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TOPOLOGICAL CONFINEMENT IN THE SCHWINGER MODEL

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1. INTRODUCTION

The problem of the quark confinement in its contemporary formulation is based on the deep inelastic scattering experiments. It has been found [1] that the inclusive cross sections, i.e., sums over all hadronic states, may be described with the help of the imaginary parts of the quark (parton) diagrams *. From a field-theoretical point of view the quark hadronization implies zero probability for the coloured-particles creation. This statement may be considered as a model-free definition of the confinement.

Nowadays the interpretation of the confinement problem - its criteria and mechanisms, is rather dependent on the model choice. The most popular criteria are the existence of a linearly rising potential between the quarks and the increasing of the Wilson loop area. In their formulation the Schwinger model-two-dimensional massless quantum electrodynamics, played an essential role [4,5]. However, recent calculations of the coloured particle Green functions made the confidence in their strictnes doubtful. It has been found that they are compatible with the existence of poles in the quark Green function. Calculation of these poles is one of the standard methods used to determine the elementary excitation spectrum in QFT and statistical physics. The existence of a pole is interpreted as the presence in this spectrum of a particle with quark quantum numbers. From such a point of view the absence of a pole may be considered as a confinement criterium (for instance, it coincides with the model-independent one following from the experiment). Such a situation takes place in the theories with topological vacuum degeneration [8].

In the present paper we discuss the confinement problem in the Schwinger model in this context. The fermionic sector of the model is insufficiently studied due to some difficulties:

1) The bosonization of the theory leads to some additional effects which have to be separated from the dynamical ones;

2) The Green functions are not gauge-invariant, that means a dependence of the results on the gauge-condition choice;

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* Quark diagram hadronization has been called the quark-hadron duality principle which is now the QCD-phenomenology basis and is used successfully in different sum-rules derivation [2,3].

3) The existence of infrared divergencies requires the corresponding regularization scheme.

The paper is organized as follows:

Section 2 is devoted to the free-fermions bosonization in two-dimensional space-time.

In Section 3 the quark Green function in Coulomb gauge is considered.

In Sections 4,5 a gauge-invariant method for the Green functions construction is proposed. The theory is quantized in a finite-volume space-time.

2. BOSONIZATION OF THE FREE FERMIONS

Bosonization provides an adequate method for the description of two-dimensional field-theoretical models with fermions [9,10]. However, considering the equivalent bosonic-theory properties one usually does not distinguish bosonization effects from the dynamical ones. To do this we shall begin with a brief review of the free-fermions bosonization in two dimensions. In this case the Lagrangian is

$$\mathcal{L}_0(x) = i\bar{\psi}(x)\gamma^{\mu}\partial_{\mu}\psi(x). \tag{1}$$

In quantum theory with such a Lagrangian there appears an anomalous term in the current-components commutator

$$[j_{50}(x), j_{51}(y)] = \frac{1}{i\pi}\partial_y\delta(x-y), \quad j_{5\mu}(x) = \bar{\psi}(x)\gamma_5\gamma_{\mu}\psi(x). \tag{2}$$

As is known, the physical reason for this anomaly is the filling off all negative-energy states, i.e., the Dirac sea [11-13].

A simple substitution

$$j_{5\mu}(x) = \frac{1}{\sqrt{\pi}}\partial_{\mu}\phi(x) \tag{3}$$

transforms relation (2) into the scalar field $\phi(x)$ commutator. Then, the current conservation law takes the form of the massless D'Alembert equation

$$\partial^{\mu}j_{5\mu}(x) = 0 \quad \Rightarrow \quad \partial^{\mu}\partial_{\mu}\phi(x) = \Box\phi(x) = 0. \tag{4}$$

Thus, the theory (1) is equivalent to the free massless scalar field one $\mathcal{L}_{0,B} = \frac{1}{2}(\partial_{\mu}\phi)^2$.

There is only this scalar particle in the spectrum; fermions apparently disappeared. An analogous situation in the Schwinger model has been interpreted in papers [10] as a manifestation of the confinement which takes place there. Following these papers, one might conclude that the free fermions are confined too.

Such a conclusion is obviously wrong, so we need a correct description of the fermions themselves in the bosonized theory. In other words, we have to find the functional dependence of the spinors $\psi(x)$ on the field $\phi(x)$.

