сообщения
объединенного
института
ядерных
исследований
дубна

E2-85-320

K.Lewin

ON THE PAIR CREATION EFFECT IN RADIATIVE CHARMONIUM TRANSITIONS

1985

1. GENERAL REMARKS

Quark pair creation as part of the internal dynamics of meson decay amplitudes rigorously must be treated relativistically including a relativistic quark model [1]. But in the case of heavy quarks it should be reasonable, to use nonrelativistic approximations correcting radiative transition amplitudes between meson bound states by pair creation terms. Such a procédure also is justified by the necessary phenomenological treatment of the soft gluon which induces the pair creation.

Working in configuration space we clarify the limits of an approximate, local description sandwiching the pair creation part of the diagram (Fig.1b) between nonrelativistic quark model wave functions (see ref. [2]) which allow a cut off in the quark momentum distribution and thus restrict the propagation velocity of the antiquark to a nonrelativistic region if its (static) mass is sufficient high. Retaining only the time-dependence of the antiquark propagator, we gain an approximate local description and the correction can be evaluated using time-dependent perturbation theory. The resulting expression contains the two wave functions, the soft gluon potential and a factor coming from the antiquark propagator which modulates the integrand of the overlap integral. At this point we go beyond the more qualitative investigation of ref. [1].

As example we study the spin flip decay $\psi(3685) \to \gamma\chi(3415)$ between two oscillator ground states with meson radius around $R \simeq 0.5$ fm. The obtained pair creation correction is relatively small (12%), but the general expression shows that it can vary essentially in similar decays because of its sensitive dependence on quark masses and photon energies.

In sections 2 and 3 we study the analytic expressions of the no-pair and the pair creation part of the overlap integral, respectively. We discuss the limits of the quasilocal approximation and rewrite it in a suitable form for numerical evaluation. Discussion of the result follows in section 4.

2. EVALUATION OF THE NO-PAIR TERM

Using the rest system of the initial meson, here and in the following section we only deal with the internal dynamics contained in the overlap integrals. Some kinematical questions will be discussed in section 4. The no-pair term as a function of photon momentum $\vec{q}$ may be written as

$$F(\vec{q}) = \int d^3r\, \psi_1^*(\vec{r}) e^{i\vec{q}\vec{r}}\, \psi_2(r) = \int d\, k\, \tilde{\psi}_1^*(\vec{k})\, \tilde{\psi}_2(\vec{k}-\vec{q}) \tag{1}$$

with obvious notation (see Fig.1a). Using oscillator ground states for the initial and final meson, we have

$$\psi_1(\vec{r}) = \psi_2(\vec{r}) = (\pi R^2)^{-3/4} \exp(-r^2/2R^2), \quad R_1 = R_2 = R \approx 0.5\,\mathrm{fm} \tag{2}$$

and evaluation of (1) gives immediately

$$F(\vec{q}) = \exp\left(-\frac{1}{16} R^2 \vec{q}^{\,2}\right), \tag{3}$$

where $F(\vec{q}) \simeq F(0)=1$ at photon energies in the region $0 < |\vec{q}| \le 300$ MeV. For comparison with the pair creation contribution it will be sufficient to use $F(\vec{q}) = 1$.

3. EVALUATION OF THE PAIR CREATION TERM

Quark pair creation in lowest order of perturbation theory is described by the time-ordered diagram of Fig.1b where the time-ordering of the antiquark propagator prevents inclusion of the soft gluon into the final bound state.

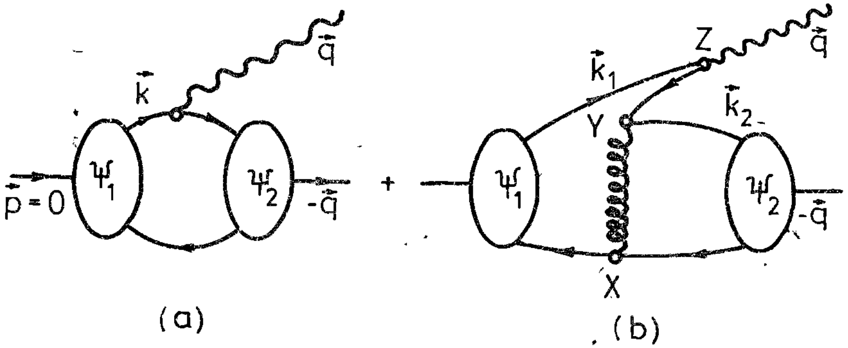

Fig.1. No-pair diagram (a) and lowest order pair creation correction (b) to radiative meson decay.

