объединенный
институт
ядерных
исследований
дубна

E2-85-363

A.Galperin, E.Ivanov, V.Ogievetsky,
E.Sokatchev

CONFORMAL INVARIANCE IN HARMONIC SUPERSPACE

Submitted to "Quantum Field Theory
and Quantum Statistics",
Essays in honour of the 60th birthday
of E.S.Fradkin

1985

I. Introduction

Old and new works by E.S.Fradkin on different topics of supergravity and conformal symmetry are widely known (/1,2,3/, etc.). This prompted us to contribute to this festschrift volume a paper devoted to conformal invariance in harmonic superspace. This new superspace has been recently proposed as an appropriate tool to deal with extended supersymmetries. We succeeded in the formulation of N=2 matter, Yang-Mills and Einstein supergravity /4/ and of N=3 Yang-Mills theories /5/ in terms of unconstrained harmonic superfields. Further, in the harmonic superspace a manifestly supersymmetric quantization scheme was developed for the N=2 rigid theories./6/

The harmonic superspace contains additional bosonic dimensions (those of the space $SU(2)/U(1)$ in the case N=2 and of $SU(3)/U(1) \times U(1)$ for N=3). They give rise to infinite towers of auxiliary and gauge degrees of freedom, which in turn helps to circumvent the so-called no-go theorems. /7/.

The above mentioned theories have so far been discussed in the context of Poincare supersymmetry. However, some of these theories are known to be superconformally invariant. In the present paper we shall show how the N=2 superconformal group is realized in harmonic superspace and shall examine the conformal invariance of the N=2 off-shell theories. It will turn out that conformal supersymmetry preserves the concept of real analytic subspace of harmonic superspace. The latter was introduced within Poincare supersymmetry and played a crucial role in the formulation of the N=2 theories. An interesting and unexpected point is that the realization constructed requires an essentially complex language and is not consistent with the standard reality condition for the zweibeins describing the sphere $SU(2)/U(1)$. It respects, instead, the combined involution introduced in our papers cited above. Thus, just the preservation of reality properties under the latter involution proves to be the fundamental principle.

The main result of this paper is the introduction of the unconstrained prepotentials and the coordinate transformation group for N=2 conformal supergravity. Conformal supergravity can be a convenient starting point for obtaining different versions of Einstein supergravity. The idea is to compensate the extra gauge transformations by adding various matter and Maxwell multiplets /8/.

Our knowledge of new unconstrained formulations of the hypermultiplets permits us to find a new version of N=2 Einstein supergravity. It has no central charge, and possessesan infinite number of auxiliary fields.

The idea of harmonization of superspace beares deep analogies with the twistor interpretation of the self-dual N=0 Yang-Mills /9,10,11/. There one harmonizes one of the $SU(2)$ subgroups of the Euclidean Lorentz group $O(4) \sim SU(2)\times SU(2)$. The concept of analytic subspace is relevant too, but now the analytic fields satisfy automatically equations of motion. In the case of extended supersymmetry the internal symmetry group ($SU(2)$ or $SU(3)$) is harmonized instead of the Lorentz one. Consequently, there the concept of analyticity helps to solve kinematical constraints, not equations of motion. We believe that the example of the N=0 self-dual Yang-Mills equations can serve as an instructive introduction into the subject of harmonic superspace and devote section II to it. In Section III the rigid N=2 conformal supersymmetry is discussed, and in section IV - its local version (i.e., N=2 conformal supergravity).

II. Analyticity and conformal symmetry on the example of self-dual N=0 Yang-Mills equations

Normally, in the 4-dimensional ordinary (N=0) Euclidean Yang-Mills theory one deals with the standard Euclidean space

$$\mathcal{P}/SO(4) \sim \{x^{\alpha\dot\alpha}\}. \qquad \text{(II.1)}$$

Here $\mathcal{P}$ is the Poincaré group, $SO(4) \sim SU(2) \times SU(2)$ is the rotation subgroup of $\mathcal{P}$ (Euclidean Lorentz group) and the coordinates $x^{\alpha\dot\alpha}$ transform as spinors with respect to the first $SU(2)$ group (index α) and the second one (index $\dot\alpha$). Obviously, in the space (II.1) there are no invariant subspaces.

x) Such an interpretation of the constraints for extended supersymmetric theories was previously discussed by Witten /12/ and Rosly /13/ .

Another realization of the same group $\mathcal{S}$ in a different space allows one to define a non-trivial subspace which will turn closely related to the solution of the self-dual YM equations. This space is

$$\mathcal{S}/SU(2) \sim R^4 \times S^3 \sim \{x^{\alpha\dot\alpha}, u^+_\alpha, u^-_\alpha\} \tag{II.2}$$

and we shall call it harmonic space.

The additional coordinates $u^\pm_\alpha$ are SU(2) spinors. They parametrize the sphere $S^3 \sim SU(2)$, if

$$\text{a)}\ u^-_\alpha = \overline{(u^{+\alpha})} \qquad \text{b)}\ u^{+\alpha}u^-_\alpha = 1. \tag{II.3}$$

In fact, we are going to use a smaller space

$$\mathcal{S}/SU(2)\times U(1) \sim R^4 \times S^2. \tag{II.4}$$

However, we prefer to consider functions defined on the bigger one (II.2) and restricted by the condition

$$D^0 f^{(q)}(x, u^\pm) = q\, f^{(q)}(x, u^\pm). \tag{II.5}$$

Here D^0 is one of the covariant derivatives on S^3

$$D^{++} = u^{+\alpha}\frac{\partial}{\partial u^{-\alpha}}, \quad D^{--} = u^{-\alpha}\frac{\partial}{\partial u^{+\alpha}},$$
$$D^0 = u^{+\alpha}\frac{\partial}{\partial u^{+\alpha}} - u^{-\alpha}\frac{\partial}{\partial u^{-\alpha}}. \tag{II.6}$$

The condition (II.5) simply means that only functions with definite U(1) charge are considered ($u^\pm_\alpha$ have charges ±1). Such functions are given by their harmonic decomposition in symmetrized products of $u^\pm_\alpha$:

$$f^{(q)}(x, u^\pm) = \sum_{n=0}^{\infty} f^{(\alpha_1\cdots\alpha_{n+q}\beta_1\cdots\beta_n)}(x)\, u^+_{(\alpha_1}\cdots u^+_{\alpha_{n+q}}u^-_{\beta_1}\cdots u^-_{\beta_n)}. \tag{II.7}$$

Clearly, such functions live in the space (II.4). We find the above description of (II.4) very convenient because it does not refer to any particular parametrization of $S^2 \sim SU(2)/U(1)$ [x].

