сообщения
объединенного
института
ядерных
исследований
дубна

E2-85-406

S.Dubnička[1], G.Georgios,[2] V.A.Meshcheryakov

PION FORM FACTOR BEHAVIOUR IN A MODIFIED QUARK LOOP MODEL

[1] Institute of Physics EPRC, Slovak Academy of Sciences, 84228, Bratislava, Czechoslovakia

[2] Higher Pedagogical Institute, Shumen, Bulgaria

1985

INTRODUCTION

At present there is a general belief (see, e.g.,[1]) that the observed hadrons are composite objects, made of the confined quarks. Therefore it is not surprising that in a theoretical description of the observables in electroweak interaction of hadrons, besides the well justified vector meson dominance (VMD) model[2], the lowest order quark-loop diagrams are taken into consideration[3-10]. What is more interesting that there is an unambiguous dual character of these two approaches[6,11,12]. The value of the electromagnetic radius of the charged pion $<r_\pi^2>$, associated with the elastic pion form factor, is a classical example of it. Really, in the context of the naive vector meson dominance model of the pion form factor in the zero width approximation one finds $<r_\pi^2> = 6/m_\rho^2$ giving the value $<r_\pi^2> = 0.38\ fm^2$. On the other hand, an alternative calculation of $<r_\pi^2>$ (6), based on the point-like pseudoscalar meson-quark-antiquark coupling and free quark propagators in the triangle diagram approximation of the pion form factor, gives $<r_\pi^2> = 3/(4\pi^2 f_\pi^2) = 0.34\ fm^2$, where factor 3 is due to the three colour degrees of freedom of quarks, the pion decay constant f_π = 93 MeV comes from the Goldberger-Treiman relation on the quark level and the soft pion limit is carried out explicitly. However, neither the naive VMD model with the correct value of ρ-mass nor quark triangle diagrams with the point-like coupling give $<r_\pi^2>$ consistent with the world averaged value. Then it is hard to expect the predicted pion form factor behaviour from these models to coincide with existing data since the slope around the point $t = 0$ is incorrect.

Recently a modified quark loop model has been proposed in which as usual the pion is assumed to couple to the electroweak currents through its constituent quarks. The basis of the modification lies in the fact that the bound state nature of the $q\bar{q}$ system (i.e., pion) is simulated by a covariant wave function like the boson propagator at the $\pi q\bar{q}$ vertex, parametrized by an effective mass M and a coupling strength $g_{\pi q\bar{q}}$. These two parameters of the model together with the effective quark mass m_q enable (utilizing at the same time also the charged and neutral pion lifetimes) to normalize the triangle diagram pion form factor contributions at the zero momentum transfer and simultaneously to predict the pion charge radius $<r_\pi^2>^{1/2}$=

= 0.677 fm, which is in a perfect agreement with the generally accepted value/14-19/. Moreover, the introduction of the bound state wave function of the pion in an ordinary boson propagator form ensures the corresponding Feynman integrals, unlike the integrals of the quark triangle diagrams with the point-like vertices, to be convergent.

The aim of the present paper is to calculate the pion form factor behaviour from the modified quark-loop diagrams and then to determine the range of an agreement of these predictions with existing data in space-like and in time-like regions.

In the next section we review the main features of the model presented in/13/. Section 3 is devoted to the calculation of the pion form factor behaviour and to its comparison with the data. Conclusions are drawn in Sec.4.

MODIFIED QUARK-LOOP MODEL

The following one-parameter wave function of boson propagator

$$\Phi(\ell) = -\frac{M^2}{\ell^2 - M^2}, \tag{1}$$

depending on the relative momentum ℓ of the quark-antiquark pair, was used in/13/ to modify the point-like $\pi q\bar{q}$ vertex with the aim to make some allowance for the bound state nature of the $q\bar{q}$ system constituting the pion. If we denote the four-momentum of the incoming pion of fig.1a by k and q and $\bar{q}$ carry momenta p and $p-k$ respectively, then $\pi q\bar{q}$ vertex is taken to be $\bar{g}_{\pi q\bar{q}} \gamma_5 \Phi(p-k/2)$, where $\bar{g}_{\pi q\bar{q}}$ is a dimensionless strength coupling constant and $p-k/2$ is the redefined (by a factor 1/2) relative momentum, $2p-k$, of the $q\bar{q}$ bound state. The wave function Φ can be thought of as simulating the effects of the strong interactions between confined q and $\bar{q}$ in an averaged way, with M a free parameter to be fitted by known data.