The potential part V sandwiched between the wave functions now contains, in addition to the radiation operator, the antiquark and the soft gluon. Depending on three space-time points x , y and z it appears as nonlocal potential and thus leads to a double-loop expression for diagram (b). Denoting it by $\Delta F(\vec{q})$ we obtain in time-dependent perturbation theory

$$\Delta F(\vec{q}) = \int dt\, e^{i\omega t}\, W_{12}(t), \quad W_{12}(t) = \int d^3r_1 d^3r_2\, \psi_1^*(\vec{r}_1) V(\vec{r}_2, \vec{r}_1 - \vec{r}_2; t) \psi_2(\vec{r}_2) \tag{4}$$

where $\vec{r}_1 = \vec{x}-\vec{z}$, $\vec{r}_2 = \vec{x}-\vec{y}$, $\vec{r}_1 - \vec{r}_2 = \vec{y}-\vec{z}$ and $t = y^\circ - z^\circ < 0$ according to Fig.1b. The substitution

$$\vec{\rho}_1 = \vec{r}_1 + \vec{r}_2, \quad \vec{\rho}_2 = \vec{r}_1 - \vec{r}_2 \tag{5}$$

transforms (4) into

$$W_{12}(t) = -\frac{1}{8}\int d^3\rho_1 d^3\rho_2\, \psi_1^*\left(\tfrac{1}{2}(\vec{\rho}_1+\vec{\rho}_2)\right) \underbrace{V_{ex}\left(\tfrac{1}{2}(\vec{\rho}_1-\vec{\rho}_2)\right) S^{(+)}(\vec{\rho}_2, t)\, e^{i\frac{\vec{q}}{2}(\vec{\rho}_1+\vec{\rho}_2)}}_{V} \times$$
$$\times\, \psi_2\left(\tfrac{1}{2}(\vec{\rho}_1-\vec{\rho}_2)\right), \tag{6}$$

where V_{ex} is a phenomenological soft gluon potential which will be discussed below, and

$$S^{(+)}(\vec{\rho}_2, t) = \left(i\gamma_0 \frac{\partial}{\partial t} - i\vec{\gamma}\nabla_{\vec{\rho}_2} + m\right) D^{(+)}(\vec{\rho}_2, t) \tag{7}$$

describes the antiquark propagator (notation of Bogolubov, Shirkov/3/). After the substitution (5) $S^{(+)}$ depends only on one of the variables of integration which is needed for splitting (4) into two terms one of which can be approximated by a local description. This term covers regions of integration satisfying

$$\vec{y} = \vec{z} \qquad \rho_2^2 \ll \rho_1^2 \qquad \rho_2^2 \ll t^2 \tag{8}$$

(note that time-ordering already requires the weaker condition $\rho_2^2 < t^2$) and after introduction of polar coordinates it reads*

$$\int_0^\infty dt\, e^{i\omega t}\, W_{12}(t) \simeq -\frac{4}{3}\frac{\pi^2 a^3}{q}\int_0^\infty d\rho_1 \rho_1^4 \sin\left(\frac{q}{2}\rho_1\right) \psi_1^*(\rho_1) \psi_2(\rho_1) \times$$
$$\times\, V_{ex}(\rho_1) \int_{\rho_1}^\infty dt\, e^{i\omega t} S^{(+)}(0, t) + \ldots \tag{9}$$

where the dots indicate the nonlocal rest of (4). The structure (9) corresponds to the first integral of the decomposition

$$\int_0^\infty d\rho_2 \ldots = \int_0^{a\rho_1} d\rho_2 \rho_2^2 + \int_{a\rho_1}^\infty d\rho_2 \ldots \qquad (0 < a^2 \ll 1) \tag{10}$$

in which the weak dependence of the integrand on ρ_2 (see formula (6)) may be neglected.

---

*From now we work with $t > 0$, i.e., $t \to -t$ in (7). Note that $D^{(+)}$ is symmetric in t.

