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N.B.Skachkov, I.L.Solovtsov,* O.Yu.Shevchenko

INFRARED ASYMPTOTICS OF A GAUGE-INVARIANT PROPAGATOR IN QUANTUM ELECTRODYNAMICS

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*Gomel Polytechnical Institute

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1. Introduction

Nowdays a lot of papers appeared where the behaviour of the Green functions was studied in the infrared limit (see, for instance, /1-4/ and the references therein). The interest in this problem is caused by a widely discussed possibility of the connection of the infrared asymptotics of QCD Green functions with the problem of quark confinement. For this purpose the standard fermion propagator

$$i\langle 0|T\Psi(x)\bar{\Psi}(y)|0\rangle \qquad (1.1)$$

has been used that, obviously, is not a gauge-invariant object. It can be shown in the framework of the exactly solvable Schwinger and Bloch-Nordsiek models /5,6,7/ that the infrared behaviour of the function (1.1) essentionally depends on the choice of a gauge.

The electron propagator in QED has a branching point at $p^2 = m^2$ in the infrared limit and the exponential explicitly depends on the choice of a gauge. The simple pole appears only in a special gauge $\alpha = 3$ (the Soloviev-Yenni gauge). The propagator (1.1) is not a good object for studying the quark confinement problem because of its gauge-dependence. In two-dimensional QCD it is not possible to solve the problem of quark confinement if one works on the basis of the gauge-dependent Green function (1.1)/8/. Besides, in a general case of the non-Abelian theories the gauge noninvariant objects suffer from infrared divergences and that is why, strictly speaking, they do not exist /9/. Due to this fact the requirement of the gauge-invariance can be considered as the condition of observability/10/. So, a self-consistent study of the structure of the gauge theories should be based on the use of gauge-invariant objects.

We shall study the gauge-invariant (G.I.) spinor propagator:

$$G(x,y) = i\langle 0|T\Psi(x)\,P\exp[ig\int_x^y d\xi^\nu A_\nu(\xi)]\,\bar{\Psi}(y)|0\rangle \qquad (1.2)$$

in the framework of the Abelian theory. The propagator (1.2) contains the exponential factor $\mathcal{P}\exp[ig\int_y^x d\xi^\mu A_\mu(\xi)]$ [1] with the contour integral over the vector field $A_\mu(\xi)$ that compensates the gauge transformations of $\psi(x)$ and $\bar{\psi}(y)$ G.I. spinor function (1.2) was considered in [12] from the viewpoint of a gauge-invariant definition of a quark mass and in [13] in the framework of exactly solvable Schwinger and Bloch-Nordsiek models.

In the present paper we shall introduce a new class of gauge-invariant fields through combining the fields of Fock and Dirac classes. It will be shown that in this new class there naturally appears a particular case of the propagator (1.2) - the propagator $G^{\text{str. line}}(x,y)$ for which as the integration contour, the only natural contour for a two-particle Green function appears, namely, a piece of the straight line that connects two points x and y of the Minkowski space. The advantage of this path over any other consists in the fact that if in the G.I. propagator $G^{str.l.}(x,y)$ one would choose the fields $\psi, \bar{\psi}, A_\mu$ in a special Fock gauge [14] $(z-(x+y)/2)^\mu A_\mu(z)=0$, then $G^{str.l.}(x,y)$ would coincide with the usual fermion propagator in this gauge. Thus, the propagator $G^{str.l.}(x,y)$ is connected with the S-matrix elements by usual reduction formulae.

In the present paper the Dyson-Schwinger equations for the gauge-invariant propagator $G^{str.l.}(x,y)$ will be derived, and the behaviour of this propagator will be studied in the infrared limit on the basis of the equations obtained as a well as of the functional integration method.

2. The construction of the gauge-invariant (G.I) spinor propagator

There are two known classes of G.I. vector fields:

$$B_\mu(x|\eta) = A_\mu(x) - \partial_\mu \int_\eta^x d\xi^\nu A_\nu(\xi), \tag{2.1}$$

$$B_\mu(x|f) = A_\mu(x) - \partial_\mu \int dy f^\nu(x-y) A_\nu(y), \tag{2.2}$$

where η is a fixed point of the Minkowski space, and f_ν in (2.2) is a real function that satisfies the condition $\partial^\nu f_\nu(z) = \delta(z)$.

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[1] Symbol $\mathcal{P}$ denotes the ordering along the contour (see [11]). In the Abelian case this symbol can be dropped if one chooses a straight-line contour.