In this manner a modification was carried out in all $\pi q\bar{q}$ vertices of the pion form factor quark triangle diagrams presented in fig.1 as well as in the quark-loop diagrams of the weak radiative decay $\pi^+ \to e^+ \nu \gamma$ and the nonradiative weak decay $\pi^+ \to \mu^+ \nu_\mu$, described by the pion decay constant f_π. Then all Feynman integrals, representing the modified quark-loop diagrams with the virtual quarks taken to have free-particle propagators, are now convergent.

The weak radiative decay $\pi^+ \to e^+ \nu \gamma$ is described by two structure form factors, $v(t)$ (vector) and $a(t)$ (axial vector), where the vector form factor $v(t)$ can be related at $t = 0$ as usual to the neutral pion decay width via an isospin rotation and the

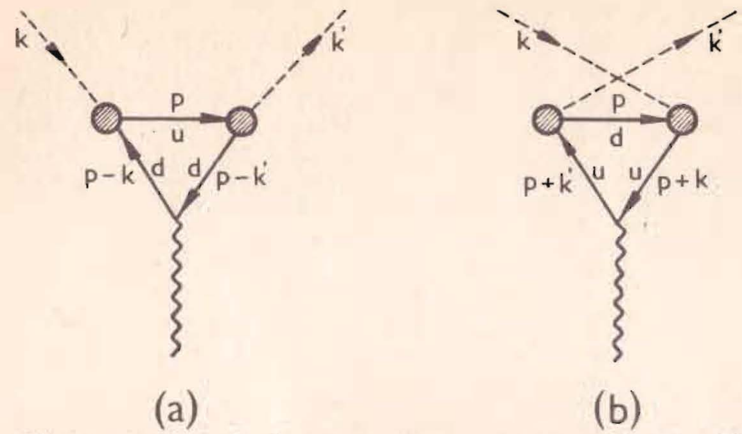

Fig.1. Modified pion form factor quark triangle diagrams.

conserved vector current hypothesis as follows

$$v(0) = -(2\Gamma_{\pi^0} / \pi \alpha^2 m_{\pi^0}^3)^{1/2}. \tag{2}$$

By using the relation (2) and the experimental data on Γ_{π^0} as well as the normalization condition of the pion form factor at $t = 0$ and the data on f_π the authors of/13/ have determined the free parameters M, $\bar{g}_{\pi q\bar{q}}$, m_q of the model as follows

$$M = 492 \text{ MeV}, \quad \bar{g}_{\pi q\bar{q}} = 4.28, \quad m_q = 243 \text{ MeV}. \tag{3}$$

Then substituting these parameters into the expression representing six times the derivative of the pion form factor at $t = 0$ they have predicted the pion charge radius value

$$\langle r_\pi^2 \rangle^{1/2} = 0.677 \text{ fm}, \tag{4}$$

which is in a perfect agreement with the world averaged value/14/ as well as with the reliable consistent values recently determined in precision experiments/15,17,19/.

Since the modified quark triangle diagrams in fig.1 with the values of the parameters/3/ are now normalized at $t = 0$ and the corresponding Feynman integrals are convergent, one can calculate the pion form factor as a function of momentum transfer squared in this approximation and compare this predicted behavious with existing data. However, this is a subject of the next section.