[x] In various geometric and algebraic instanton constructions /9-11,14/ as a rule, employs a complex, CP^1 parametrization of $SU(2)/U(1)$ with $u^{+i} = (\lambda, 1)$, $u^{-i} = (1+\lambda\bar\lambda)^{-1}(-1, \bar\lambda)$.

Now we are prepared to define the invariant subspace of (II.2) (or (II.4), if the restriction (II.5) is imposed). Consider the new set of variables

$$\{x^{\pm\dot\beta} = x^{\alpha\dot\beta}u^\pm_\alpha,\ u^\pm_\alpha\}. \tag{II.8}$$

It is easy to check that under the Poincaré group transformations

$$\delta x^{\alpha\dot\beta} = a^{\alpha\dot\beta} + L^{(\alpha}{}_{\gamma)}x^{\gamma\dot\beta} + L^{(\dot\beta}{}_{\dot\gamma)}x^{\alpha\dot\gamma}$$
$$\delta u^{\pm\alpha} = L^{(\alpha}{}_{\gamma)}u^{\pm\gamma} \tag{II.9}$$

the following subset of (II.8)

$$\{x^{+\dot\beta},\ u^\pm_\alpha\} \tag{II.10}$$

forms an invariant subspace

$$\delta x^{+\dot\beta} = a^{\alpha\dot\beta}u^+_\alpha + L^{(\dot\beta}{}_{\dot\gamma)}x^{+\dot\gamma}$$
$$\delta u^\pm_\alpha = L^{(\alpha}{}_{\gamma)}u^{\pm\gamma}. \tag{II.11}$$

We shall refer to it as to the analytic subspace of the harmonic space (II.2) because only $x^{+\dot\beta}$ appears in (II.10), and not its complex conjugate $x^{-\dot\beta}$. Nevertheless, the analytic subspace is closed under the following combination of complex conjugation and the mapping $\overset{*}{(u^\pm)} = \pm u^\mp$

$$\overline{\overset{*}{(x^\pm_{\dot\beta})}} = x^{\pm\dot\beta}, \quad \overline{\overset{*}{(u^\pm_\alpha)}} = u^{\pm\alpha}. \tag{II.12}$$

On the space (II.10) one can define analytic functions $f^{(q)}(x^+, u^\pm)$ which can also be made real with respect to (II.12) for even q:

$$\overline{\overset{*}{f^{(q)}}} = f^{(q)}, \quad q = 2n. \tag{II.13}$$

Thus, the operation $\overline{*}$ is sufficient for establishing the reality properties of the analytic subspace and of the functions on it. Moreover, as we shall see later, it is the only conjugation compatible with the conformal group. The functions $f^{(q)}(x^+, u^\pm)$ automatically obey the analyticity (or Cauchy-Riemann) condition

$$\partial^+_{\dot\beta} f^{(q)} \equiv \frac{\partial}{\partial x^{-\dot\beta}} f^{(q)} = 0. \tag{II.14}$$

It is important to realize that from (II.14) follows the equation of motion

$$\Box f \equiv \frac{\partial}{\partial x^{+\dot\beta}}\frac{\partial}{\partial x^{-}_{\dot\beta}} f = 0. \tag{II.15}$$

The significance of the existence of the analytic subspace becomes clear in the context of gauge theory. Indeed, suppose that the field $f(x, u^{\pm})$ transforms under a gauge group with parameters $\tau(x)$:

$$f'(x,u) = e^{i\tau(x)} f(x,u). \tag{II.16}$$

Then one can define the covariant derivative with the Lie algebra valued gauge connection $A_{\alpha\dot\beta}$

$$\mathcal{D}_{\alpha\dot\beta} = \partial_{\alpha\dot\beta} + i A_{\alpha\dot\beta}$$

$$A'_{\alpha\dot\beta} = e^{i\tau}(A_{\alpha\dot\beta} - i\partial_{\alpha\dot\beta})e^{-i\tau} \tag{II.17}$$

and the field strength tensors $F_{\alpha\beta}, F_{\dot\alpha\dot\beta}$

$$[\mathcal{D}_{\alpha\dot\beta}, \mathcal{D}_{\gamma\dot\rho}] = i(\varepsilon_{\alpha\gamma}F_{\dot\beta\dot\rho}(x) + \varepsilon_{\dot\beta\dot\rho}F_{\alpha\gamma}(x)). \tag{II.18}$$

Now, let us impose the condition of covariant analyticity on the field f :

$$\mathcal{D}^{+}_{\dot\beta} f \equiv u^{+\alpha}\mathcal{D}_{\alpha\dot\beta} f = 0. \tag{II.19}$$

The integrability condition for (II.19) is

$$0 = [\mathcal{D}^{+}_{\dot\alpha}, \mathcal{D}^{+}_{\dot\beta}] \iff F_{\alpha\beta} = 0 \tag{II.20}$$

which is nothing but the well-known self-duality equation. So, the existence of covariantly analytic fields is equivalent to the self-duality of the field-strenth tensor.

We can go even further. Equation (II.20) has the obvious "pure gauge" solution

$$\mathcal{D}^{+}_{\dot\alpha} = e^{-iv(x,u)}\,\partial^{+}_{\dot\alpha}\, e^{iv(x,u)},$$

i.e.,

$$A^{+}_{\dot\alpha} = -ie^{-iv}(\partial^{+}_{\dot\alpha} e^{iv}). \tag{II.21}$$

Here $v(x,u)$ is defined up to the gauge freedom

$$e^{iv'(x,u)} = e^{i\lambda(x,u)} e^{iv(x,u)} e^{-i\tau(x)}. \tag{II.22}$$

where $\tau(x)$ is the gauge parameter (II.17) and $\lambda(x,u)$ is a "pregauge" parameter satisfying the analyticity condition

$$\partial^{+}_{\dot\alpha}\lambda(x,u) = 0 \Rightarrow \lambda = \lambda(x^{+},u). \tag{II.23}$$