The axial-current component $j_{50}(x)$ is proportional to the canonically conjugated momentum for the field $\phi(x)$

$$j_{50}(x) = \frac{1}{\sqrt{\pi}}\partial_0\phi(x) = \frac{1}{\sqrt{\pi}}\pi(x).$$

So, the following relations take place:

$$[j_{50}(x), f(\phi(y))] = \frac{1}{\sqrt{\pi}}[\pi(x), f(\phi(y))] = \frac{1}{i\sqrt{\pi}}\frac{\delta}{\partial\phi(x)}f(\phi(y))$$

that lead to the equation on $\psi(x)$:

$$[j_{50}(x), \psi(\phi(y))] = \frac{1}{i\sqrt{\pi}}\frac{\delta}{\delta\phi(x)}\psi(\phi(y)) = \delta(x-y)\psi(\phi(y))\gamma_5.$$

Its solution has the form

$$\psi(x) = \exp\{i\sqrt{\pi}\gamma_5\phi(x)\}\chi(x), \quad \psi^{+}(x) = \exp\{-i\sqrt{\pi}\gamma_5\phi(x)\}\chi^{+}(x), \tag{5}$$

where $\chi(x)$ is a function which does not depend on the field $\phi(x)$. An additional requirement for reproducing the free two-point fermion Green function in this language may be used for defining $\chi(x)$:

$$\langle\psi(x)\bar{\psi}(y)\rangle = e^{-i\pi\Lambda_0(x-y)}\langle\chi(x)\bar{\chi}(y)\rangle \tag{6}$$

(here $\Lambda_0(x-y)$ is Green's function of the free massless scalar field). This task may be achieved if we put

$$\chi(x) = \exp\{i\sqrt{\pi}\gamma_5\Sigma(x)\}\chi_0(x), \tag{7}$$

where $\Sigma(x)$ is a free massless scalar field quantized with an indefinite metrics and $\chi_0(x)$ is a free fermion field. Thus, the fermion Green function may be obtained from the generating functional with an action

$$S_B = \int d^2x\,\mathcal{L}_B,$$

$$\mathcal{L}_B = \frac{1}{2}(\partial_{\mu}\phi)^2 - \frac{1}{2}(\partial_{\mu}\Sigma)^2 + i\bar{\chi}_0\gamma^{\mu}\partial_{\mu}\chi_0.$$

$$+\bar{\eta}(x)\, e^{i\sqrt{\pi}\gamma_5(\phi+\Sigma)}\chi_0 + \bar{\chi}_0\, e^{-i\sqrt{\pi}\gamma_5(\phi+\Sigma)}\eta(x), \tag{8}$$

$\bar{\eta}(x)$ and $\eta(x)$ being the fermion sources.

3. SCHWINGER'S MODEL IN THE COULOMB GAUGE

Let us now turn to the Schwinger model:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu})\psi, \quad F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{9}$$

We shall choose the gauge $A_1 = 0$ which is both the two-dimensional version of axial and radiation (Coulomb) gauges. Then, the equation of motion for A_0 (for instance, a constraint equation) takes the form

$$\frac{\partial S}{\partial A_0} = 0 \implies \partial_1^2 A_0 = e j_0. \tag{10}$$

Substitution of its formal solution in (9) leads us to the following action:

$$S = \int d^2x\,[\,i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + \frac{e^2}{4}\int dy_1\, j_0(x)|x_1 - y_1|\, j_0(y)] =$$

$$= \int d^2x\,[\,i\bar{\psi} j^{\mu}\partial_{\mu}\psi + \frac{e^2}{2} j_0(x)\frac{1}{\partial_1^2} j_0(x)]. \tag{11}$$

It is this Coulomb interaction in (11) which causes the anomalous axial current divergence. In terms of the bosonic field $\phi(x)$ its partial conservation law takes the form of the massive Klein-Gordon equation $(\Box - m^2)\phi(x) = 0$, where $m = e\pi^{-1/2}$ is the field $\phi(x)$ mass. Its appearance is an entirely quantum effect, caused by the gauge field induced "polarization" of Dirac's vacuum [11].