In the case of a straight-line integration contour in (1) the field $B_\mu(x|\eta)$ coincides with the field taken in the Fock gauge [14]: $(x-\eta)^\mu A_\mu(x)=0$ [2]. Due to this fact we shall call these fields the fields of the Fock class. The fields (1.2) were introduced by Dirac [16] (the Dirac class fields). It is important to emphasize that the fields (1.2) coincide with the fields A_μ taken in the gauge $f^\mu(P)A_\mu(P)=0$.

The field $B_\mu(x|\eta)$ (1.1) in case of the straight-line integration path can be expressed through the tension tensor $F_{\mu\nu}$ by the inversion formula [14,15]

$$B_\mu(x|\eta) = \int_0^1 ds\, s\,(x-\eta)^\nu F_{\mu\nu}(\eta+s(x-\eta)). \tag{2.3}$$

The inversion formula connecting the fields with $F_{\mu\nu}$ has been derived by us for the Dirac class in [17]

$$B_\mu(x|f) = \int dy f^\nu(x-y) F_{\mu\nu}(y). \tag{2.4}$$

It was shown that the fields (1.1) and (1.2) considered on the equations of motion satisfy the Lorentz gauge condition $\partial^\mu B_\mu = 0$. This condition according to the Dirac terminology [18] appears here as a secondary constraint.

The G.I. Fock and Dirac fields are obtained from the ordinary electromagnetic fields $A_\mu(x)$ taken in an aribtrary gauge with the help of the gauge transformation, $A_\mu(x) \to A_\mu^\lambda(x) = A_\mu(x) + \partial_\mu\lambda(x)$ with the choice $\lambda = \Lambda(x|\eta) = -\int_0^1 ds(x-\eta)^\nu A_\nu(\eta+s(x-\eta))$ for the fields (1.1) and $\lambda = \Lambda(x|f) = -\int dy f^\nu(x-y) A_\nu(y)$ for the fields (1.2). With the help of the local phase transformation with the same choice of λ we find the form of the G.I. spinor fields in Fock and Dirac classes, respectively,

$$\Psi(x|\eta) = \exp[-ig\int_\eta^x d\xi^\nu A_\nu(\xi)]\psi(x), \tag{2.5}$$

$$\Psi(x|f) = \exp[-ig\int dy f^\nu(x-y) A_\nu(y)]\psi(x). \tag{2.6}$$

It is easy to see that the spinor fields of the Fock class (2.5) are G.I. fields up to the global transformations (they are multiplied by the factor $\exp[i\lambda(\eta)]$ under the gauge transformations).

Besides, due to the fact that the point η is fixed for the fields of Fock (2.1) and (2.5) classes, the translation invariance of the spinor and vector propagators constructed with the help of fields (2.1) and (2.5) is broken. (If one would take in (2.1)

[x] This gauge condition has also been considered in [15].

and (2.5) $\eta = \eta(x)$, then the gauge invariance of Fock vector and spinor fields would be broken.) These difficulties can be avoided by taking the limit $\eta \to \infty$.

Here we propose a new way of introducing the G.I. fields that is free of the difficulties mentioned above and allows us to construct a G.I. spinor propagator $G^{str.l.}(x,y)$ of the type discussed previously in the Introduction.

We introduce the field

$$B_\mu(x|\eta;f) = A_\mu(x) - \frac{\partial}{\partial x^\mu}\Big[\int_{\eta(x)}^{x} dz^\nu A_\nu(z) + \int d\tau f^\nu(\eta(x)-\tau) A_\nu(\tau)\Big], \tag{2.7}$$

where f_ν satisfies the same conditions as the function f_ν in the Dirac class (2.2). The inversion formula for the field (2.7) looks like

$$B_\mu(x|\eta;f) = \int_0^1 dS\, S\,(x-\eta)^\nu F_{\mu\nu}(\eta+S(x-\eta)) + \frac{\partial \eta^\rho(x)}{\partial x^\mu}\Big\{\int_0^1 dS(1-S)(x-\eta)^\nu F_{\rho\nu}(\eta+S(x-\eta)) + \int d\tau f^\nu(\eta-\tau) F_{\rho\nu}(\tau)\Big\}. \tag{2.8}$$

Let us note that in the framework of perturbation theory the gauge conditions are introduced for the free vector fields to be quantized later. In this case from (2.8) with the account of antisymmetry of the tensor $F_{\mu\nu}$, the Maxwell equations, and the properties of the f_ν function it is easy to find[3] that the field $B_\mu(x|\eta;f)$ satisfies the Lorentz condition

$$\partial^\mu B_\mu(x|\eta;f) = 0, \tag{2.9}$$

that once more appears as the secondary constraint.

The G.I. spinor field corresponding to (2.7) is introduced by the phase transformation

$$\psi(x) \mapsto \Psi(x|\eta;f) = \exp(ig\Lambda(x|\eta;f))\psi(x),$$

where $\Lambda(x|\eta;f)$ is the expression that stands in square brackets in (2.7).