Further, the gauge connection $A_{\alpha\dot\beta}(x)$ in (II.17) is real ($\overline{A_{\alpha\dot\beta}} = A_{\alpha\dot\beta}$) and does not depend on $u^{\pm}$, so one can derive the following restrictions on $v(x,u)$

i) $\overset{*}{\overline{(A^{+}_{\dot\alpha})}} = A^{+\dot\alpha} \Rightarrow \overset{*}{\overline{v}} = v,\ \overset{*}{\overline{\lambda}} = \lambda,$ (II.24)

ii) $D^{0}A^{+}_{\dot\alpha} = A^{+}_{\dot\alpha} \Rightarrow D^{0}v = D^{0}\lambda = 0.$ (II.25)

In fact, in (II.24), (II.25) part of the λ -gauge freedom has been already used to obtain the simple result that both v and λ are real functions of zero charge ;

iii) $D^{++}A^{+}_{\dot\alpha} = 0 \Rightarrow$

$$\partial^{+}_{\dot\alpha}(e^{iv}(D^{++}e^{-iv})) = 0.$$

In other words,

$$e^{iv}(D^{++}e^{-iv}) = iV^{++}, \tag{II.26}$$

where V^{++} is an arbitrary real analytic field of charge +2 :

$$\overset{*}{\overline{V^{++}}} = V^{++},\ D^{0}V^{++} = 2V^{++},\ V^{++} = V^{++}(x^{+},u). \tag{II.27}$$

So, we have reduced the problem of finding the solutions of the self-duality equations (II.20) to the solution of the linear differential equation

$$D^{++}e^{iv} = -iV^{++}e^{iv} \tag{II.28}$$

for any given V^{++}.

The quantity e^{iv} has the following geometric meaning. Given a field f_{τ} transforming under the τ - gauge group (II.16) we can convert it with the help of the "bridge" e^{iv} into a field

$$f_{\lambda} = e^{iv}f_{\tau},\ f'_{\lambda} = e^{i\lambda}f_{\lambda}$$

which transforms under the pregauge λ -group. In this new, λ -frame, the analyticity condition (II.19) becomes simply $\partial^{+}_{\dot\beta}f_{\lambda} = 0$ and has the solution $f_{\lambda} = f_{\lambda}(x^{+},u)$. In other words, in the λ -frame analyticity is manifest. At the same time, in the λ - frame the derivative D^{++} acquires a connection,

$$\mathcal{D}^{++} = D^{++} + iV^{++},\ V^{++\prime} = e^{i\lambda}(V^{++} - iD^{++})e^{-i\lambda}. \tag{II.29}$$

The unconstrained analytic connection V^{++} becomes the basic object in the theory. This phenomenon, the replacement of the constrained ordinary connections (or vielbeins) by the unconstrained analytic harmonic connections (or vielbeins), is the key point in the extended supersymmetric gauge theories (see below).

The equation (II.28) which allows to express the bridge e^{iv} in terms of V^{++} can be solved perturbatively /6/.
The problem of finding non-perturbative solutions is, of course, a difficult one, especially a selection of regular solutions. Here we shall give just one simple example, that of the 1-instanton solution for the gauge group SU(2). The corresponding choice for V^{++} is

$$(V^{++})_i{}^{\dot{j}} = -i\,\frac{1}{\rho^2}\, x^+_i x^{+\dot{j}} \equiv -\frac{i}{\rho^2}\,\delta^{\dot\alpha}_i \delta^{\dot{j}}_{\dot\beta}\, x^+_{\dot\alpha} x^{+\dot\beta} . \qquad \text{(II.30)}$$

One can verify that

$$(e^{iv})_i{}^{\dot{j}} = \frac{1 + x^2/2\rho^2}{\sqrt{1 + x^2/\rho^2}} \left[\delta_i{}^{\dot{j}} - \frac{x^+_{(i} x^{-\dot{j})}}{\rho^2 + \frac{1}{2}x^2} \right], \quad x^2 = x^{+k} x^-_k = \frac{1}{2} x^{\alpha\dot\alpha} x_{\alpha\dot\alpha} \qquad \text{(II.31)}$$

is a solution of (II.28). From here one finds the gauge connection

$$(A_{\alpha\dot\beta})_i{}^{\dot{j}} = -i\,\frac{x_{\alpha\dot\gamma}\, \delta^{\dot\gamma}{}_{(i}\, \delta_{\dot\beta}{}^{\dot{j})}}{\rho^2 + x^2} \qquad \text{(II.32)}$$

which coincides with the well-known 1-instanton solution of Belavin et al /15/.

So far we have concerned Poincaré invariance of the self-duality equation. However, it is well known that this equation is also conformally invariant. Above we have described an interpretation of this equation based on the concept of harmonic space (II.8) and its analytic subspace (II.10). Therefore a natural question arises: How is the conformal group realized in (II.8) and is the subspace (II.10) conformally invariant?

In the ordinary space (II.1) the dilatations (ℓ) and the conformal boosts $(k^{\alpha\dot\alpha})$ are realized in the familiar way

$$\delta x^{\alpha\dot\alpha} = \ell x^{\alpha\dot\alpha} + k_{\beta\dot\beta}\, x^{\alpha\dot\beta} x^{\beta\dot\alpha} . \qquad \text{(II.33)}$$

In the harmonic space, however, some of the conformal transformations look rather unusually

$$\begin{aligned}
\delta x^{+\dot\alpha} &= a^{\alpha\dot\alpha} u^+_\alpha + \ell x^{+\dot\alpha} - k_{\beta\dot\beta} x^{+\dot\beta} u^{-\beta} x^{+\dot\alpha} \\
&\quad + L^{(\dot\alpha}{}_{\dot\beta)} x^{+\dot\beta} - L^{(\gamma}{}_{\beta)} u^+_\gamma u^{-\beta} x^{+\dot\alpha} \\
\delta u^+_\alpha &= -k_{\beta\dot\beta} x^{+\dot\beta} u^{+\beta} u^-_\alpha - L^{(\gamma}{}_{\beta)} u^{+\beta} u^+_\gamma u^-_\alpha \\
\delta u^-_\alpha &= 0 \\
\delta x^{-\dot\alpha} &= a^{\alpha\dot\alpha} u^-_\alpha + \ell x^{-\dot\alpha} + L^{(\dot\alpha}{}_{\dot\beta)} x^{-\dot\beta} + L^{(\alpha}{}_{\beta)} (x^{-\dot\alpha} u^{+\beta} - x^{+\dot\alpha} u^{-\beta}) u^-_\alpha + \\
&\quad + k_{\beta\dot\beta} (x^{-\dot\alpha} u^{+\beta} - x^{+\dot\alpha} u^{-\beta}) x^{-\dot\beta} .
\end{aligned} \qquad \text{(II.34)}$$