PION FORM FACTOR BEHAVIOUR IN THE MODIFIED QUARK-LOOP DIAGRAM APPROXIMATION

By using the standard methods of quantum field theory the pion electromagnetic vertex in the modified quark-loop diagram approximation represented by fig.1 can be written in the following form

$$\Gamma_\mu (k,k') = ie(k+k')_\mu F_\pi(t) = 3(\sqrt{2})^2 e \bar{g}^2_{\pi q \bar{q}} \int \frac{d^4p}{(2\pi)^4} \times$$

$$\times \{ \frac{2}{3} Tr[\gamma_5 \Phi(p+k/2) \frac{1}{\not p - m} \gamma_5 \Phi(p+k'/2) \frac{1}{\not p - \not k' - m} \gamma_\mu \frac{1}{\not p + \not k - m}] - \qquad (5)$$

$$- \frac{1}{3} Tr[\gamma_5 \Phi(p-k/2) \frac{1}{\not p - \not k - m} \gamma_\mu \frac{1}{\not p - \not k' - m} \gamma_5 \Phi(p-k'/2) \frac{1}{\not p - m}]\},$$

where e is the electric charge, $F_\pi(t)$ is the electromagnetic form factor of the pion and $m = m_u = m_d = m_q$ is the constituent quark mass given by (3). If we carry out the calculation of traces explicitly, we obtain

$$i(k+k')_\mu F_\pi(t) = \frac{8M^4 \bar{g}^2_{\pi q \bar{q}}}{(2\pi)^4} \{ (J_5 - 2J_1) - (k+k')_\mu (J_6 + 2J_2) +$$

$$+ (k'_\nu k_\mu + k_\nu k'_\mu)(J_7 - 2J_3) + (kk')(J_8 + 2J_4) \}. \qquad (6)$$

where

$$J_1 = \int d^4p \frac{p_\mu}{D_a^+}, \quad J_2 = \int d^4p \frac{1}{D_a^+}, \quad J_3 = \int d^4p \frac{p_\nu}{D_b^+}, \quad J_4 = \int d^4p \frac{p_\mu}{D_b^+}$$

$$J_5 = \int d^4p \frac{p_\mu}{D_a^-}, \quad J_6 = \int d^4p \frac{1}{D_a^-}, \quad J_7 = \int d^4p \frac{p_\nu}{D_b^-}, \quad J_8 = \int d^4p \frac{p_\mu}{D_b^-}$$

and

$$D_a^+ = \{(p+\frac{k}{2})^2 - M^2\}\{(p+\frac{k'}{2})^2 - M^2\}\{(p+k')^2 - m^2\}\{(p-\frac{k'}{2})^2 - M^2\}$$

$$D_b^+ = D_a^+ (p^2 - m^2)$$

$$D_a^- = \{(p-\frac{k}{2})^2 - M^2\}\{(p-k)^2 - m^2\}\{(p-k')^2 - m^2\}\{(p-\frac{k'}{2})^2 - M^2\}$$

$$D_b^- = D_a^- (p^2 - m^2).$$

In order to calculate the integrals $J_1, \dots, J_8$ first we use the identity

$$- \frac{1}{k^2 - m^2 + i\epsilon} = i \int_0^\infty e^{i\alpha_n (k^2 - m^2 + i\epsilon)} d\alpha_n \qquad (7)$$

and then we carry out the integration over d^4p. As a result the quadrupole and quintuple integrals in the variables α_n have appeared which, in terms of the new variables x_n

$$\alpha_1 = x_1 A$$

$$\alpha_2 = x_2 A$$

$$\dots\dots\dots\dots \qquad (8)$$

$$\alpha_n = (1 - x_1 - \dots - x_{n-1}) A,$$

where $A = \alpha_1 + \alpha_2 + \dots + \alpha_n$ and $n = 4$ or 5, can be reduced to triple and quadrupole integrals, respectively. Then the pion form factor (6) takes the form

$$(k+k')_\mu F_\pi(t) = \frac{M^4 \bar{g}^2_{\pi q \bar{q}}}{2\pi^2} \{ k_\mu (F_1 - 2m_\pi^2 E_3 + m_\pi^2 E_2) +$$