Up to now this splitting is formal. From physical point of view it becomes interesting only in such cases where the quasilocal term (9) represents a reasonable approximation of $\Delta F(\vec q)$. Investigation of this point needs physical interpretation of the dimensionless parameter a. Looking at the light-cone picture of Fig.2, we notice that the parameter a determines the angle of the narrow cone which limits the propagation velocity of the antiquark in agreement with (8) and (9). Such a limit corresponds to a cut-off in the internal relative momentum distribution governed by the bound state wave functions. In the oscillator model these are Gaussian distributions and therefore admit a reasonable cut-off. For $R = 0.5$ fm they give an expectation value $\langle|\vec k|\rangle \approx 800$ MeV, so that a cut-off near 1500 MeV for $|\vec k_1 - \vec q|$ should be sufficient. Relating the parameter a to the maximum of $|\vec k_1 - \vec q|$, one obtains

$$\frac{\max|\vec k_1 - \vec q|}{m_{\bar q}} = \sigma = \frac{\beta_{max}}{\sqrt{1-\beta_{max}^2}} \qquad \beta_{max} = \frac{\sigma}{\sqrt{1+\sigma^2}} = a . \tag{11}$$

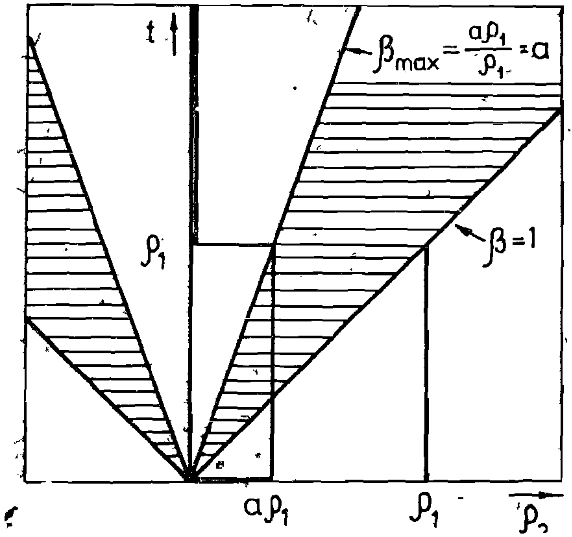

Fig. 2. Illustration of the parameter a *as maximal propagation velocity of the antiquark in the quasilocal term.*

It can be seen from Fig.3, how $\beta_{max} = a$ depends on the maximum momentum of the antiquark and on its static mass $m_{\bar q}$. It shows that the quasilocal approximation fails if $m_{\bar q} = m_{u,d}$, and that it works for $m_{\bar q} \geq m_c$. In the case of the charmed quark $m_{\bar q} = m_c$ which we study below, a rough evaluation with $a \approx 0.5$ should be possible.

For further evaluation of (9) the time integral must be calculated. Its light-cone singularity when $\rho_1 \to 0$ brings no problem because of the high power of ρ_1 in the integrand. Using

$$S^{(+)}(0,t) = (i\gamma_0 \frac{\partial}{\partial t} + m)\, D^{(+)}(0,t) \tag{7a}$$

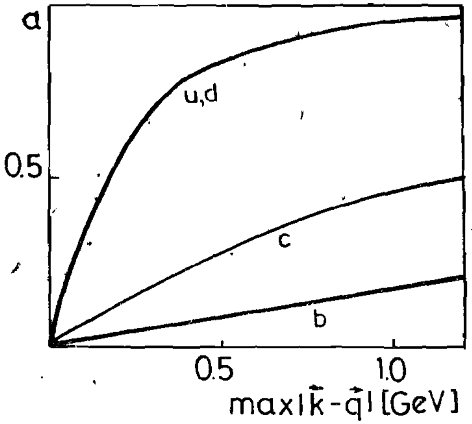

Fig.3: $a = \beta_{max}$ as function of $\max|\vec k_1 - \vec q|$ with $m_{\bar q}$ as curve parameter ($m_{u,d} = 0.3$ GeV, $m_c = 1.5$ GeV, $m_b = 5.8$ GeV).

we obtain by partial integration ($\kappa = mt$)

$$\int_{\rho_1}^{\infty} dt\, e^{i\omega t} S^{(+)}(0,t) = -\gamma_0 D^{(+)}(0,\rho_1)\sin(\omega\rho_1) + i\gamma_0 D^{(+)}(0,\rho_1)\cos(\omega\rho_1) + \frac{m-\gamma_0\omega}{m}\int_{m\rho_1}^{\infty} d\kappa\, e^{i(\omega/m)\kappa} D^{(+)}(\kappa) . \tag{12}$$