We see, first of all, that these transformations are not compatible with the ordinary complex conjugation. However, they respect the unimodularity condition (II.3b) and the conjugation $\ast$ which is sufficient to establish reality x). Further, the Lorentz subgroup of the conformal group is realized in (II.34) unusually (c.f. (II.11)). Nevertheless, the ordinary coordinate $x^{\alpha\dot\alpha} = x^{-\dot\alpha} u^{+\alpha} - x^{+\dot\alpha} u^{-\alpha}$ transforms in a standard way and so do such u-independent functions as the vector connection $A^{\alpha\dot\beta}(x)$.

It is most remarkable that the conformal group leaves the analytic subspace $\{x^{+\dot\alpha}, u^{\pm\alpha}\}$ invariant. So, interpretation of the self-dual equation given above is compatible with conformal symmetry. The preservation of the analytic subspace by the superconformal group will be a key factor in N=2 conformal supergravity (see sect.IV) .

III. Conformal invariance in N=2 harmonic superspace

The introduction of harmonic superspace proved especially fruitful in N=2 supersymmetry /4,6/ xx). There we harmonize the automorphism group SU(2) of the N=2 supersymmetry algebra (and not the Lorentz group, as in the previous section). The harmonic variables $u^\pm_i$ /4/ are similar to the ones in (II.3) (i is an SU(2) isospinor index). In addition, the "central basis" of the N=2 harmonic superspace contains space-time ($x^{\alpha\dot\alpha}$) and fermionic ($\theta^\alpha_i, \bar\theta^{\dot\alpha i}$) coordinates

---

x) Let us emphasize the crucial importance of preserving the conditions (II.12). Without them, one might realize on $x^\pm_{\dot\beta}, u^\pm_\alpha$ a complex extension of the conformal group with SL(2,C) instead of SU(2). However, only the ordinary conformal group survives in this extension upon imposing (II.12).

xx) See also Galperin et al. /5/ for applications of harmonic superspace in N=3 Yang-Mills theory.

corresponding to the translation and supertranslation generators. Again, the most important feature of the harmonic N=2 superspace is the existence of an invariant analytic subspace. To obtain it one has to make the change of variables

$$x_A^{\alpha\dot\alpha} = x^{\alpha\dot\alpha} - 4i\,\theta^{i\alpha}\bar\theta^{\dot\alpha j} u^+_{(i} u^-_{j)}, \qquad \text{(III.1)}$$

$$\theta^\pm_{A\alpha} = \theta^i_\alpha u^\pm_i, \quad \bar\theta^\pm_{A\dot\alpha} = \bar\theta^i_{\dot\alpha} u^\pm_i .$$

Then one can see that the subspace

$$\{ \underbrace{x_A^{\alpha\dot\alpha}, \theta_A^{+\alpha}, \bar\theta_A^{+\dot\alpha}}_{\zeta_A}, u^\pm_i \} \qquad \text{(III.2)}$$

is closed under the following Poincaré N=2 supersymmetry transformations

$$\delta x_A^{\alpha\dot\alpha} = -4i\,(\varepsilon^{i\alpha}\bar\theta_A^{+\dot\alpha} + \theta_A^{+\alpha}\bar\varepsilon^{i\dot\alpha}) u^-_i, \qquad \text{(III.3)}$$

$$\delta\theta^+_{A\alpha} = \varepsilon^i_\alpha u^+_i, \quad \delta\bar\theta^+_{A\dot\alpha} = \bar\varepsilon^i_{\dot\alpha} u^+_i, \quad \delta u^\pm_i = 0 .$$

The significance of this analytic (only $\theta^+_{A\alpha,\dot\alpha}$, but not $\theta^-_{A\alpha,\dot\alpha}$ are present) subspace is in the fact that all the N=2 supersymmetric theories are naturally formulated in it. Indeed, the N=2 matter multiplets (hypermultiplets) are described by the analytic superfields $\omega(\zeta_A, u)$ ($U(1)$ charge 0) and $q^+(\zeta_A, u)$ ($U(1)$ charge +1) with the following free actions

$$S_\omega = \int d\zeta^{(-4)} du\, \omega (D^{++})^2 \omega, \qquad \text{(III.4)}$$

$$S_q = \int d\zeta^{(-4)} du\, \overset{*}{q}{}^+ D^{++} q^+ .$$

Here

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} - 4i\,\theta_A^{+\alpha}\bar\theta_A^{+\dot\alpha}\frac{\partial}{\partial x_A^{\alpha\dot\alpha}} + \theta_A^{+\alpha}\frac{\partial}{\partial\theta_A^{-\alpha}} + \bar\theta_A^{+\dot\alpha}\frac{\partial}{\partial\bar\theta_A^{-\dot\alpha}} \qquad \text{(III.5)}$$

enters instead of the harmonic derivative D^{++} (II.6); the definition of the integral in (III.4) see in Galperin et al.[14]. Next, the N=2 gauge theory is described by an analytic superfield $V^{++}(\zeta_A, u)$ (charge +2). It occurs in close analogy with V^{++} in (II.26). Indeed, the principal constraints determining the geometry of N=2 gauge theory are

$$\{\mathcal{D}^+_\alpha, \mathcal{D}^+_\beta\} = \{\bar{\mathcal{D}}^+_{\dot\alpha}, \bar{\mathcal{D}}^+_{\dot\beta}\} = \{\mathcal{D}^+_\alpha, \bar{\mathcal{D}}^+_{\dot\beta}\} = 0 . \qquad \text{(III.6)}$$

Their solution has again the pure gauge form ((II.21)) with the bridge e^{iv} obeying the equations analogous to (II.24-26). The major difference is that eq. (III.6) does not imply any equations of motion as was the case with (II.20). The analyticity condition $\mathcal{D}^+_\alpha \varphi = \bar{\mathcal{D}}^+_{\dot\alpha}\varphi = 0$ is also manifestly solved in the analytic subspace but again it does not imply dynamical equations. The gauge transformation law for the prepotential V^{++} in N=2 case parallels (II.29), the gauge invariant action can be found in Galperin et al.[6]