To construct the $G_\nu^{str.l.}(x,y)$, let us choose $\eta = \frac{x+y}{2}$ and[4] $f^\nu(\eta-\tau) = -(x-y)/2 \int_0^\infty dS\, \delta((x+y)/2 + S(x-y)/2 - \tau)$

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[3] We are interested in the case when η is a function linear in x .

[4] With this choice of η and f the field B_μ coincides with the field A_μ taken in the gauge $(z-(x+y)/2)^\mu A_\mu(z) = 0$ used previously in /19/ in deriving the dynamical equations for the two-particle relativistic wave function.

In this case the G.I. spinor propagator is defined as follows

$$G^{str.l.}(x,y) = i\langle 0|T\psi(x)\exp[ig\int_x^y d\xi^\nu A_\nu(\xi)]\bar\psi(y)|0\rangle, \tag{2.10}$$

where the integration is performed along the piece of the straight line that connects the points x and y , i.e.

$$\xi^\mu = x^\mu + S(y-x)^\mu, \quad 0 \leq S \leq 1. \tag{2.11}$$

3. Schwinger equations

Let us derive the Schwinger-Dyson equations for the G.I. propagator $G^{str.l.}(x,y)$ defined by (2.10) and (2.11). (In what follows we shall omit the symbol str.l.) Let us add the Lagrangian by a term with the vector source J_μ : $\Delta\mathcal{L} = J_\mu(x)A^\mu(x)$. In the interaction representation the Green function (1.2) can be represented as follows:

$$G(x,y|J) = \frac{1}{S_0[J]} g(x,y|J), \tag{3.1}$$

where

$$S_0[J] = \langle 0|TS[J]|0\rangle. \tag{3.2}$$

(3.3)

Here $S[J]$ is the S-matrix in the presence of the source J_μ . The G.I. Green function (2.4) that contains the vacuum diagrams will be rewritten in the form

$$g(x,y|J) = \mathcal{P}\exp[ig\int_x^y d\xi^\mu \big(-i\frac{\delta}{\delta J^\mu(\xi)}\big)]\tilde g(x,y|J), \tag{3.4}$$

where

$$\tilde g(x,y|J) = i\langle 0|T\psi(x)\bar\psi(y)S[J]|0\rangle. \tag{3.5}$$

It is easy to check that the function (3.5) satisfies the equation /6/

$$\Big[i\gamma_\mu\Big(\frac{\partial}{\partial x^\mu_y} - g\frac{\delta}{\delta J^\mu(x)} - g\Big[\frac{\partial}{\partial x^\mu}\int_x^y d\xi^\nu \frac{\delta}{\delta J^\nu(\xi)}\Big] - ig\mathcal{U}_\mu(x) - ig\Big[\frac{\partial}{\partial x^\mu}\int_x d\xi^\nu \mathcal{U}_\nu(\xi)\Big]\Big) - m\Big]G(x,y|J) = -\delta(x-y). \tag{3.6}$$

With the account of (3.4) we derive from (3.6) the equation for function (3.3)

$$\left[i\gamma_\mu\left(\frac{\partial}{\partial x^\mu}-g\frac{\delta}{\delta J_\mu(x)}-g\left[\frac{\partial}{\partial x_\mu}\int_x^y d\xi^\nu\frac{\delta}{\delta J^\nu(\xi)}\right]-m\right]\cdot \tag{3.7}$$
$$\cdot g(x,y/J)=-\delta(x-y)\langle 0/P\exp ig\int_x^y d\xi^\mu A_\mu(\xi)S[J]/0\rangle =$$
$$=-\delta(x-y)S_0[J].$$

Defining the vacuum expectation value of the vector field A_μ

$$\mathcal{U}_\mu(x)=S_0^{-1}[J]\langle 0/TA_\mu(x)S[J]/0\rangle , \tag{3.8}$$

we find with the help of (3.1) the equation for the G.I. function (1.2)

$$i\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-g\frac{\delta}{\delta J_\mu(x)}-g\left[\frac{\partial}{\partial x_\mu}\int_x^y d\xi^\nu\frac{\delta}{\delta J^\nu(\xi)}\right]-\right. \tag{3.9}$$
$$\left.-ig\mathcal{U}^\mu(x)-ig\left[\frac{\partial}{\partial x_\mu}\int_x^y d\xi^\nu\,\mathcal{U}_\nu(\xi)\right]\right)-m\Big]G(x,y/J)=-\delta(x-y).$$

The gauge-invariance is easy to check of the obtained equation.