$$+ k'_\mu (F_2 - 2m_\pi^2 E_5 + m_\pi^2 E_1) + (k-k')_\mu (F_3 + 4m_\pi^2 E_4) + 2(kk')k_\mu E_1 + 2(kk')k'_\mu E_2 \}, \qquad (9)$$

where

$$F_1 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \{ \frac{x_1 + x_2}{f_1^2} + \frac{1 - x_2}{f_3^2} \}$$

$$F_2 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \{ \frac{2}{f_1^2} + \frac{x_2 - x_3 + 1}{2f_3^2} \}$$

$$F_3 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \{ \frac{x_2 + 2x_3}{f_1^2} - \frac{x_1}{2f_3^2} \}$$

$$E_1 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4 \{ \frac{x_1 + 2x_2}{f_4^3} \}$$

$$E_2 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4 \{ \frac{2x_3 + x_4}{f_4^3} \}$$

$$E_3 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4 \left\{ \frac{x_3}{f_2^3} \right\}$$

$$E_4 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4 \left\{ \frac{x_4}{f_2^3} \right\}$$

$$E_5 = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4 \left\{ \frac{x_1+2x_2-2}{f_2^3} \right\}$$

$$f_1 = \{-x_1 \tfrac{k}{2} + x_2(\tfrac{k'}{2}-k) + x_3(k'-k) + k\}^2 + \{(x_1+x_2)(M^2-m^2+\tfrac{3}{4}m_\pi^2) + (m^2-m_\pi^2)\}$$

$$f_2 = \{-x_1 \tfrac{k}{2} - x_2 k + x_3(\tfrac{k'}{2}-k) + x_4(k'-k) + k\}^2 + (x_1+x_3)(M^2-m^2+\tfrac{3}{4}m_\pi^2) + \\ + x_2 m_\pi^2 + (m^2-m_\pi^2)\}$$

$$f_3 = \{x_1\tfrac{(k-k')}{2} + x_2(k-\tfrac{k'}{2}) + x_3\tfrac{k'}{2} + \tfrac{k'}{2}\}^2 + \{-(x_2+x_3)(M^2-m^2+\tfrac{3}{4}m_\pi^2) + (M^2 - \tfrac{m_\pi^2}{4})\}$$

$$f_4 = \{x_1\tfrac{k}{2} + x_2 k + x_3 k' + x_4\tfrac{k'}{2}\}^2 + \{(x_1+x_4)(M^2-m^2-\tfrac{m_\pi^2}{4}) - \\ -(x_2+x_3)m_\pi^2 + m^2\}$$

and m_π is the pion mass. If we multiply both sides of (9) by $(k+k')_\mu$ then the contribution of the term with $(k-k')_\mu$ is automatically equal to zero and the pion form factor is given by the following expression

$$F_\pi(t) = \frac{M^4 g^2_{\pi q\bar q}}{4\pi^2} \Big\{ \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \Big[\frac{x_1+x_2+2}{f_1^2} - \frac{x_2+x_3-3}{2f_3^2} \Big] + \\ + 2m_\pi^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4 \Big[\frac{(3-t/m_\pi^2)(x_1+2x_2+2x_3+x_4)}{2f_4^3} - \\ - \frac{x_1+2x_2+x_3-2}{f_2^3} \Big]\Big\} \qquad (10)$$

in which the integrals cannot be calculated explicitly and expressed through elementary functions. Therefore, we have used the standard program MIKOR [20] on the Dubna CDC-6500 computer in order to evaluate them numerically at different values of t. With this aim it is convenient to modify the integrals of the expression (10) into the integrals with constant boundaries. The latter is achieved by means of the substitution

$$y_1 = x_1, \quad y_2 = \frac{x_2}{1-x_1}, \ldots, \quad y_n = \frac{x_n}{1-x_1-\ldots-x_{n-1}} \qquad (11)$$

which reduces the integrals in consideration into the following form

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 G(x_1,x_2,x_3) = \\ = \int_0^1 dy_1 \int_0^1 dy_2 \int_0^1 dy_3\, G[y_1,(1-y_1)y_2,(1-y_1)(1-y_2)y_3](1-y_1)^2(1-y_2), \qquad (12)$$

