O}_{|\Delta S|=2} | K^0 \rangle_{vac}}. \qquad (7)$$

In the QL-model the matrix element $\langle \bar{K}^0 | \hat{O}_{|\Delta S|=2} | K^0 \rangle_{vac}$ is of the form

$$\langle \bar{K}^0 | \hat{O}_{|\Delta S|=2} | K^0 \rangle_{vac} = (4/3)(F_\pi^2 m_K/4\pi^3)[N_1 + (\lambda-1)(N_1-N_2)]^2. \qquad (8)$$

Thereby the B -parameter takes the form

$$B = \frac{3}{4} + \frac{1}{4}\left[\frac{N_1 + (\lambda-1)N_2}{N_1 + (\lambda-1)(N_1-N_2)}\right]^2 \geq 1. \qquad (9)$$

Write down the numerical values of B for three values of λ:

$$B = \begin{cases} 1{,}000 \; , & \lambda = 1{,}0 \, , \\ 1{,}002 \; , & \lambda = 1{,}5 \, , \; [8] \\ 1{,}005 \; , & \lambda = 1{,}8 \, , \; [7] \end{cases} \qquad (10)$$

The chosen values λ correspond to unbroken and broken unitary symmetry.

3. Conclusion

The numerical value of B -parameter obtained in the QL-model differs from unity slightly. It means that the $K^0 - \bar{K}^0$ matrix element is completely saturated by the vacuum intermediate state. Emphasize that B -parameter slightly depends on the broken unitary symmetry parameter λ. This result contradicts the statement obtained in the framework of soft kaon method (chiral perturbation theory) /9/. However the soft kaon method for the calculation of the $K^0 - \bar{K}^0$ matrix element is possibly incorrect. The point is that in the $K^0 - \bar{K}^0$ transition in the framework of the soft kaon method there are no states with masses which are much larger than kaon mass. The absence of these states makes the limit $m_K^2 \to 0$ unsubstantiated.

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References

1. Gault F.D., Webb J.N. Z.Phys., 1984, C22, 93;
De Rafael E. Lectures on quark flavour mixing in the standard model, MPI-PAE/PTh 72/84, October 1984.
2. Donoghue J.F., Holstein B. Phys. Rev., 1984, D29, 1088;
Machet B. Z.Phys., 1984, C26, 449;
Bijnens J. et al. Preprint, CALT-68-1193, 1984;
Cea P., Nardulli G., Preparata G. Phys. Letters, 1984, 148B, 477;
Trampetic J. Preprint, CERN-TH 41 02/85, 1985.
3. Kobayashi M., Maskawa K. Progr. Theor. Phys., 1973, 49, 652.
4. Glashow S.L., Nucl.Phys., 1961, 22, 579;
Weinberg S. Phys.Rev. Letters, 1967, 19, 1264;
Salam A. Proc. 8th Nobel Symposium ed. N.Swartholm, Stockholm, 1968, p. 367.
5. Vysotsky M.I. Yad. Fis., 1980, 31, 1535;
Guberina B., Peccei R., Nucl.Phys., 1980, B163, 289;
Gilman F.J., Wise M.B. Phys. Letters, 1980, 93B, 139; Phys.Rev., 1983, D27, 1128;
Linge-Li Chau et al. Phys.Rev., 1983, D27, 2145.
6. Buras A.J., Phys.Rev. Letters, 1981, 46, 1359;
Gaiser B.D., Tsao T., Wise M.B. Ann. of Phys. (N.Y.), 1981, 132, 66;
Oh S.R. Nucl.Phys., 1984, G10, 1335.
7. Volkov M.K., Ebert D. Yad.Fiz., 1982, 36, 1265; Z.Phys., 1983, C16, 205;
Volkov M.K. Ann. of Phys. (N.Y.-, 1983, 157, 282; Proc. of the International Conf. on the Problems of Quantum Field Theory, Alushta, Cremea, 20-25 April (1984), D2-84-366, Dubna, 1984, p.281.
Ebert D., ibid., p. 300.
8. Ivanov A.N., Schechter V.M., Yad.Fiz., 1980, 31, 530;
Ivanov A.N., Yad.Fiz., 1981, 33, 1679;
Ivanov A.N., Troitskaya N.I. Yad. Fiz., 1982, 36, 220;
Transactions of the scientific seminars of V.A.Steklov Math. Inst. (Leningrad Department), vol. 131, 1983; vol. 132, 1985.
9. Bijnens. J.Phys. Rev. Letters, 1984, 53, 2367.

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Вычисление матричного элемента $K^\circ - \bar{K}^\circ$
перехода в модели кварковых петель /КП-модель/

В рамках модели кварковых петель вычислен матричный элемент $K^\circ - \bar{K}^\circ$ перехода. Параметр В слабо отличается от единицы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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The Calculation of the $K^\circ - \bar{K}^\circ$ Matrix Element
in the Quark Loops Model (the QL-Model)

The $K^\circ - \bar{K}^\circ$ matrix element calculation is carried out in the QL-model. The B-parameter slightly differs from unity.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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