All the N=2 theories listed above are known to be conformally invariant. To see this in the framework of harmonic superspace one has to know how the conformal group acts. Again, the problem is to find such a realization of the conformal group which preserves the notion of analyticity and the unimodularity condition $u^{+i}u^-_i = 1$. It can be obtained by supplementing the well known transformation rules for $(x^{\alpha\dot\alpha}, \theta^{\alpha i}, \bar\theta^{\dot\alpha}{}_i)$ [16] by the following transformations of the harmonic coordinates

$$\delta u^+_i = \Lambda^{++} u^-_i$$

$$\delta u^-_i = 0, \qquad \text{where} \qquad \text{(III.7)}$$

$$\Lambda^{++} \equiv \lambda^{ij} u^+_i u^+_j + 4i\, k_{\alpha\dot\alpha}\theta_A^{+\alpha}\bar\theta_A^{+\dot\alpha} + 4i(\theta_A^{+\alpha}\eta^i_\alpha + \bar\eta^i_{\dot\alpha}\bar\theta_A^{+\dot\alpha}) u^+_i, \quad \overset{*}{\overline{(\Lambda^{++})}} = \Lambda^{++}$$

contains the parameters of SU(2) (λ^{ij}), conformal boosts ($k_{\alpha\dot\alpha}$) and the second N=2 supersymmetry ($\eta^i_{\alpha\dot\alpha}$). The λ, k, η transformations of the coordinates $x_A^{\alpha\dot\alpha}, \theta_A^{+\alpha,\dot\alpha}$ of the analytic subspace follow from (III.1) and the transformations of $x^{\alpha\dot\alpha}, \theta^{\alpha,\dot\alpha i}$

$$\delta x_A^{\alpha\dot\alpha} = k_{\beta\dot\beta} x_A^{\alpha\dot\beta} x_A^{\beta\dot\alpha} + 4i(x_A^{\alpha\dot\beta}\bar\theta_A^{+\dot\alpha}\bar\eta^i_{\dot\beta} - x_A^{\dot\alpha\beta}\theta_A^{+\alpha}\eta^i_\beta) u^-_i -$$

$$- 4i\,\lambda^{(ij)} u^-_i u^-_j \theta_A^{+\alpha}\bar\theta_A^{+\dot\alpha} \qquad \text{(III.8)}$$

$$\delta\theta_A^{+\alpha} = k_{\beta\dot\beta} x^{\alpha\dot\beta}\theta_A^{+\beta} - 2i(\theta_A^+)^2\eta^{\alpha i} u^-_i + x_A^{\alpha\dot\beta}\bar\eta^i_{\dot\beta} u^+_i +$$

$$+ \lambda^{(ij)} u^+_i u^-_j \theta_A^{+\alpha}, \quad \delta\bar\theta_A^{+\dot\alpha} = -\overset{*}{\overline{(\delta\theta_A^{+\alpha})}} .$$

One clearly sees that the analytic subspace remains invariant x). Note the peculiar way in which the SU(2) group acts in (III.7,8)

x) For completeness we present also the transformation law for $\theta_A^{-\alpha}$ $\left(\delta\bar\theta_A^{-\dot\alpha} = -\overset{*}{\overline{(\delta\theta_A^{-\alpha})}}\right)$

$$\delta\theta_A^{-\alpha} = k_{\beta\dot\beta} x_A^{\alpha\dot\beta}\theta_A^{-\beta} - 2i\,k^{\alpha\dot\beta}(\theta_A^-)^2\bar\theta^+_{A\dot\beta} + 4i\,\eta^i_\beta\theta_A^{-\beta}(\theta_A^{-\alpha}u^+_i - \theta_A^{+\alpha}u^-_i) +$$

$$+ \bar\eta^i_{\dot\beta}(x_A^{\alpha\dot\beta} + 4i\,\theta_A^{-\alpha}\bar\theta_A^{+\dot\beta}) u^-_i + \lambda^{(ij)} u^-_i (u^-_j\theta_A^{+\alpha} - u^+_j\theta_A^{-\alpha})$$

(recall the similar situation in (II.34)). The implications of this will be explained at the end of this section.

Let us now examine the superconformal invariance of the hypermultiplet models (III.4). First of all, the volume element $d\zeta^{(-4)}du$ produces the weight factor

$$\mathrm{Ber}\frac{\partial(\zeta_A', u')}{\partial(\zeta_A, u)} \simeq 1 + \frac{\partial}{\partial x_A^{\alpha\dot\alpha}}\delta x_A^{\alpha\dot\alpha} - \frac{\partial}{\partial \theta_A^{+\alpha}}\delta\theta_A^{+\alpha} - \frac{\partial}{\partial \bar\theta_A^{+\dot\alpha}}\delta\bar\theta_A^{+\dot\alpha} + u^{-i}\frac{\partial}{\partial u^{+i}}\Lambda^{++} = 1 - 2\Lambda; \quad D^{++}\Lambda = \Lambda^{++}; \tag{III.9}$$

$$\Lambda = -\ell + \lambda^{ij}u_i^+u_j^- + 4i(\theta_A^{+\alpha}\eta_\alpha^i + \bar\eta^i_{\dot\alpha}\bar\theta_A^{+\dot\alpha})u_i^- - k_{\alpha\dot\alpha}x_A^{\alpha\dot\alpha}.$$

Second, the derivative D^{++} transforms as follows

$$D^{++\prime} \simeq D^{++} - \Lambda^{++}D^0; \quad D^{0\prime} = D^0, \tag{III.10}$$

where D^0 is the operator counting charges (cf. (II.5))

$$D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta_A^{+\alpha,\dot\alpha}\frac{\partial}{\partial \theta_A^{+\alpha,\dot\alpha}} - \theta_A^{-\alpha,\dot\alpha}\frac{\partial}{\partial \theta_A^{-\alpha,\dot\alpha}}.$$

So D^{++} ceases to be covariant by itself. Nevertheless, the actions (III.4) are superconformally invariant, provided the superfields ω and q^+ transform with weights correlated with their dimension

$$[\omega] = [q^+] = [\mathrm{mass}]^1 \Rightarrow \omega' = (1+\Lambda)\omega, \; q^{+\prime} = (1+\Lambda)q^+. \tag{III.11}$$

Note that $D^{++}q^+$ and $(D^{++})^2\omega$ are covariant objects (just these enter the equations of motion) but $D^{++}\omega$ is not.