From eqs. (3), (8), (9) we obtain the total bosonized action of the theory

$$S_{tot,B} = \int d^2x\,[\,\frac{1}{2}(\partial_{\mu}\phi)^2 - \frac{1}{2}(\partial_{\mu}\Sigma)^2 + i\bar{\chi}_0 j^{\mu}\partial_{\mu}\chi_0 - \frac{m^2}{2}\phi^2 + \bar{\eta}\, e^{i\sqrt{\pi}\gamma_5(\phi+\Sigma)}\chi + \bar{\chi}\, e^{-i\sqrt{\pi}(\phi+\Sigma)}\eta\,]. \tag{12}$$

The action (12) determines the generating functional for the exact Green functions in the Schwinger model (in the gauge $A_1 = 0$):

$$Z[\bar{\eta}, \eta] = \langle vac\,|\,T\exp\{-i\int H\, dx_0\}\,|\,vac\rangle.$$

So, for the two-current correlator we find

$$\langle j_{5\mu}(p)\, j_{5\nu}(q)\rangle \sim \frac{p_{\mu}p_{\nu}}{m^2 - p^2 - i\epsilon}\,\delta^2(p+q). \tag{13}$$

This result points to the existence of a massive uncharged scalar particle in the model spectrum.

For the two-point fermions Green function with the help of eqs.(6), (7), (12) we obtain the following expression:

$$G(x-y) = G_0(x-y)\, e^{-i\pi[\Delta_m(x-y) - \Delta_0(x-y)]}, \tag{14}$$

where $G_0(x-y)$ is the free fermions Green function and $\Delta_m(x-y)$ is the massive scalar field one. The asymptotic behaviour of the function (14) in the momentum space is

$$G(p)_{p\to\infty} \sim \frac{\hat{p}}{p^2}, \quad G(p)_{p\to 0} \sim \frac{\hat{p}}{(p^2 + i\epsilon)^{5/4}} \tag{15}$$

(see Appendix and also paper [14]). In the literature there are contradictory opinions if such a behaviour ensures confinement or not. Note, that in $QED_{(3+1)}$ there are some gauges in which an analogous behaviour of Green function of the observable electrons takes place. At least, function (15) has a singularity at the point $p = 0$ [15].

4. GAUGE-INVARIANT VARIABLES AND VACUUM TOPOLOGICAL DEGENERATION IN THE SCHWINGER MODEL

As is known, the choice of the gauge has an essential influence on the properties of Green functions of the coloured objects [16]. So, it would be convenient to formulate the latter in terms of gauge-invariant quantities only.

Equation (10), without any gauge-fixing, takes the form

$$\partial_1^2 A_0 = \partial_1\partial_0 A_1 + e j_0. \tag{16}$$

So, instead of (11) we have the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_0 A_1^I)^2 + i\bar{\psi}^{-I}\gamma^{\mu}\partial_{\mu}\psi^I - \frac{e^2}{2}(\partial_1^{-1} j_0^I)^2. \tag{17}$$

where the variables $A^I(A), \psi^I(\psi, A)$ are defined as

$$A_1^I(A) = A_1 - \partial_1 \frac{1}{\partial_1^2} \partial_1 A_1 = 0, \quad \psi^I(\psi, A) = \exp(-ie\partial_1^{-1} A_1)\psi$$

and are invariant under gauge transformations

$$\left.\begin{aligned} A_1^{(\lambda)} &= A_1 + \frac{\partial_1 \lambda}{e} \\ \psi^{(\lambda)} &= e^{i\lambda} \psi \end{aligned}\right\} \Rightarrow \begin{aligned} &A_1^I(A^{(\lambda)}) = A_1^I(A) \\ &\psi^I(A^{(\lambda)}, \psi^{(\lambda)}) = \psi^I(A, \psi). \end{aligned} \tag{18}$$

The Schwinger model action in terms of the gauge-invariant variables (18) and the sources

$$\eta^I = \exp(-ie\partial_1^{-1} A_1)\eta, \quad \bar{\eta}^I = \bar{\eta} \exp(ie\partial_1^{-1} A_1) \tag{19}$$

coincides with the action in the gauge

$$\partial_1 A_1 = 0, \quad \partial_1 \partial_0 A_1 = 0 \to A_1 = 0. \tag{20}$$

The choice of the variables (18) is fixed by the dynamics (i.e., by the constraint equation) and by the requirement of gauge invariance. However, in the construction based on the explicit solution of the constraint equation (16) there is a functional ambiguity: the action of the inverse operator $(\partial_1)^{-2}$ in (16) is determined up to a function which satisfies the D'Alembert equation

$$\partial_1^2 \lambda = 0, \quad \partial_1^2 \partial_0 \lambda = 0. \tag{21}$$

So, the variables (18) take the form

$$(A_1^I)^\lambda = A_1 - \partial_1 \frac{1}{\partial_1^2} \partial_1 A_1 + \frac{\partial_1 \lambda}{e} = A_1^I + \frac{\partial_1 \lambda}{e}, \quad (\psi^I)^\lambda = e^{i\lambda} \psi^I. \tag{22}$$

In the gauge (20) this is the well-known Gribov gauge ambiguity [17]. It is caused by the existence of gauge transformations

$$A_1 = e^{i\lambda} (A_1 + i \frac{\partial_1}{e}) e^{-i\lambda} = A_1 + \frac{\partial_1 \lambda}{e},$$

that leave invariant gauge conditions (20). This is possible if the gauge function $\lambda(x)$ satisfies eqs.(21) called Gribov's equations.

