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# COMMENTS ON THE ELECTROMAGNETIC PROPERTIES OF MAJORANA FERMIONS

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There is now interest in Majorana particles which occur commonly in grand unified and supersymmetric theories, as well as in connection with some aspects of neutrino physics. The problem of the possible electromagnetic structure of such truly neutral fermions reveals some interesting but apparently less known and investigated features of these (not yet detected) objects. In the lowest spin $J = 1/2$-case it has been shown [1] that the most general expression for the matrix element of the electromagnetic current of a Majorana fermion is

$$<p + k; J = \frac{1}{2}, J_z^{(f)} | j_\mu(0) | p; J = \frac{1}{2}, J_z^{(i)} > = \tag{1}$$

$$= iA(k^2)\bar{u}(p + k; J_z^{(f)}) [k^2 \gamma_\mu - (k \cdot \gamma) k_\mu] \gamma_5 u(p; J_z^{(i)})$$

and it has been subsequently realised [2] that this conclusion comes in fact from CPT-invariance alone. The particular Lorentz structure from Eq.(1), now known as the "Zeldovich's anapole", had been long ago [3] considered in connection with a new kind of electromagnetic interaction (invariant under time reversal but odd under parity) and had been interpreted in terms of a toroid current (i.e., a closed current circulating through a toroidal solenoid). This particular current configuration represents a new dipole characteristic of the system (a "toroid" dipole moment), different from the usual electric or magnetic dipoles. What is special about a spin 1/2-Majorana fermion is that for it there is nothing else left except this structure and therefore it can be viewed as the cleanest elementary carrier of the toroid dipole. As is seen from Eq.(1), another distinct feature of the anapole (toroid dipole) vertex is that it gives rise to a contact interaction with the external electromagnetic field (i.e., the spin 1/2 self-conjugate fermion interacts with the external field only if it overlaps with the source of the latter). One may ask whether the afore-mentioned peculiarities of the spin 1/2 Majorana particles would hold in the general case of Majorana fermions of arbitrary (half integral) spin (in the spin 3/2 case the matter becomes even more actual if one recalls the central role played by gravitino in supersymmetry). In the present Communication we answer this question in the affirmative in the form of a theorem, already stated in the first sentence of the abstract. In the proof we have to use as two lemmas some results previously obtained by other authors. Before stating the first lemma, we mention that in a series of papers, summarized in the reviews[4], it has been shown that in both classical and quantum electrodynamics the toroid dipole moment is only the first element of an independent (third) family of "toroid" multipoles which together with the usual electric and magnetic ones achieve a complete description of a general configuration of charges and currents. What we need here as a first lemma is the following (most general) multipole parametrization[4] of the matrix element of the electromagnetic current $j_\mu(x) = (j_0(x), \vec{j}(x))$ taken between one particle states of mass m, spin J, spin projections $J_z^{(i)}$, $J_z^{(f)}$ and momenta $\vec{p}^{(i)} = -\vec{k}/2$, $\vec{p}^{(f)} = +\vec{k}/2$:

$$\langle +\frac{\vec{k}}{2}; J, J_z^{(f)} | j_0(x=0) | -\frac{\vec{k}}{2}; J, J_z^{(i)} \rangle = \sum_{L,M} \frac{[4\pi(2L+1)]^{1/2}}{(2L+1)!!} |\vec{k}|^L \frac{C^{J,L,J}_{J_z^{(i)},M,J_z^{(f)}}}{C^{J,L,J}_{J,0,J}} Y_{LM}(\vec{n}) Q_{L,J}(-\vec{k}^2), \tag{2a}$$

$$\langle +\frac{\vec{k}}{2}; J, J_z^{(f)} | \vec{j}(x=0) | -\frac{\vec{k}}{2}; J, J_z^{(i)} \rangle = \sum_{L,M} \frac{[4\pi(2L+1)(L+1)/L]^{1/2}}{(2L+1)!!} |\vec{k}|^L \cdot \frac{C^{J,L,J}_{J_z^{(i)},M,J_z^{(f)}}}{C^{J,L,J}_{J,0,J}} \times \{\vec{F}^{(0)}_{LM}(\vec{n}) M_{L,J}(-\vec{k}^2) + |\vec{k}| \vec{F}^{(+)}_{LM}(\vec{n}) T_{L,J}(-\vec{k}^2)\}. \tag{2b}$$