Analogously, the dimensionless N=2 gauge prepotential has weight zero. This leaves the matter couplings, e.g.,

$$\int d\zeta^{(-4)}du\, \overset{*}{q}{}^{+}\, iV^{++}q^{+}$$

invariant. One can show that the N=2 SYM action is superconformally invariant as well. /6/

Above we mentioned that the SU(2) subgroup of the superconformal group is peculiarly realized, in particular, on $u^\pm_i$. This manifests itself in the transformation laws of the components of the superfields. For instance, consider $q^+(\zeta_A(z,u),u)$ as a function of the central basis coordinates. It has the following harmonic expansion

$$q^+(\zeta_A(z,u),u) = \sum_{n=0}^{\infty} q^{i_1\ldots i_{2n+1}}(z)\, u^+_{(i_1}\cdots u^+_{i_{n+1}}u^-_{i_{n+2}}\cdots u^-_{i_{2n+1})},$$

where the coefficients $q^{i_1\ldots i_{2n+1}}(z)$ are ordinary superfields. Under the SU(2) transformations (III.7) these superfields get mixed up. At the same time, there exists another SU(2) group under which all the above theories are also invariant and which rotates $u^\pm_i$ and, correspondingly, $q^{i_1\ldots i_n}(z)$ in the standard way

$$\delta_{St}u^{\pm i} = \alpha^i{}_j u^{\pm j}. \tag{III.12}$$

It is not the subgroup of the N=2 superconformal group. Instead it forms a semi-direct product with the latter. Remarkably, these two SU(2) groups coincide for the on-shell q^+ hypermultiplets and N=2 gauge theory. Indeed, a SU(2) variation of q^+ taken at a fixed point is:

$$\delta^*_{SU(2)}q^+ = \delta^*_{St}q^+ - \lambda^{ij}u_i^-u_j^-D^{++}q^+, \tag{III.13}$$

where we have singled out the standard (III.12) type part. However on-shell in the free case

$$D^{++}q^+ = 0 \Rightarrow q^+ = q^i(z)u_i^+$$

and a difference between two SU(2) variations disappears, i.e., on-shell (III.13) means that

$$\delta_{SU(2)}q^i(z) = \lambda^i{}_j q^j(z).$$

Similarly, $\delta^*_{SU(2)}V^{++}$ can be represented as

$$\delta^*_{SU(2)}V^{++} = \delta^*_{St}V^{++} + (D^{++}\rho + i[V^{++},\rho]), \quad \rho \equiv -\lambda^{ij}u_i^-u_j^-V^{++} = \overset{*}{\rho}, \tag{III.14}$$

thus demonstrating that in the pure N=2 gauge theory two SU(2) groups coincide modulo a gauge transformation. This property extends to the case of on-shell q^+ minimally coupled to V^{++}. Indeed, in this case the equation of motion $(D^{++} + iV^{++})q^+ = 0$ implies that the second term in (III.13) is gauge transformation with the same parameter ρ as in (III.14)

$$\delta^*_{SU(2)}q^+ = \delta^*_{St}q^+ - i\rho q^+.$$

For ω hypermultiplet, the situation is more peculiar, even on -shell two SU(2)'s are realized in essentially different ways.

Concluding this section, we point out that the transformations of the N=2 superconformal group in harmonic superspace are easily generalized for the case N=3. Again, the leading principle is the preservation of the corresponding analytic subspace /6/.

The off-shell N=3 gauge theory proves superconformally invariant, as could be expected.

IV. N=2 conformal supergravity

N=2 harmonic superspace provides the framework for the unconstrained superfield formulation of N=2 Einstein supergravity in its version given in Fradkin and Vasiliev (1979) and de Wit et al. (1980). At the same time, as was shown in de Wit et al. (1981), this and other versions of Einstein supergravity can be obtained from conformal supergravity by compensation of the extra gauge transformations. Therefore the question of finding an unconstrained superfield formulation of N=2 conformal supergravity is of considerable interest.

Here we shall present, for the first time, the unconstrained prepotentials and the full nonlinear coordinate group for N=2 conformal supergravity.

The gauge group of this theory is a local extension of the rigid one (III.7,8) which preserves the fundamental concept of analytic subspace, U(I)-charge, the unimodularity condition $u^{+i}u^-_i = 1$ and the reality properties with respect to conjugation $\underline{*}$

$$
\begin{aligned}
&\delta x_A^m = \lambda^m(\zeta_A, u)\\
&\delta \theta_A^{+\mu} = \lambda^{+\mu}(\zeta_A, u)\\
&\delta \bar\theta_A^{+\dot\mu} = \bar\lambda^{+\dot\mu}(\zeta_A, u)\\
&\delta u^+_i = \lambda^{++}(\zeta_A, u)\, u^-_i\\
&\delta u^-_i = 0\\
&\delta \theta_A^{-\mu} = \lambda^{-\mu}(\zeta_A, u, \theta_A^-, \bar\theta_A^-),\ \delta\bar\theta_A^{-\dot\mu} = \bar\lambda^{-\dot\mu}(\zeta_A, u, \theta_A^-, \bar\theta_A^-).
\end{aligned}
\tag{IV.1}
$$

In order to introduce the prepotentials we consider the harmonic derivative $\mathcal{D}^{++}$ and postulate the following generalization of the rigid transformation law (III.10):

$$
\begin{aligned}
&\mathcal{D}^{++\prime} = \mathcal{D}^{++} - \lambda^{++}\mathcal{D}^0,\\
&\mathcal{D}^{0\,\prime} = \mathcal{D}^0.
\end{aligned}
\tag{IV.2}
$$