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int_0^{1-x_1-x_2-x_3} dx_4\, H(x_1,x_2,x_3,x_4) = \\ = \int_0^1 dy_1 \int_0^1 dy_2 \int_0^1 dy_3 \int_0^1 dy_4\, H[y_1,(1-y_1)y_2,(1-y_1)(1-y_2)y_3,(1-y_1)(1-y_2)(1-y_3)y_4] \times \\ \times (1-y_1)^3(1-y_2)^2(1-y_3), \qquad (13)$$

where

$$G(x_1,x_2,x_3) = \frac{x_1+x_2+2}{f_1^2} - \frac{x_1+x_3-3}{2f_3^2}$$

$$H(x_1,x_2,x_3,x_4) = \frac{(3-t/m_\pi^2)(x_1+2x_2+2x_3+x_4)}{2f_4^3} - \frac{x_1+2x_2+x_3-2}{f_2^3}$$

and $(1-y_1)^2(1-y_2)$, $(1-y_1)^3(1-y_2)^2(1-y_3)$ are the corresponding Jacobians.

The numerical prediction of (10) with transformed integrals (12), (13) and the values of parameters (3) is presented by the full line in fig.2, where the inverse pion form factor is drawn as a function of the momentum transfer squared t.

It is very convenient for a comparison of our predictions with the data to use the inverse pion form factor, since the data (mainly in the low momentum transfer values) of the space-like region [21] are rather scattered, and their perfect fie by the inverse naive VMD model formula

$$[F_\pi^{(\rho)}(t)]^{-1} = \frac{m_\rho^2 - t}{m_\rho^2} \qquad (14)$$

with $m_\rho = 721$ MeV is a straight line (presented by the dashed line in fig.2) in this case. The deviation of the full line from the dashed one in fig.2 determines a region of agreement of, the predicted pion form factor behaviour given by (10) with existing data. The latter is given, roughly speaking, by the following range of momenta $-0.08\ \mathrm{GeV}^2 \le t \le 4m_\pi^2$, where $t = 4m_\pi^2$ is the