$Q_{L,J}(-\vec{k}^2)$, $M_{L,J}(-\vec{k}^2)$, $T_{L,J}(-\vec{k}^2)$ are respectively the charge, magnetic and toroid multipole ($2^L$-pole) form factors (which at zero momentum transfer ($\vec{k}^2 = 0$) give the corresponding multipole moments of the considered system); $Y_{LM}(\vec{n})$ are the usual spherical functions while

$$\vec{F}^{(0)}_{LM}(\vec{n}) = \vec{Y}_{LLM}(\vec{n}), \quad \vec{F}^{(+)}_{LM}(\vec{n}) = i\vec{F}^{(0)}_{LM} \times \vec{n}, \quad \vec{n} = \frac{\vec{k}}{|\vec{k}|};$$

the (spherical basis) components of the vector $\vec{Y}_{LLM}$ are

$$\{\vec{Y}_{LLM}\}_m = \sum_{M'} C^{L,1,L}_{M',m,M} Y_{LM'}(\vec{n});$$

the summation over L and $M = -L, \dots, +L$ in Eqs.(2a), (2b) is restricted by the appearing (in view of the Wigner-Eckart theorem) Clebsch-Gordan coefficients (it starts, in fact, with $L = 0$ in Eq.(2a) and $L = 1$ in Eq.(2b)).



In the case of a Majorana fermion of (half integral) spin J, TCP invariance restricts the general form of the electromagnetic vertex displayed in Eqs.(2a), (2b) to a more particular one. To find the resulting selection rules, we need as our second lemma a certain non-trivial phase condition imposed by TCP on single-particle Majorana states which we formulate according to Kayser and Goldhaber[2] as follows: Defining the Majorana particle as a self-conjugate fermion under TCP (rather than under charge conjugation C, which may not always be appropriate) and writing the effect of TCP on the Majorana single particle state of momentum $\vec{p}$, spin J, spin projection $J_z$, as

$$TCP | \vec{p}; J, J_z \rangle = \eta(J_z) | \vec{p}; J, -J_z \rangle, \tag{3}$$

the phase factor $\eta(J_z)$ ($|\eta(J_z)| = 1$) is constrained by TCP-invariance alone to satisfy the condition[2]

$$\eta(-J_z) = (-1)^{2J} \eta(J_z) = -\eta(J_z). \tag{4}$$

Since it is known that, if the electromagnetic interaction is to conserve TCP, $j_\mu(x=0)$ must be TCP-odd also, the second lemma just stated leads then to the following TCP-condition on the vertex:

$$\langle +\frac{\vec{k}}{2}; J, J_z^{(f)} | j_\mu(0) | -\frac{\vec{k}}{2}; J, J_z^{(i)} \rangle = (-)\eta^*(J_z^{(i)}) \eta(J_z^{(f)}) \cdot \langle -\frac{\vec{k}}{2}; J, -J_z^{(i)} | j_\mu(0) | \frac{\vec{k}}{2}; J, -J_z^{(f)} \rangle. \tag{5}$$

Inserting now Eqs.(2a), (2b) into Eq.(5), taking into account the phase condition Eq.(4) and performing some simple manipulations among which the hardest are the use of the parity property of $Y_{LM}(\vec{n})$ and of the elementary relation

$$C^{J,L,J}_{-J_z^{(f)},M,-J_z^{(i)}} = (-1)^{L+M} C^{J,L,J}_{J_z^{(i)},M,J_z^{(f)}},$$