In the finite-volume space-time eqs.(21) have nontrivial solutions in the class of smooth gauge transformations, $\exp\{i\lambda(x)\}$. These transformations are characterized by an integer degree of mapping of the space-time boundary onto the group U(1) [11]

$$\nu = \frac{e}{4\pi} \int_{-T/2}^{T/2} dx_0 \int_{-R/2}^{R/2} dx_1 \epsilon_{\mu\nu} F^{\mu\nu} = n_+ + n_- \tag{23}$$

$$n_\pm = \frac{1}{2\pi} \int_{-R/2}^{R/2} dx_1 \partial_1 \lambda_\pm (x_1, x_0 = \pm \frac{T}{2}) = \pm(0, 1, 2, \ldots) \tag{24}$$

(The number ν is also called the Pontryagin index [18]).

The solution of eqs. (21) with boundary conditions (23), (24) takes the form

$$\lambda(x) = \lambda(x_1 | N(x_0)) = 2\pi N(x_0) \frac{x_1}{R}, \tag{25}$$

where the function $N(x_0)$ takes as initial and final values the numbers $n_\pm$:

$$N(x_0 = \pm \frac{T}{2}) = n_\pm. \tag{26}$$

The gauge field $A_1(x) = \frac{1}{e} \partial \lambda(x)$ interpolates between purely gauge fields at the ends of the time interval

$$A_{1(\pm)} = \frac{1}{e} \partial_1 \lambda_\pm \tag{27}$$

which are called "classical vacua" [19].

The existence of smooth solutions of eq.(21) represents the vacuum topological degeneration in Schwinger's model. It is not connected in any way with the chiral invariance breaking (which is the usual explanation of this phenomenon [10,20])*, but with the nontrivial topology of the gauge-field configuration space [11,22]. The dynamics of this degeneration is described by the action

$$S_T = \frac{1}{2} \int_{(V)} d^2x \left(\frac{\partial_1 \partial_0 \lambda}{e}\right)^2 = \int_{-T/2}^{T/2} dx_0 \frac{\dot{N}^2}{2} I, \quad I = \frac{1}{R} \left(\frac{2\pi}{e}\right)^2. \tag{28}$$

Quantization of the action (28) is not difficult

$$K = \frac{\delta L_T}{\delta \dot{N}} = \dot{N} I, \quad [K, N] = i, \quad L_T = \int_{-R/2}^{R/2} dx_1 S_T.$$

The topological momentum K spectrum is easily found by taking into account the physical equivalence of the states

$$\langle p | N \rangle = e^{-ipN} \quad \text{and} \quad \langle p | N + n \rangle = e^{-ip(N+n)}.$$

The real state represents Bloch's wave, which is an average over this degeneration with a weight $\exp(in\theta)$

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*Our results confirm the conclusion made in paper [21] about nonexistence of nontrivial vacuum structure in the Schwinger model, but in the frames of the conventional approach to it.

$$<K|N> = \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} e^{in\theta} e^{-iK(N+n)} =$$

$$= \begin{cases} e^{-i(2\pi k + \theta)N}, & K = 2\pi k + \theta \\ 0, & K \neq 2\pi k + \theta \end{cases} \qquad (29)$$

$k = 0, \pm 1, \pm 2, \dots; \ |\theta| \le \pi.$

This spectrum leads to the discrete spectrum of a constant electric field

$$\frac{R}{2}\hat{E} = \frac{K^2}{2I} = (2\pi k + \theta)^2 \frac{e^2 R}{8\pi^2}, \quad \hat{E} = \frac{1}{i}\frac{\delta}{\delta A_1}.$$

Its minimal (in modulo) value $E_{min} = e\theta/2\pi$ coincides with Coleman's constant electric field he introduced [20] to explain the θ-vacuum in the Schwinger model. He considered θ as a simple additional parameter. In our approach θ is connected with the new topological variable, so it has a dynamical content and these constant electric fields represent the real infrared vacuum of the theory.