γ_0 as rest of the Dirac structure in our approach acts on a Pauli spinor of the nonrelativistic final meson state and can be replaced by unity. The final expression for (12) may be written as

$$\int_{\rho_1}^{\infty} dt\, e^{i\omega t} S^{(+)}(0,t) = \phi_1(m\rho_1) + i\phi_2(m\rho_1), \tag{13}$$

where $m = m_{\bar q}$ from now. With $m\rho_1 = x$ one obtains

$$\phi_1(x) = \frac{m^2}{8\pi x}\left[\cos(\frac{\omega}{m}x) N_1(x) - \sin(\frac{\omega}{m}x) J_1(x)\right] + (1-\frac{\omega}{m})\int_x^{\infty}\frac{d\kappa}{\kappa}\left[\cos(\frac{\omega}{m}\kappa) J_1(\kappa) + \sin(\frac{\omega}{m}\kappa) N_1(\kappa)\right] \tag{13a}$$

$$\phi_2(x) = \frac{m^2}{8\pi x}\left[\sin(\frac{\omega}{m}x) N_1(x) + \cos(\frac{\omega}{m}x) J_1(x)\right] + (1-\frac{\omega}{m})\int_x^{\infty}\frac{d\kappa}{\kappa}\left[\sin(\frac{\omega}{m}\kappa) J_1(\kappa) - \cos(\frac{\omega}{m}\kappa) N_1(\kappa)\right] . \tag{13b}$$

The appearing integrals need numerical evaluation ($J_1(x)$ and $N_1(x)$ are Bessel functions in usual notation).

The last factor in the integrand of the expression (9) to be discussed is the $q\bar{q}$ potential $V_{ex}(\rho_1)$. Following Richardson[4] we use

$$V_{ex}(r) = \frac{8\pi}{33 - 2n_f}\,\Lambda\left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r}\right), \tag{14}$$

where

$$f(t) = \frac{4}{\pi}\int_0^\infty dp\,\frac{\sin(pt)}{p}\left[\frac{1}{\ln(1+p^2)} - \frac{1}{p^2}\right] = 1 - 4\int_1^\infty \frac{dp}{p}\,\frac{e^{-pt}}{[\ln(p^2-1)]^2 + \pi^2}. \tag{15}$$

This structure is obtained by Fourier transformation from

$$\tilde{V}(\vec{p}^{\,2}) = -\frac{4}{3}\,\frac{12\pi}{33 - 2n_f}\cdot\frac{1}{\vec{p}^{\,2}}\cdot\frac{1}{\ln(1+\vec{p}^{\,2}/\Lambda^2)} \tag{16}$$

which appears as interpolating form of

$$-p^2 \gg \Lambda^2:\quad \tilde{V}(p^2) = \frac{4}{3}\,\alpha_s(p^2)\frac{1}{p^2} = \frac{16\pi}{33 - 2n_f}\,\frac{1}{p^2}\cdot\frac{1}{\ln(-p^2/\Lambda^2)} \tag{17}$$

and

$$-p^2 \ll \Lambda^2:\qquad \tilde{V}(p^2) \to \mathrm{const}\,\frac{1}{(p^2)^2} \tag{18}$$

after dropping the retardation. The notation is obvious from QCD. This choice of potential has the advantage to be free of parameters except the dependence on the energy scale Λ. It has been used with success in charmonium spectroscopy where the scale was fitted to be $\Lambda \simeq 400$ MeV[4]. For sufficient small r one obtains from (14) and (15)

$$V_{ex}(r \to 0) \sim \frac{1}{\Lambda r \ln(\Lambda r)}, \tag{19}$$

i.e., V_{ex} behaves softer than a Coulomb potential.

4. NUMERICAL RESULT AND DISCUSSION

Now we are able to evaluate numerically the pair creation term in the quasilocal approximation (9). Studying as example the spin flip transition $\psi'(3685) \to \gamma\chi\ (3415)$ between charmonium ground states (see ref.[6]) we describe the initial and final meson bound states by the oscillator ground state wave functions (2) with $R \simeq 0.5$ fm (see ref.[4]) and $|\vec{q}| = \omega = 270$ MeV. The value of the parameter a should be near 0.5 in accordance with Fig.3 in order to cover the essential part of

the relative internal momentum distribution of the quarks. Insertion of eqs. (2), (13) and (14) into eq. (9) leads to the numerical result

$$\mathrm{Re}(\Delta F) = 2.9\cdot 10^{-2}\qquad \mathrm{Im}(\Delta F) = 0.3\cdot 10^{-2}, \tag{20}$$

where the error due to the approximations should remain within ±50%. Insertion into the transition probability then gives

$$\frac{|F + \Delta F|^2}{|F|^2} = 1.12 \pm 0.06. \tag{21}$$

This correction is transferred to the decay width since each term of the kinematical decomposition contains the same overlap integral (see, for example, ref.[5]). The correction (21) is relatively small, but as can be seen from the expressions (13a) and (13b), it depends sensitively on quark masses and photon energies and thus cannot be ignored in quantitative investigations of similar processes.