one finally proves the theorem expressed in the abstract: for a Majorana fermion of spin J only the toroid form factors $T_{L,J}(-\vec{k}^2)$ (with $L = 1, 2, \dots, 2J$, i.e., all of those appearing in the right-hand side of Eq.(2b)) survive in general; all the other electric and magnetic multipole form factors $Q_{L,J}(-\vec{k}^2)$ and $M_{L,J}(-\vec{k}^2)$ in Eqs.(2a), (2b) are ruled out exclusively on TCP-invariance grounds. That the number of the remaining (toroid) form factors makes up the right required number of independent transitions in the most general parametrization of the electromagnetic vertex of a spin J Majorana particle, one may convince himself also by an easy count (in the non-relativistic approximation) of the independent transitions in the crossed vertex $\langle 0|j(0)|M,M\rangle$, where $|M,M\rangle$ represents a pair of identical Majorana particles of spin $J=(2\ell-1)/2$ ($\ell=1,2,\ldots$). Antisymmetry at the permutation of the two fermions requires $(-1)^{L+S+1}$ to be $-1$ ($\vec{L}$, $\vec{S}$ are the orbital angular momentum and the total spin in the MM system). S may be $0,1,2,\ldots,2\ell-1$ and, for each of these values of S, L must be such that its coupling with this particular S give the total angular momentum 1 carried by the concerved current $j_\mu(x)$. For S given, only states with $L=S-1,\ S,\ S+1$ can do that, but for these only $L=S$ will render $(-1)^{L+S+1}$ negative. Therefore only $|M,M\rangle$ states with $L=S=1,2,\ldots,2\ell-1$ are allowed and the number of independent form factors is confirmed so to be $2\ell-1=2J$.

For spin $J=3/2$ Majorana fermions it is more convenient practically to have at hand a covariant form of the electromagnetic vertex rather than the expression

$$\langle +\frac{\vec{k}}{2};\frac{3}{2},J_z^{(f)}|\vec{j}(x=0)|-\frac{\vec{k}}{2};\frac{3}{2},J_z^{(i)}\rangle =$$

$$= \sum_{\substack{L=1,2,3\\ M=-L,\ldots,+L}} \frac{[4\pi(2L+1)(L+1)/L]^{1/2}}{(2L+1)!!}\cdot\frac{C^{3/2,L,3/2}_{J_z^{(i)},M,J_z^{(f)}}}{C^{3/2,L,3/2}_{3/2,0,3/2}}\cdot \tag{6}$$

$$\times |\vec{k}|^{L+1}\,\vec{F}^{(+)}_{LM}(\vec{n})\,T_{L,3/2}(-\vec{k}^2),$$

$$\langle +\frac{\vec{k}}{2};\frac{3}{2},J_z^{(f)}|j_0(x=0)|-\frac{\vec{k}}{2};\frac{3}{2},J_z^{(i)}\rangle = 0,$$

obtained as a particular case from the above considerations: It is:

$$\langle p+k;J=\frac{3}{2},J_z^{(f)}|j_\mu(x=0)|p;J=\frac{3}{2},J_z^{(i)}\rangle =$$

$$= i\bar{u}_\rho(p+k;J_z^{(f)})\{A(k^2)[k^2\gamma_\mu-(k\cdot\gamma)k_\mu]\gamma_5 g^{\rho\sigma} + \tag{7}$$

$$+[B(k^2)+iC(k^2)\gamma_5][k_\mu k^\rho k^\sigma - \frac{1}{2}k^2(g_\mu^\rho k^\sigma + g_\mu^\sigma k^\rho)]\}u_\sigma(p;J_z^{(i)}).$$

Eq. (7) may be obtained by the same procedure which has been used by Kayser and Goldhaber [2] to get Eq. (1), i.e., using the



TCP requirement Eq. (5) and the relation (similar to its correspondent in the spin 1/2 case)

$$u_\rho(p;-J_z) = (-1)^{J_z-1/2}\,\gamma_1\gamma_3\,\bar{u}^T_\rho(p;J_z).$$

As far as the interpretation of the three form factors $A(k^2)$, $B(k^2)$, $C(k^2)$ is concerned, it is easy to show that $A(0)$, $B(0)$ and $C(0)$ express respectively the dipole, quadrupole and octupole toroid moments of the considered spin 3/2 Majorana particle. All one has to do is to evaluate the corresponding expectation values of the multipole toroid moments [5]

$$T_{L,M}(t) = \left(\frac{4\pi}{2L+1}\right)^{1/2}\frac{i}{2(L+1)(2L+3)}\times$$
$$\times\int \vec{j}(\vec{x},t)\,\vec{\nabla}\times\{r^{L+2}(-i\vec{r}\times\vec{\nabla})\,Y^*_{LM}(\frac{\vec{r}}{r})\}\,d^3x \tag{8}$$