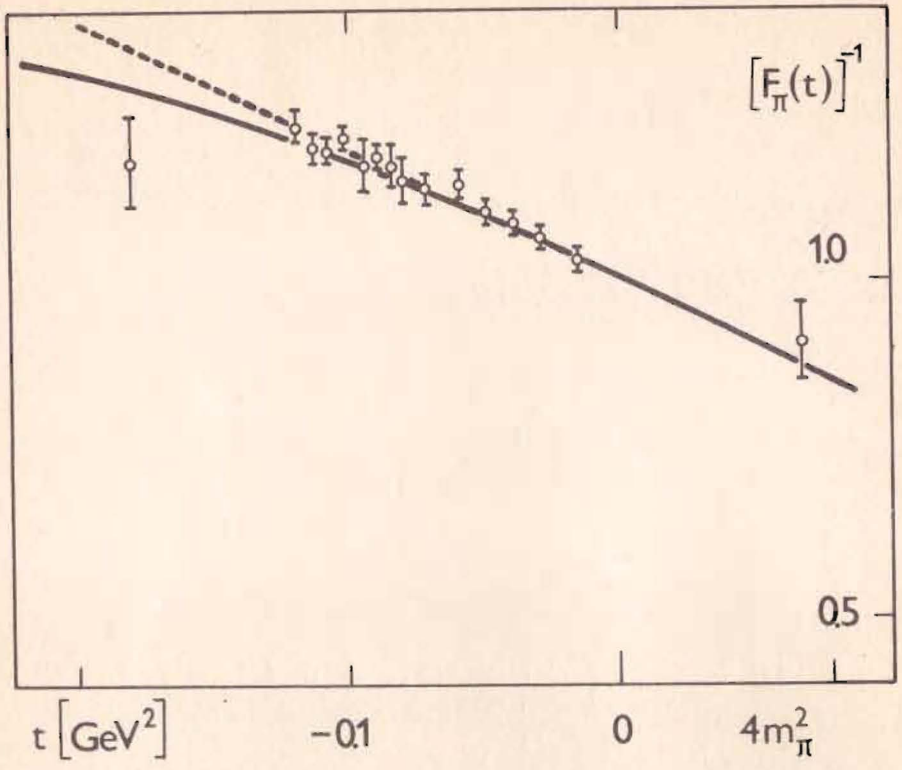

Fig.2. Inverse pion form factor versus t. *The numerical prediction of (10) with transformed integrals (12), (13) and the values of parameters (3) is presented by the full line. The dashed line represents the inverse naive VMD model formula (14) with* $m_\rho = 721$ *MeV.*

elastic threshold from which the pion form factor is starting to be complex.

CONCLUSION

By using the modified quark loop model proposed in [13] which predicts the pion charge radius to be in a perfect agreement with the generally accepted value, we have calculated the pion form factor behaviour and compared it with existing data. The agreement is achieved only in the range of momenta $-0.08\ \mathrm{GeV}^2 \leq t \leq 4m_\pi^2$. This result indicates that the modification of the point-like $\pi q\bar{q}$ vertex by the simple wave function of the type of boson propagator (1) is enough only for the description of low energy parameters of the pion. For the pion form factor

in the whole space-like region one has to look for a more realistic vertex function which will depend besides the relative momentum ℓ of the quark-antiquark pair, on the total four-momentum and on the spin, but it will have the same asymptotic behaviour as (1). This will be the aim of our further investigations.

ACKNOWLEDGEMENTS

The authors are indebted to Dr.S.B.Gerasimov, Dr.R.Lednický and Dr.A.I.Saltykov for discussions.

REFERENCES

1. Close F.E. Introduction to Quarks and Partons. Academic Press, London, New York, San Francisco, 1979.
2. Sakurai J.J. Currents and Mesons (in Russian). Atomizdat, Moscow, 1972.
3. Moreno M., Pestieau J. Phys.Rev., 1976, D13, p.175.
4. Resnick L., Kim J.H. Can.J.Phys., 1976, 54, p.158.
5. Pascual P., Tarrach R. Nucl.Phys., 1978, B146, p.509.
6. Gerasimov S.B. Yad.Fiz., 1979, 29, p.513.
7. Tarrach R. Z.Phys.C - Particles and Fields, 1979, 2, p.221.
8. Espriu D., Tarrach R. J.Phys.G: Nucl.Phys., 1980, 6, p.1055.
9. Ebert D., Volkov M.K. Fortschr.Phys., 1981, 29, p.35.
10. Ametller Ll., Ayala G., Bramon A. Phys.Rev., 1981, D24, p.233.
11. Bramon A., Massó E. Phys.Lett., 1981, 104B, p.311.
12. Volkov M.K., Kreopalov D.V. JINR, P4-81-697, Dubna, 1981.
13. Kim J.H., Resnick L. Can.J.Phys., 1980, 58, p.964.
14. Zovko N. Fortschr.Phys., 1975, 23, p.185.
15. Quenzer A. et al. Phys.Lett., 1978, 76B, p.512.
16. Dubnicka S., Meshcheryakov V.A., Milko J. J.Phys.F: Nucl. Phys., 1981, 7, p.605.
17. Dally E.B. et al. Phys.Rev.Lett., 1982, 48, p.375.
18. For a criticism of the first two Soviet-American experimental results on the pion charge radius, which are inconsistent with the value presented in (17), see the paper Dubnička S., Dubničkova A.Z. Z.Phys.C - Particles and Fields, 1982, 15, p.149.
19. Amendolia S.R. et al. Phys.Lett., 1984, 146B, p.114; CERN Preprint EP, 84-59, Geneva, 1984.
20. Saltykov A.I. Algorithms and Programs for Solution of Some Problems in Physics. KFKI-74-34, Budapest, 1974, p.133.
21. Bebek C.J. et al. Phys.Rev., 1978, D17, p.1693.

Received by Publishing Department
on May 30, 1985.