The second Schwinger equation for the vacuum expectation value of the vector field can be found in a standard way[6]. It coincides with the usual equation because the latter contains the spinor Green function with the coinciding arguments. In this case and with our choice of the integration contour the exponentional factor in (1.2) dissappears. Thus, the second Schwinger equation acquires the form

$$\mathcal{U}_\mu(x)=\int dz\,\mathcal{D}_{\mu\nu}(x,z)\cdot\left[J^\nu(z)+ig\,Sp\left(\gamma^\nu G(z,z/J)\right)\right], \tag{3.10}$$

where $\overset{\circ}{\mathcal{D}}_{\mu\nu}$ is the vector particle propagator. Equations (2.10) and (2.11) are analogs of Schwinger equations for the G.I. propagator (1.2) that we are looking for.

4. Dyson equations

Let us transform equations (3.9), and (3.10) so that they compose a system of integral equations. To do this, we shall perform the transition to a new functional variable, $\mathcal{U}_\mu(x)$. Making use of the relation

$$\frac{\delta}{\delta J^\nu(y)}=\int dz\,\frac{\delta\mathcal{U}_\alpha(z)}{\delta J^\nu(y)}\,\frac{\delta}{\delta\mathcal{U}_\alpha(z)}=-\int dz\,\mathcal{D}_{\nu\alpha}(z,y)\frac{\delta}{\delta\mathcal{U}_\alpha(z)}, \tag{4.1}$$

We introduce in analogy with [6] the vertex function

$$\Gamma_\mu(x,y/z)=\frac{\delta G^{-1}(x,y/\mathcal{U})}{\delta\mathcal{U}^\mu(z)}. \tag{4.2}$$

Hence we have

$$\frac{\delta G(x,y/\mathcal{U})}{\delta\mathcal{U}^\mu(z)}=\int dx'dy'\,G(x,x'/\mathcal{U})\Gamma_\mu(x',y'/z)G(y',y/\mathcal{U}), \tag{4.3}$$

then equation (3.9) takes the form:

$$\left[i\partial_x-m+g\,\hat{\mathcal{U}}(x)+\int_x^y d\xi^\nu\,\mathcal{U}_\nu(\xi)\right]G(x,y/\mathcal{U})+ \tag{4.4}$$
$$+ig\gamma^\mu\int dx'dy'dz\left[\mathcal{D}_{\mu\nu}(x/z)+\frac{\partial}{\partial x^\mu}\int_x^y d\xi^\alpha\mathcal{D}_{\alpha\nu}(\xi,z)\right]\cdot$$
$$\cdot G(x,x'/\mathcal{U})\Gamma^\nu(x',y',z)G(y',y/\mathcal{U})=-\delta(x-y).$$

Defining the G.I. vector field

$$B_\mu(x,y)=\mathcal{U}_\mu(x)+\frac{\partial}{\partial x^\mu}\int d\xi^\nu\,\mathcal{U}_\nu(\xi), \tag{4.5}$$

and the mass operator

$$\hat{M}(x,y/\mathcal{U})=-ig\gamma^\mu\int dx'dz\,\mathcal{D}_{\mu\nu}(x,z)+ \tag{4.6}$$
$$+\frac{\partial}{\partial x^\mu}\int_x^y d\xi^\alpha\mathcal{D}_{\alpha\nu}(\xi,z)\Big]G(x,x'/\mathcal{U})\Gamma^\nu(x',y/z),$$

we rewrite equation (3.4) as follows

$$[i\hat{\partial}_x-m+g\,\hat{B}(x/y)]G(x,y/\mathcal{U})- \tag{4.7}$$
$$-\int dy'M(x,y'/\mathcal{U})G(y',y/\mathcal{U})=-\delta(x-y).$$

It is easy to cast equation (4.7) into the following form

$$G(x,y/\mathcal{U})=S_c(x-y)+g\int dy'S_c(x-y')\hat{B}(y',y)G(y',y/\mathcal{U})-$$
$$-\int dx'dy'S_c(x-x')M(x',y'/\mathcal{U})G(y',y/\mathcal{U}). \tag{4.8}$$

The mass operator (4.6) has a G.I. form. To see this, we shall give it the form

$$M(x,y|u) = ig\gamma^{\mu}\int dy'\langle 0|\,T\psi(x)\exp[ig\int_x^{y'} d\xi^{\nu}A_{\nu}(\xi)]\cdot \cdot\bar{\psi}(y')B_{\mu}(x,y')S[u]\,|0\rangle. \tag{4.9}$$