in the cases of interest $L=1,2,3$ by taking for the appearing matrix element $\langle +\frac{\vec{k}}{2};J=\frac{3}{2},J_z=+\frac{3}{2}|\vec{j}(\vec{x},t)|-\frac{k}{2};J=\frac{3}{2},J_z=+\frac{3}{2}\rangle$ displayed in Eq. (7). After some straightforward calculations one finds so indeed

$$\langle\vec{0};\frac{3}{2},+\frac{3}{2}|\begin{Bmatrix} T_{L=1,M=0}\\ T_{L=2,M=0}\\ T_{L=3,M=0}\end{Bmatrix}|\vec{0};\frac{3}{2},+\frac{3}{2}\rangle = (2\pi)^3\delta^3(\vec{k}=0)\begin{Bmatrix} A(k^2=0)\\ -B(k^2=0)\\ \frac{3}{2m}C(k^2=0)\end{Bmatrix} \tag{9}$$

Having seen from the previous discussion that Majorana particles may still be left, in general, with a certain electromagnetic structure, identified before as consisting in toroid moments and distributions, the question arises of whether such neutral fermions, on account of this type of structure, might give rise to Cherenkov or transition radiation. Ginzburg and Tsytovich [5] have recently analyzed the Cherenkov and transition radiation of a moving elementary toroid dipole (constant in time in its rest frame; if there is time dependence, there will be some radiation anyway), complaining at some place (the end of Section 6 of their paper) that "we are not aware of any real problems in which the ... radiation of relativistic toroid dipole moments would be of any importance". We point out that if Majorana particles are really there, the above complaint may lose much of its sadness. Indeed, relying on the classical electrodynamics analysis of ref. [5], we are led to the suggestion that Majorana particles (with non vanishing toroid structure) might, in principle, give rise to Cherenkov and transition radiation, albeit small by the toroid character of the source, by the actual values of the toroid moments themselves (determined by small parity or time reversal violating interactions), small again because the toroid current, as noted in ref.[5], would emit this kind of radiation mainly in as much as it is filled up by the medium through which it travels, thus making the effect conditional on both the type of media and the actual spatial extension of the travelling Majorana particle. Concerning this latter point, it seems that for more or less normal media much would depend upon the Majorana particles in question being very extended objects, which should not be a priori excluded. For extremely dense (nuclear) matter, the requirement of large extension might be less prohibiting and even much lesser for such a medium as the vacuum itself in the presence of strong electromagnetic fields.

We end with the remark that while Majorana fermions are allowed to possess only toroid multipole moments and distributions as intrinsic electromagnetic characteristics, in general there is nothing to prevent them from having electric, magnetic, or toroid[6] polarizabilities as distinct characteristics describing their behaviour in external electric (and magnetic) fields or currents; in other words, they may get induced electric, magnetic or toroid moments when external fields are present, irrespective of whether or not parity or time reversal invariance are good symmetries, because in this case a new direction, that of the external field or current is available, making thus inoperative (for the induced moments) the arguments leading to selection rules.

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Радеску Е.Е. E2-85-341

Об электромагнитных свойствах майорановских фермионов

На основе только CPT-инвариантности показано, что майорановская частица со спином $J$ /$J$ – полуцелое/ может обладать только тороидными моментами /и распределениями/ с порядком мультипольности $L = 1,2,\ldots,2J$; все другие обычные электрические и магнитные мультипольные характеристики запрещены. В случае спина 3/2 получена ковариантная электромагнитная вершина в терминах дипольного, квадрупольного и октупольного тороидных формфакторов. Отмечено, что майорановские фермионы с неисчезающими тороидными моментами, в принципе, могут, в определенных условиях, испускать черенковское или переходное излучение.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Comments on the Electromagnetic Properties of Majorana Fermions

It is proved that CPT-invariance alone requires the electromagnetic structure of a spin $J$ Majorana particle ($J$ – half-integral) to consist at most in toroid multipole moments and distributions with the multipolarity order $L = 1,2,\ldots,2J$; all other usual electric and magnetic multipole characteristics are forbidden. In the spin $J = 3/2$ case the covariant form of the electromagnetic vertex is obtained in terms of dipole, quadrupole and octupole toroid form factors. It is suggested that under certain circumstances Majorana fermions with non-vanishing toroid multipole moments might give rise to Cherenkov or transition radiation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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