The derivative

$$
\mathcal{D}^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta_A^{+\mu,\dot\mu}\frac{\partial}{\partial\theta_A^{+\mu,\dot\mu}} - \theta_A^{-\mu,\dot\mu}\frac{\partial}{\partial\theta_A^{-\mu,\dot\mu}}
\tag{IV.3}
$$

is the same as in the rigid case because it just "counts" the U(I) charges which are strictly respected by the group (IV.1). However, the derivative $\mathcal{D}^{++}$ acquires non-trivial vielbeins

$$
\begin{aligned}
\mathcal{D}^{++} = {} & u^{+i}\frac{\partial}{\partial u^{-i}} + H^{(+4)}u^{-i}\frac{\partial}{\partial u^{+i}} + H^{++m}\frac{\partial}{\partial x_A^m} +\\
& + H^{++\mu+}\frac{\partial}{\partial\theta_A^{+\mu}} + H^{++\dot\mu+}\frac{\partial}{\partial\bar\theta_A^{+\dot\mu}} + H^{++\mu-}\frac{\partial}{\partial\theta_A^{-\mu}} + H^{++\dot\mu-}\frac{\partial}{\partial\bar\theta_A^{-\dot\mu}}\,.
\end{aligned}
\tag{IV.4}
$$

In the rigid case $D^{++}\Phi$ is analytic if Φ is analytic and we should preserve this important property in the local case. Then we have for prepotentials

$$
\begin{aligned}
&H^{(+4)} = H^{(+4)}(\zeta_A, u),\ H^{++m} = H^{++m}(\zeta_A, u)\\
&H^{++\mu+} = H^{++\mu+}(\zeta_A, u),\ H^{++\dot\mu+} = H^{++\dot\mu+}(\zeta_A, u),
\end{aligned}
\tag{IV.5}
$$

i.e., they must be analytic superfields (while $H^{++-\mu}, H^{++-\dot\mu}$ are general ones since $\theta_A^-, \bar\theta_A^-$ is not a part of the analytic subspace). This requirement is, of course, compatible with the transformation properties of the vielbeins following from (IV.1-3):

$$
\begin{aligned}
&\delta H^{(+4)} \equiv H^{(+4)\prime}(\zeta_A', u') - H^{(+4)}(\zeta_A, u) = \mathcal{D}^{++}\lambda^{++}\\
&\delta H^{++m} = \mathcal{D}^{++}\lambda^m\\
&\delta H^{++\mu,\dot\mu+} = \mathcal{D}^{++}\lambda^{+\mu,\dot\mu} - \theta_A^{+\mu,\dot\mu}\lambda^{++}\\
&\delta H^{++\mu,\dot\mu-} = \mathcal{D}^{++}\lambda^{-\mu,\dot\mu} + \theta_A^{-\mu,\dot\mu}\lambda^{++}.
\end{aligned}
\tag{IV.6}
$$

Now we can claim that the vielbeins H are the unconstrained prepotentials for N=2 conformal supergravity. Indeed, the analytic superfields $H^{(+4)}$, H^{++m} and $H^{++\mu,\dot\mu+}$ have the following superspin (Y) and superisospin (I) content (see /4/)

$$
\begin{aligned}
&H^{(+4)}: Y = 0;\ I = 1, 2, 3, \ldots\\
&H^{++m}: Y = 1, 0;\ I = 0, 1, 2, \ldots\\
&H^{++\mu,\dot\mu+}\ Y = \tfrac{1}{2};\ I = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \ldots
\end{aligned}
\tag{IV.7}
$$

On the other hand, the analytic parameters $\lambda^{++}, \lambda^m, \lambda^{+\mu,\dot\mu}$ contain:

$$\begin{aligned} &\lambda^{++} : Y = 0\,;\ I = 0, 1, 2, \dots \\ &\lambda^{m} : Y = 0, 1\,;\ I = 1, 2, 3, \dots \\ &\lambda^{+\mu,\dot\mu} : Y = \tfrac{1}{2}\,;\ I = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \dots \end{aligned} \tag{IV.8}$$

Taking into account (IV.6–IV.8) one sees that the gauge-independent part of the analytic prepotentials is just the superspin 1, superisospin 0 multiplet of N=2 conformal supergravity. (The non-analytic prepotentials $H^{++\mu,\dot\mu-}$ can be completely gauged away by the corresponding parameters $\lambda^{-\mu,\dot\mu}$). More explicitly, the Wess-Zumino gauge for the prepotentials is

$$\begin{aligned} H^{(+4)} &= (\theta^+)^2(\bar\theta^+)^2 D(x) \\ H^{++m} &= \theta^+\sigma_a\bar\theta^+ e^{am}(x) + (\bar\theta^+)^2\theta^{+\alpha}\psi^m_{\alpha i}(x)u^{-i} + \\ &+ (\theta^+)^2\bar\theta^+_{\dot\alpha}\bar\psi^{\dot\alpha m}_i(x)u^{-i} + (\theta^+)^2(\bar\theta^+)^2 V^m_{(ij)}(x)u^{-i}u^{-j} \\ H^{++\mu+} &= (\theta^+)^2\bar\theta^+_{\dot\mu}A^{\mu\dot\mu}(x) + (\bar\theta^+)^2\theta^{+\nu}t_{(\nu}{}^{\mu)}(x) + (\theta^+)^2(\bar\theta^+)^2\chi^\mu_i u^{-i} \\ H^{++\dot\mu+} &= (\bar\theta^+)^2\theta^+_\mu A^{\mu\dot\mu}(x) + (\theta^+)^2\bar\theta^{+\dot\nu}t_{(\dot\nu}{}^{\dot\mu)}(x) + (\theta^+)^2(\bar\theta^+)^2\bar\chi^{\dot\mu}_i u^{-i}. \end{aligned} \tag{IV.9}$$

Here one can identify all the components of off-shell N=2 conformal supergravity /8/.

As mentioned in the beginning of this section, the final aim when considering conformal supergravity is to be able to construct Einstein supergravities. The method developed by de Wit et al. /8/ consists in the compensation of the extra gauge transformations (e.g., γ_5, Weyl, SU(2)) by various matter and Maxwell multiplets. Thus, with the help of a Maxwell multiplet one compensates the γ_5 and Weyl transformations and gets the so-called "almost simple" N=2 supergravity /18,8/.
Then, adding the so-called "non-linear", "tensor" or "scalar" multiplets one obtains the three known off-shell versions of Einstein N=2 supergravity /2,8,17,19/. In fact, the scalar multiplet of de Wit et al. is a form of the hypermultiplet with central charge /20,21/ . However now we know also another version of the hypermultiplet described by the analytic superfield ω (III.4). It has no central charge, instead it has an infinite number of auxiliary fields. Using

it as compensator allows us to construct a new version of N=2 Einstein supergravity. Here we shall sketch this construction. First, we add a new bosonic coordinate x^5 which has been invented in Sohnius /21/ for the description of superfields with central charge. However, in our case nothing will depend on it, even its own transformation parameter

$$\delta x^5 = \lambda^5(\zeta_A, u). \tag{IV.10}$$

Then we add a new term in (IV.4),

$$\mathcal{D}^{++}_{modif.} = \mathcal{D}^{++}_{IV.4} + H^{++5}(\zeta_A, u)\frac{\partial}{\partial x^5}. \tag{IV.11}$$