Let us study the second Schwinger equation (3.10). Calculating the functional derivative $\delta/\delta J_{\nu}(y)$ of both sides of (3.10) we find with the help of (4.1)

$$\mathcal{D}_{\mu\nu}(x,y) = \overset{\circ}{\mathcal{D}}_{\mu\nu}(x,y) - -ig\int dz\,d\tau\,\overset{\circ}{\mathcal{D}}_{\mu\alpha}(x,z)\,Sp[\gamma^{\alpha}\frac{\delta G(z,z|u)}{\delta u_{\beta}(\tau)}\,\mathcal{D}_{\beta\nu}(\tau,y)]. \tag{4.10}$$

Defining the polarization operator

$$P_{\alpha\beta}(z,\tau) = ig\,Sp[\gamma^{\alpha}\frac{\delta G(z,z|u)}{\delta u_{\beta}(\tau)}] = = ig\,Sp[\gamma^{\alpha}\int dz'dz''G(z,z'|u)\Gamma^{\beta}(z',z''|\tau)G(z'',z|u), \tag{4.11}$$

we represent (4.10) as follows

$$\mathcal{D}_{\mu\nu}(x,y) = \overset{\circ}{\mathcal{D}}_{\mu\nu}(x,y) - -\int dz\,d\tau\,\overset{\circ}{\mathcal{D}}_{\mu\alpha}(x,z)P_{\alpha\beta}(z,\tau)\mathcal{D}_{\beta\nu}(\tau,y). \tag{4.12}$$

5. Infrared asymptotics (Dyson-Schwinger equations)

To study the behaviour of the G.I. propagator (1.2) in the infrared region, we shall apply the method developed in [20].

Equation (3.9) with the account of (4.1) and the contour parametrization (2.11) can be represented at $y=0$ in the form

$$\{i\gamma^{\mu}[\frac{\partial}{\partial x^{\mu}} + g\Big(\hat{\varphi}_{\mu}(x) - \frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,\hat{\varphi}_{\nu}(sx)\Big) - -ig\Big(u_{\mu}(x) - \Big(\frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,u_{\nu}(sx)\Big)\Big)] - m\}\,G(x,0|u) = -\delta(x), \tag{5.1}$$

where $\hat{\varphi}_{\mu}(x) = \int d\tau\,\mathcal{D}_{\mu\nu}(x-\tau)\dfrac{\delta}{\delta u_{\nu}(\tau)}$.

$$\hat{\varphi}_{\mu}(x) = \int d\tau\,\mathcal{D}_{\mu\nu}(x-\tau)\frac{\delta}{\delta u_{\nu}(\tau)}\,. \tag{5.2}$$

Let us transform to the momentum representation and consider the object

$$\Phi_{\mu}(p|u) = \int dx\,\exp(ipx)\,[\hat{\varphi}_{\mu}(x) - \frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,\hat{\varphi}_{\nu}(sx)]\cdot \cdot G(x,0|u). \tag{5.3}$$

With the definition of the Fourier-transform $\hat{\varphi}_{\mu}(k)$

$$\hat{\varphi}_{\mu}(x) = \int\frac{dk}{(2\pi)^4}\exp(-ikx)\,\hat{\varphi}_{\mu}(k), \tag{5.4}$$

we obtain

$$\Phi_{\mu}(p|u) = \int dx\frac{dk}{(2\pi)^4}\int_0^1 ds\,e^{ipx}[e^{-ikx}\hat{\varphi}_{\mu}(k) - e^{-ikx}\hat{\varphi}_{\mu}(k) + ix^{\nu}k_{\mu}s\,e^{-iksx} + ix^{\nu}k_{\mu}s\,e^{-ikxs}\hat{\varphi}_{\nu}(k)]\cdot G(x,0|u). \tag{5.5}$$

Performing the integration by parts in the last term of (5.5) we come to

$$\Phi_{\mu}(p|u) = \int_0^1 ds\int\frac{dk}{(2\pi)^4}[\hat{\varphi}_{\mu}(k)G(p-k|u) + k_{\mu}\frac{\partial\hat{\varphi}_{\nu}(k)}{\partial k_{\nu}}\cdot \cdot G(p-sk|u)]. \tag{5.6}$$

In accordance with [20] in the infrared limit, in (5.6) the following approximations can be done:

$$\Phi_{\mu}(p|u) \simeq \int_0^1 ds\int\frac{dk}{(2\pi)^4}[\hat{\varphi}_{\mu}(k) + k_{\mu}\frac{\partial\hat{\varphi}_{\nu}(k)}{\partial k_{\nu}}]G(p|u) = = \int\frac{dk}{(2\pi)^4}\frac{\partial^0}{\partial k_{\nu}}[k_{\mu}\hat{\varphi}_{\nu}(k)]G(p|u) = 0. \tag{5.7}$$