Concluding, we note that the pair creation effect also should be important in the case of electroweak radiative decays (see[7,8]) especially if two- and three-quark weak interactions are taken into account. Then the calculations depend on the internal quark dynamics via overlap integrals which are more involved than in the above example. Our investigation indicates that also in such cases one cannot expect quantitative results if quark pair creation is ignored.

The author would like to thank S.B.Gerasimov for valuable discussions. Useful remarks by D.Ebert, W.Kallies and G.B.Motz at the first stage of this work should be mentioned gratefully.

REFERENCES

1. Feinberg G., Sucher J. Phys.Rev.Lett., 1975, 35, p.1740; Phys.Rev.Lett., 1971, 26, p.680.
2. Close F.E. An Introduction to Quarks and Partons. Academic Press, London, 1979.
3. Bogolubov N.N., Shirkov D.V. Introduction to the Theory of Quantized Fields. Interscі.Publ., New York-London, 1959.
4. Richardson J.L. Phys.Lett., 1979, 82B, p.272.
5. Gilman F.J., Wise M.B. Phys.Rev., 1979, D19, p.976.
6. Particle Data Group. Rev.of Mod.Phys., 1984, 56, No.2, part 2.
7. Close F.E., Rubinstein H.R. Nucl.Phys., 1980, B173, p.477.
8. Abdullah S., Singh S.K. J.Phys.C: Particles and Fields, 1983, 18, p.31.

Received by Publishing Department
on May 6, 1985.

В Объединенном институте ядерных исследований начал выходить сборник *"Краткие сообщения ОИЯИ"*. В нем будут помещаться статьи, содержащие оригинальные научные, научно-технические, методические и прикладные результаты, требующие срочной публикации. Будучи частью "Сообщений ОИЯИ", статьи, вошедшие в сборник, имеют, как и другие издания ОИЯИ, статус официальных публикаций.

Сборник "Краткие сообщения ОИЯИ" будет выходить регулярно.

The Joint Institute for Nuclear Research begins publishing a collection of papers entitled *JINR Rapid Communications* which is a section of the JINR Communications and is intended for the accelerated publication of important results on the following subjects:

- Physics of elementary particles and atomic nuclei.
- Theoretical physics.
- Experimental techniques and methods.
- Accelerators.
- Cryogenics.
- Computing mathematics and methods.
- Solid state physics. Liquids.
- Theory of condensed matter.
- Applied researches.

Being a part of the JINR Communications, the articles of new collection like all other publications of the Joint Institute for Nuclear Research have the status of official publications.

JINR Rapid Communications will be issued regularly.

Левин К. E2-85-320

Об эффекте рождения пар
в радиационных переходах чармониев

Исследуется вклад в амплитуду радиационного распада чармония, связанный с диаграммами Фейнмана, учитывающими рождение внутренней пары кварков $c\bar{c}$. Целью работы является вычисление поправки, обусловленной этим рождением пар, к интегралу перекрытия волновых функций для амплитуды перехода в квазилокальном, полурелятивистском приближении, которое законно для очарованных и более тяжелых кварков. Применение к ширине распада перехода $\psi/3685/\to\gamma\chi/3415/$ с переворотом спина дает поправку в 12% к результату расчета, проведенного без учета рождения пар при условии, что мезонный радиус берется равным примерно 0,5 фм, а для параметра Λ принимается значение 400 МэВ, извлеченное из данных по спектроскопии чармония. Ошибка приближения оценивается на уровне ниже $\pm 50\%$. Исследования показывают, что и в случае электрослабых распадов мезонов для вычислений без учета рождения пар нельзя ожидать надежных количественных результатов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Lewin K. E2-85-320

On the Pair Creation Effect in Radiative
Charmonium Transitions

The paper deals with the contributions to radiative charmonium decay amplitudes which come from Feynman diagrams containing creation of internal $c\bar{c}$ quark pairs. The aim of the work is calculation of this pair creation correction to the wave function overlap integral of the transition amplitude in a quasilocal semirelativistic approximation which works for charmed and heavier quark pairs. The application to the decay width of the spin flip transition $\psi(3685)\to\gamma\chi(3415)$ gives a 12% correction to the no-pair term using a meson radius near 0.5 fm and a scale parameter $\Lambda =$ = 400 MeV taken from fits in charmonium spectroscopy. The error of the approximations is estimated to be smaller than 50%. The investigation indicates that also in the case of electroweak meson decays quantitative results cannot be expected from the no-pair contribution alone.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985