As explained in Galperin et al. /4/ , the analytic superfield H^{++5} with its transformation law

$$\delta H^{++5} = \mathcal{D}^{++}\lambda^5 \tag{IV.12}$$

describes a Maxwell multiplet. In the same paper it was shown also that H^{++5} has a non-vanishing flat limit,

$$H^{++5} = i(\theta^+_A\theta^+_A - \bar\theta^+_A\bar\theta^+_A) + h^{++5}(\zeta_A, u). \tag{IV.13}$$

So, it serves as a compensator for some of the parameters in $(\lambda^{+\mu,\dot\mu}(\zeta_A, u)$ (IV.1), in particular, for the γ_5 and dilatation ones. The remaining extra gauge transformations (in particular, SU(2)) are compensated by a superfield $\omega(\zeta_A, u)$ which has flat limit 1:

$$\omega(\zeta_A, u) = 1 + \omega(x) + \omega^{(ij)}(x)u^+_i u^-_j + \dots \tag{IV.14}$$

It transforms as follows

$$\begin{aligned} \omega'(\zeta'_A, u') &= \mathrm{Ber}^{-\frac{1}{2}}\left(\frac{\partial(\zeta'_A, u')}{\partial(\zeta_A, u)}\right)\omega(\zeta_A, u) \\ \delta\omega &= -\frac{1}{2}\left(\frac{\partial}{\partial x^m_A}\lambda^m - \frac{\partial}{\partial\theta^{+\mu}_A}\lambda^{+\mu} - \frac{\partial}{\partial\bar\theta^{+\dot\mu}_A}\bar\lambda^{+\dot\mu} + u^{-i}\frac{\partial}{\partial u^{+i}}\lambda^{++}\right)\omega. \end{aligned} \tag{IV.15}$$

One can easily see that the component $\omega^{(ij)}(x)$ in (IV.14) compensates the local SU(2) transformations. The component $\omega(x)$ is intended to become the Lagrange multiplier for the component $D(x)$ in (IV.9). The action for this version of N=2 Einstein supergravity is the sum of the properly superconformally covariantized actions for H^{++5} and ω and will be presented elsewhere.

Authors are sincerely grateful to Yu.Manin, O.Ogievetsky and B.Zupnik for discussion of instantons in harmonic space .

References

1. Fradkin E.S., Palchik M.Ya. Phys.Rep. 1978, 44, p243.
2. Fradkin E.S., Vasiliev M.A. Phys.Lett., 1979, 85B, p.47.
3. Fradkin E.S., Tseytlin A.A. Phys.Lett., 1984, 134B, p.301.
4. Galperin A., Ivanov E., Kalitzin S., Ogievetsky V., Sokatchev E. Class.Quantum Grav. 1984, 1, p.469.
5. Galperin A., Ivanov E., Kalitzin S., Ogievetsky V., Sokatchev E. Class.Quantum Grav. 1985, 2, p.155. Phys.Lett. 1985, 151B, p.215.
6. Galperin A., Ivanov E., Ogievetsky V., Sokatchev E., Preprints JINR, E2-85-127, 128, 1985.
7. Rocek M., Siegel W. Phys.Lett., 1981 , 105B, p.275.
   Rivelles V., Taylor J.G. J.Phys. 1982, A15, p.163.
   Stelle K.S. Prepr.Santa Barbara, NSF-ITP-85-01.
   Howe P.S., Stelle K.S., West P.C. Kings College Prepr., 1985.
8. de Wit B., van Holten J.W., van Proyen A. Nucl.Phys.1981, B184, p.77.
9. Ward R.S. Phys.Lett., 1977, 61A, p.81.
10. Atiyah M.F. Geometry of Yang-Mills Fields; Lezioni Fermiane, Pisa. 1979.
11. Manin Yu.I., 1984, Fields and Complex Geometry, Moscow, Nauka.
12. Witten E. Phys.Lett., 1978, 78B, p.394.
13. Rosly A.A. 1982, Proc. of the Intern.Seminar on Group Theoretical Methods in Physics, Nauka, Moscow, v.I, p.263.
14. Belavin A.A, Zacharov V.E. Phys.Lett., 1977, 73B, 53.
15. Belavin A.A., Polyakov A.M., Schwartz A.S., Tyupkin Yu.S. Phys.Lett., 1975, 59B, p.85.
16. Ferber A. Nucl. Phys. 1978, B132, p.55.
17. de Wit B., van Holten J.W., van Proeyen A. Nucl. Phys., 1980, B167, p.186.
18. Breitenlohner P., Sohnius M.F., Nucl. Phys., 1980, B165, p.483.
19. de Wit B., Philippe R., van Proeyen A. Nucl. Phys., 1983, B219, p.143.
20. Fayet P. Nucl. Phys., 1976, B113, p.135.
    Salam A., Strathdee J. Nucl. Phys., 1975, B97, p.293.
21. Sohnius M.F. Nucl. Phys., 1978, B138, p.109.

Received by Publishing Department
on May 15, 1985

Гальперин А. и др.   E2-85-363

Конформная инвариантность
в гармоническом суперпространстве

В гармоническом суперпространстве реализована N = 2 конформная суперсимметрия и проанализированы ее отличительные черты. Найдены координатная группа и аналитические препотенциалы для N = 2 конформной супергравитации. Предложена новая версия N = 2 эйнштейновской супергравитации с бесконечным числом вспомогательных полей. В ней используется в качестве компенсатора гипермультиплет без центральных зарядов и без связей.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Galperin A. et al.   E2-85-363

Conformal Invariance
in Harmonic Superspace

N = 2 conformal supersymmetry is realized in harmonic superspace, its peculiarities are analyzed. The coordinate group and the analytic prepotentials for N = 2 conformal supergravity are found. Using the inconstrained hypermultiplet without central charges as a compensator we pbtain a new version of N = 2 Einstein supergravity with infinitely many auxiliary fields.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985