It can be shown analogously that in the infrared limit

$$\int dx\,e^{ipx}[u_{\mu}(x) - \frac{\partial}{\partial x^{\mu}}x^{\nu}\int_0^1 ds\,u_{\nu}(sx)]G(x,0|u) \simeq 0. \tag{5.8}$$

Thus, we see that in the infrared region $G(p)$ obeys the equation

$$(\hat{p}-m)G(p) \simeq -1\,,\quad p^2 \simeq m^2, \tag{5.9}$$

i.e., in the infrared limit the G.I. fermion Green function has the simple pole

$$G(P) = (m-\hat{P})^{-1}, \quad P^2 \simeq m^2. \tag{5.10}$$

Assuming that for renormalization of the G.I. Green function the counterterms will be needed of the same structure as for renormalization of the S-matrix, we find for the renormalized Green function

$$G'(x,y) = Z_2^{-1} G(x,y). \tag{5.11}$$

Thus, in the infrared limit the renormalized G.I. propagator has the simple-pole singularity

$$G'(P) \simeq \frac{Z_2^{-1}}{m-\hat{P}}, \quad P^2 \simeq m^2. \tag{5.12}$$

6. Infrared asymptotics (functional method)

Let us represent the G.I. spinor propagator (1.2) in the form of a functional integral over the fermion and vector fields

$$G(x,y) = i\int \mathcal{D}[\Psi,\bar{\Psi}]\mathcal{D}A\, \Psi(x) \exp\Big[ig\int_x^y d\xi^\nu A_\mu(\xi)\Big]\bar{\Psi}(y)\cdot \exp\{iS[\Psi,\bar{\Psi};A]\}, \tag{6.1}$$

where $S[\Psi,\bar{\Psi},A]$ is an action functional of QED and the functional integration measures $\mathcal{D}[\Psi,\bar{\Psi}]$ and $\mathcal{D}A$ are renormalized so as to obtain from (6.1) at $g=0$ a free spinor propagator. Besides, the measure $\mathcal{D}A$ includes some gauge condition whose particular form is not essential here. Performing integration over the fermionic fields in (6.1) we get

$$G(x,y) = \int \mathcal{D}A \frac{\det[i\hat{\partial}+g\hat{A}-m]}{\det[i\hat{\partial}-m]} \exp\{iS_0[A]\}\cdot G(x,y|A)\exp\Big[ig\int_x^y d\xi^\nu A_\mu(\xi)\Big], \tag{6.2}$$

where $S_0[A]$ is an action of a free electromagnetic field A_μ, and $G(x,y|A)$ is the Green fnnction of the fermion in an external field that satisfies the equation

$$[i\hat{\partial}_x + g\hat{A}_x - m]G(x,y|A) = -\delta(x-y). \tag{6.3}$$

A formal representation of the solutions of equations like (6.3) in the form of functional integrals was proposed in /21/. Following to /21/ it has the form

$$\hat{G}(x,y|A) = [i\hat{\partial}_x + g\hat{A}(x) + m]\tilde{G}(x,y|A), \tag{6.4}$$

where

$$\tilde{G}(x,y|A) = i\int_0^\infty ds \exp[-is(m^2-i0)]\int \mathcal{D}B\, \delta\Big[x-y-2\int_0^s d\xi B(\xi)\Big]\cdot \exp\Big\{-i\int_0^s d\xi\Big[B_\mu(\xi)B^\mu(\xi) - g\big(2B_\mu(\xi)+\sigma_{\mu\nu}(\xi)i\hat{\partial}_x^\nu(\xi)\big)\cdot A^\mu\Big(x-2\int_\xi^s d\eta B(\eta)\Big)\Big]\Big\}. \tag{6.5}$$

The integration measure DB in (6.5) is normalized as follows

$$\int \mathcal{D}B \exp\Big[-i\int_0^s d\xi B_\mu(\xi)B^\mu(\xi)\Big] = 1. \tag{6.6}$$

In the infrared limit the following approximations are valid /20-22/. First, it is possible to put in (6.2),

$$\frac{\det[i\hat{\partial}+gA-m]}{\det[i\hat{\partial}-m]} \simeq 1, \tag{6.7}$$

which means the neglect of vacuum-polarization effects. Second, the term $\sigma_{\mu\nu}$ can be omitted in (6.5), which corresponds to the neglect of spin effects. It is important to emphasize that these approximations do not break the gauge-invariance property of the initial spinor propagator.