5. TOPOLOGY AND CONFINEMENT

The effective Lagrangian is modified when we take into account vacuum topological degeneration. So, replacing (16),(22), (25) into (9), we find

$$L = L_T(\dot{N}) + L_{Sch}(\psi^I) + L_S(\dot{N}, j_0^1, \lambda) +$$

$$+ \int_{-R/2}^{R/2} dx_1 [\bar{\eta}(x_1|N(x_0))\psi^I + \bar{\psi}^I \eta(x_1|N(x_0))],$$

$$L_S(\dot{N}, j_0^1, \dot{\lambda}) = \int_{-R/2}^{R/2} dx_1 \{\dot{N}\partial_1[(\partial_1^{-1} j_0^1)\partial_0\lambda] + e^2\partial_1(\partial_1^{-1} j_0^1 \partial_1^{-2} j_0^1)\}, \qquad (30)$$

$$\eta(x_1|N(x_0)) = g(x_1|N(x_0))\eta(x) = e^{2\pi i N(x_0)\frac{x_1}{R}}\eta(x),$$

$L_{Sch}(\psi^I)$ and $L_T(\dot{N})$ determined by eqs.(17) and (28). It is important to notice that the additional dynamical variable $N(x_0)$, which takes part in the current interaction, causes a change in the phases of coloured field sources in (30) (factor $g(x_1|N(x_0))$. Though the function $g(x_1|N(x_0))$ itself is a smooth

one, after taking an average over degeneration, which it describes, there appears a singularity

$$<g(x_1|N(x_0))> = \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} <n|n + \frac{x_1}{R}> = \delta_{\frac{x_1}{R}, 0}, \qquad (31)$$

where $\delta_{\frac{x_1}{R}, 0}$ is the Croneker symbol.

This singularity does not affect the two-current correlator structure, because the phase factors extinguish each other. There remains the pole at the point $p^2 = e^2\pi^{-1}$, representing the existence of a massive scalar particle in the spectrum.

At the same time the fermion Green function vanishes:

$$G(p) = \lim_{R,T\to\infty} \int_{(V)} d^2x d^2y \, e^{ip(x-y)} e^{-i\pi[\Lambda_m(x-y) - \Lambda_0(x-y)]} G_0(x-y) \times$$

$$\times \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} \sum_{s=-\infty}^{\infty} <n|n + s - \frac{x_1}{R}> <n - s + \frac{y_1}{R}|n> = \qquad (32)$$

$$= \lim_{R,T\to\infty} \int_{(V)} d^2x d^2y \, e^{ip(x-y)} e^{-i\pi(\Lambda_m - \Lambda_0)} G_0(x-y)\delta_{\frac{x_1}{R}, \frac{y_1}{R}} = 0$$

that means the existence of confinement in the sense of the model-independent criterium formulated in section 1. As we have seen, confinement is caused by the topological degeneration of the gauge field vacuum. Note, that the limit procedures in (32) (on R, T and on L) cannot be replaced, the correct way they are following one another being determined as in the quantum statistics [16].

CONCLUSION

We have tries to analyze the reasons for confinement on the example of Schwinger's model. In its conventional interpretation the charged-particles confinement is problematic because of the existence of a singularity in the quanrk Green function. When theory is quantized in terms of gauge-invariant variables in finite-volume space-time there appears topological degeneration of the "coloured" field phases. After taking an average over this degeneration the quark Green function vabishes, but the "neutral" current correlator (in the limit of an infinite volume) coincides with the one in the standard approach to the model [23].

Thus, the assumptions about the topological degeneration of the gauge field vacuum as a reason for the confinement in two-dimensional QED becomes well motivated. We would like to exmphasize that the topological structure of the Schwinger model coincides with the one of a non-abelian gauge theory in four dimensions /24/. So, the conclusion about the existence of confinement takes place in QCD, too /8/.

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APPENDIX

Let us remind the form of the (right) fermions Green's function in two-dimensional momentum space

$$\underset{\sim}{G}_{0R}(p) = \theta(p_+)\theta(-p_-)\frac{1}{p_- + i\epsilon} + \theta(-p_+)\theta(p_-)\frac{1}{p_- - i\epsilon} = \frac{p_+}{p_+p_- + i\epsilon}, \quad (A.1)$$

where the "cone" components are defined as usual $p_\pm = p_0 \pm p_1$.