After these approximations the functional integral (6.2) can be calculated explicitly. Finally, we obtain

$$G(x,y) = i\int_0^\infty ds \exp[-is(m^2-i0)]\int \mathcal{D}B \exp\Big[i\int_0^s d\xi B^2(\xi)\Big]\cdot\Big[i\hat{\partial}_x + m - i\gamma_\mu\frac{\delta}{\delta j_\mu(x)}\Big]\delta\Big(x-y-2\int_0^s d\eta B(\eta)\Big)\cdot \exp\Big[\frac{i}{2}g^2\int dw_1 dw_2 J^\mu(w_1)\mathcal{D}_{\mu\nu}(w_1,w_2)J^\nu(w_2)\Big], \tag{6.8}$$

where

$$J_\mu(w) = j_\mu(w) + \int_x^y d\xi_\mu \delta(w-\xi) + \int_0^s d\xi\, 2B_\mu(\xi)\cdot \delta\Big(w-x+2\int_\xi^s d\eta B(\eta)\Big) \tag{6.9}$$

and $\mathcal{D}_{\mu\nu}(w_1,w_2)$ is the photon propagator in an arbitrary gauge. Upon some simple calculations expression (6.8) takes the form

$$G(x,y) = i\int_0^\infty ds \exp[-is(m^2-i0)]\int \mathcal{D}B \exp[-i\int_0^s d\xi B^2(\xi)]\cdot$$
$$\cdot[i\hat{\partial}_x + m + g^2 K(x,y/B)\delta(x-y-2\int_0^S d\eta B(\eta))\cdot \tag{6.10}$$
$$\cdot \exp[\frac{i}{2} g^2 \Phi(x,y/B)],$$

where

$$\hat{K}(x,y/B) = \int_x^y d\xi^\nu \gamma^\mu \mathcal{D}_{\mu\nu}(x-\xi) + 2\int_0^S d\xi \gamma^\mu \mathcal{D}_{\mu\nu}[2\int_\xi^S d\eta\cdot \tag{6.11}$$
$$\cdot B(\eta)]B^\nu(\xi),$$

$$\Phi(x,y/B) = 4\int_0^S d\xi_1 \int_0^S d\xi_2 B_\mu(\xi_1)\mathcal{D}_{\mu\nu}[2\int^{\xi_2} d\eta B(\eta)]B^\nu(\xi_2) +$$
$$+\int_x^y d\xi_1^\mu \int_x^y d\xi_2^\nu \mathcal{D}_{\mu\nu}(\xi_1-\xi_2) + 2\int_x^y d\xi^\mu \int_0^S d\xi B^\nu(\xi)\mathcal{D}_{\mu\nu}[\xi - x + 2\int_\xi^S d\eta\cdot \tag{6.12}$$
$$\cdot B(\eta) - \xi].$$

In the course of our calculations we suppose that there is performed the regularization that does not destroy the gauge invariance (for example, the dimensional regularization). In this case all the integrals we meet are meaningful.

For a straight-line path it is easy to see that the functions $K(x,y/B)$ and $\Phi(x,y/B)$ depend only on the difference of x and y. Passing in (6.10) to the momentum representation, i.e., performing the Fourier transformation over the variable $x-y$ and performing a shift of the functional-integration variable

$$B_\mu(\eta) = P_\mu + \omega_\mu(v), \quad \eta = sv, \tag{6.13}$$

we obtain the G.I. Green function in the momentum representation

$$G(P) = i\int_0^\infty ds \exp[is(P-m^2+i0)\int \mathcal{D}\omega \exp[-is\int_0^1 dv\, \omega^2(v)]\cdot$$
$$\cdot[\hat{P}+m+g^2\tilde{K}(P/\omega)]\exp[\frac{i}{2}g^2\tilde{\Phi}(P/\omega)]. \tag{6.14}$$

In the infrared region it is possible to neglect the functional argument in functionals $\tilde{K}(P/\omega)$ and $\tilde{\Phi}(P/\omega)$. The functions $\tilde{K}(P/0)$ and $\tilde{\Phi}(P/0)$, as it is easy to show, are equal to zero. Thus, finally, we find in the infrared limit $P^2 \simeq m^2$ that

$$G(P) \simeq i(\hat{P}+m)\int_0^\infty ds\, e^{is(P^2-m^2+i0)} = \frac{\hat{P}+m}{m^2-P^2+i0}. \tag{6.15}$$

7. Conclusion

Thus, we have derived an analog of the Dyson-Schwinger equations for a gauge-invariant propagator. The mass operator that appears here has an explicit gauge-invariant form.

On the basis of the Dyson-Schwinger equations and functional methods the behaviour of the gauge-invariant spinor Green function is studied in the infrared limit. It is shown that unlike a gauge-non-invariant Green function that has a branch point in α -gauge, the gauge-invariant spinor propagator has a simple-pole singularity in the infrared limit. Note that this result is completely consistent with the results obtained previously in the framework of the Block--Nordsick model/13/.

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References

1. Mandelstam S. - Phys.Rev. 1979, v. D20, p. 3223;
Pagels H. - Phys.Rev. 1977, v. D15, p. 2991.
2. Baker M., Ball J.S., Lachariasen F. - Nucl.Phys. 1981, v. B186, p. 531; - Nucl.Phys. 1983, v. B229, p. 445.
3. Arbuzov B.A. - Phys.Lett., 1983, v. 125B, N 6, p. 497;
Alekseev A.I., Arbuzov B.A., Baykov V.A. - Sov. Journ. Nucl.Phys. 1981, v. 34, p. 1374;
Nekrasov M.L., Rochev V.E. - Sov.Journ.Nucl.Phys. 1984, v. 39, p. 1275.
4. Efimov G.V. - JINR P2-84-716, Dubna, 1984.
5. Schwinger J. - Phys.Rev. 1962, v. 128, p. 2425.
Fried H.M. - Functional Methods and Models in Quantum Field Theory, Cambridge, MIT Press, 1972;
Bloch F., Nordsiek A. - Phys.Rev. 1937, v. 52, p. 54.
6. Bogolubov N.N., Shirkov D.V. - The introduction to the Theory of Quantized Fields, Interscience Publ., Inc., New York, 1959.
7. Coleman S. - 1/N. Ettore Majorana Int. School of Subnuclear Physics, Erice, 1979.
8. Matveev V.A. - The Proc. of XIV Int. School on High Energy Physics. JINR D2-81-158, JINR, Dubna, 1981, p. 242.

9. Slavnov A.A. - ibid., p. 36.
10. Fock V.A. - Izvestiy of the USSR Academy of Sciences, OMEN, 1937, p. 551; The Collection of the Works on Quantum Field Theory, Leningrad Univ. Pub., 1957, p.141 (in Russian).
11. Dubovikov M.S., Smilga A.V. - Nucl.Phys. 1981, v. B185, p. 109; Cronström C. - Phys.Lett., 1980, v. 90B, p. 267; Schwinger J. - Particles, Sources and Fields, Addison-Wesley Pub.Comp.Mass. 1970.
12. Kapshay V.N., Skachkov N.B., Solovtsov I.L. - JINR, E2-83-26, Dubna, 1983; In proc. of VI-Int. Seminar on the Problems of High Energy Phys. and Quantum Field Theory, Protvino, 1983, v. 2, p. 262.
13. Kanaya K. et al. - Phys.Lett. 1982, v. 111B, N 1, p. 61; Kanaya K. - Phys.Rev. 1982, v. D26, p. 1758.
14. Solovtsov I.L. - Izvestiy Vuzov, Fizika, 1985, N 1, p. 65; Solovtsov I.L., Solovtsova O.P. - Izvestiy Vuzov, Fizika, 1984, N 12, p. 49.
15. Logunov A.A. - Sov. Journ. ZETP, 1955, v. 29, p. 871.
16. Barbashov B.M. - Sov.Journ.ZETP, 1965, vol.48, p.607. Blochintsev D.I., Barbashov B.M. - Sov. Journ.Uspekhi of Physical Sciences, 1972, v. 106, p. 593.
17. Fradkin E.C. - Sov. Journ.Proc. of PhIAN, 1965, v. 29, p. 7.
18. Bloch F., Nordsiek A. - Phys.Rev. 1937, v. 52, p. 54.
19. Dirac P.A.M. - Canad.Jour. of Physics, 1955, v. 33, N 11, p. 650.
20. Dirac P.A.M. - Lectures on Quantum Mechanics, Yechiva Univ., New York, 1964.

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Скачков Н.Б., Соловцов И.Л., Шевченко О.Ю. E2-85-462

Инфракрасная асимптотика калибровочно-инвариантного пропагатора в квантовой электродинамике

Введен новый класс калибровочно-инвариантных полей. Для калибровочно-инвариантного обобщения спинорного пропагатора получены уравнения Дайсона-Швингера. На основе этих уравнения, а также с помощью функциональных методов показано, калибровочно-инвариантный спинорный пропагатор в инфракрасной области имеет особенность в виде простого полюса.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Skachkov N.B., Solovtsov I.L., Shevchenko O.Yu. E2-85-462

Infrared Asymptotics of a Gauge-Invariant Propagator in Quantum Electrodynamics

A new class of the gauge-invariant field is introduced. For the gauge-invariant propagator of a spinor field the analog of the Dyson-Schwinger equations is derived. By using these equations as well as the functional intergation method it is shown that the gauge-invariant spinor propagator has a simple pole singularity in the infrared region.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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