We shall now calculate the exact Green function of the (right) "quark" in the Schwinger model, using relation (14). As we are going to discuss the confinement problem, we are interested in the behaviour of this function when $p^2 \to 0$. With the corresponding asymptotics of propagators $\Delta_m(x)$ and $\Delta_0(x)$ (entering into eq.(14)) taken into account we find

$$\underset{\sim}{G}_R(p^2 \to 0) = -\frac{1}{4\pi}\int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} dx_-\, e^{i\frac{1}{2}(p_+x_- + p_-x_+)} \frac{(x_+x_-)^{1/4}}{(x_- - i\epsilon)m^{1/2}}$$

$$= -\frac{1}{4\pi m^{1/2}}\Big[\int_0^{\infty} dx_+ x_+^{1/4} e^{\frac{1}{2}ip_-x_+} \int_{-\infty}^{\infty} dx_- \frac{x_-^{1/4}}{x_- - i\epsilon} e^{\frac{1}{2}ip_+x_-} + \quad (A.2)$$

$$+ \int_0^{\infty} dx_+ (-x_+)^{1/4} e^{-\frac{1}{2}ip_-x_+} \int_{-\infty}^{\infty} dx_- \frac{x_-^{1/4}}{x_- + i\epsilon} e^{-\frac{1}{2}ip_+x_-}\Big].$$

The integrals in the first term of (A.2) can be easily calculated

$$\int_0^{\infty} dx_+ x_+^{1/4} e^{\frac{1}{2}ip_-x_+} = \left(\frac{2i}{p_- + i\epsilon}\right)^{5/4} \frac{1}{4}\Gamma\left(\frac{1}{4}\right)\theta(-p_-).$$

$$\int_{-\infty}^{\infty} dx_- \frac{x_-^{1/4}}{x_- - i\epsilon} e^{\frac{1}{2}ip_+x_-} = (-2i)^{1/2}\left(\frac{2i}{p_+ + i\epsilon}\right)^{1/4}\Gamma\left(\frac{1}{4}\right)\theta(p_+)$$

that finally gives us

$$\underset{\sim}{G}_{R(1)}(p^2 \to 0) = C\,\theta(p_+)\theta(-p_-)\left(\frac{1}{p_- + i\epsilon}\right)\left(\frac{1}{p_+p_- + i\epsilon}\right)^{1/4}, \quad C = \frac{\Gamma^2(1/4)}{4\pi m^{1/2}}. \quad (A.3)$$

In an analogous way for the second term we obtain

$$\underset{\sim}{G}_{R(2)}(p^2 \to 0) = C\,\theta(-p_+)\theta(p_-)\left(\frac{1}{p_- - i\epsilon}\right)\left(\frac{1}{p_+p_- + i\epsilon}\right)^{1/4}. \quad (A.4)$$

Comparing (A.3), (A.4) with (A.1) we are led to the conclusion that

$$\underset{\sim}{G}_R(p^2 \to 0) = C\,\underset{\sim}{G}_{0R}(p)\left(\frac{1}{p^2 + i\epsilon}\right)^{1/4}.$$

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Топологический конфайнмент в модели Швингера

Проанализированы причины конфайнмента на примере модели Швингера. В общепринятой ее трактовке удержание заряженных частиц проблематично, так как кварковая функция Грина имеет особенность. При квантовании теории в терминах калибровочно-инвариантных переменных в конечном пространстве-времени возникает топологическое вырождение фазы "цветных" полей. После усреднения по этому вырождению кварковая функция Грина исчезает, а нейтральные корреляторы токов /в пределе бесконечного объема/ совпадают с выражениями, получаемыми в стандартном подходе. Таким образом, причиной конфайнмента в безмассовой КЭД$_{(1+1)}$ можно считать топологическое вырождение калибровочного вакуума.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Topological Confinement in the Schwinger Model

The reasons for confinement are analysed on the example of the Schwinger model. In the conventional interpretation of this model the charged-particles confinement is problematic because of the existence of a singularity in the quark Green function. When theory is quantized in terms of gauge-invariant variables in finite-volume space-time there appears topological degeneration of the "coloured"-field phases. After taking an average over this degeneration the quark Green function vanishes but the "neutral"-current correlator (in the limit of an infinite volume) coincides with the one in the standard approach to the model. So, the topological degeneration of the gauge-field vacuum appears as a reason for confinement in QED$_{(1+1)}$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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