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STRUCTURE FUNCTIONS OF PION AND NUCLEON DETERMINED FROM HIGH MASS MUON PAIR PRODUCTION

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1. INTRODUCTION

One of the main goals of the experiments on direct muon pair production is the determination of the structure functions of hadrons. The basis of all known phenomenological analysis is the classical annihilation model proposed by Drell and Yan[1], and in some cases - the additional contributions to this annihilation, required by higher order QCD subprocesses, which contribute to the massive lepton pair production[2].

During the last years two experiments at CERN SPS have published the highest statistical data on double differential cross-section $d\sigma/dMdx_f$. In the first one - the NA3 experiment 30 000 events have been collected with masses 4.5-8.5 GeV/c from 400 GeV/c protons hitting Platinum target[3]. The second one - NA10 experiment - has collected a sample of 155 000 events with masses higher than 4.07 GeV/c by 194 GeV/c negative pions hitting Tungsten target[4].

The aim of the present work is determination of the quark densities of nucleon and pion through simultaneous fit of the shape of the normalized cross-section, measured in both experiments. The normalization is performed in short mass intervals, which gives a hope that the estimations are independent of the so-called K-factor[5].

2. MODEL DESCRIPTION AND PARAMETRIZATION

It is well known that in the framework of the quark-parton annihilation model the double differential cross-section of massive muon pair production may be written as

$$d\sigma / dMdx_f = K(M, x_f) \cdot (4\pi\alpha^2/9) \cdot (1/M^3\sqrt{x_f + 4\tau}) \cdot Q(x_1, x_2, M), \qquad (1)$$

with

$$Q(x_1, x_2, M) = \sum_i e_i^2 \{ qh_1^i(x_1, M) \cdot \overline{qh}{}_2^i(x_2, M) + (1 \to 2)\}, \qquad (2)$$

where M is the mass of muon pair; x_f, the Feynman variable; x_1 and x_2 are the fractional momenta of hadrons carried by the quarks of flavour i, e_i is the quark electric charge, qh_j^i and $\overline{qh}{}_j^i$ are the densities of quarks and antiquarks in hadrons, $\alpha = 1/137.036$, $\tau = x_1 \cdot x_2$ and $K(M, x_f)$ is a function introduced to describe the differences between the theoretically predicted cross-section and the observed one. The argument M in the quark densities reflects the scaling violation of the structure functions, a fact which is well established for the nucleon through the deep inelastic scattering (D.I.S.) experiments [6]. An indication of scaling violation for the pion structure function has been reported in [2].

Assuming validity of Gross-Llewellyn Smith sum rules [7], the condition of momentum conservation carried by valence and sea quarks and taking the parametrization of the quark densities proposed by Buras and Gaemers [8], the relation (2) in the case of NA3 experiment may be written as

$$Q(x_1, x_2, M) = E(x_1, M)\cdot S(x_2, M) + F(x_2, M)\cdot S(x_1, M) \quad (3)$$

with

$$E(x_1, M) = \{4\cdot U^p(x_1, M) + D^p(x_1, M)\}/9,$$

$$F(x_2, M) = \{2.199\cdot U^p(x_2, M) + 2.801\cdot D^p(x_2, M) + 10.5\cdot S^p(x_2, M)\}/9.$$

In the case of NA10 experiment $Q(x_1, x_2, M)$ will be as

$$Q(x_1, x_2, M) = V^\pi(x_1, M)\cdot G(x_2, M) + H(x_2, M)\cdot S^\pi(x_1, M), \quad (4)$$

with

$$G(x_2, M) = \{1.61\cdot U^p(x_2, M) + 2.39\cdot D^p(x_2, M) + 5\cdot S^p(x_2, M)\}/9,$$

$$H(x_2, M) = \{2.208\cdot U^p(x_2, M) + 2.792\cdot D^p(x_2, M) + 11\cdot S^p(x_2, M)\}/9.$$

In (3) and (4) U^p, D^p and S^p are the distributions of up and down valence quarks and sea quarks of the nucleon respectively, $V^\pi = D^\pi = \bar{U}^\pi$ and S^π are the distributions of valence quarks and sea quarks of pion. According to the assumptions mentioned above this distributions can be written as

$$U^p = 2\cdot x^{\alpha_p(M)}\cdot(1-x)^{\beta_p(M)}/B(\alpha_p, \beta_p + 1),$$

$$D^p = x^{\alpha_p(M)}\cdot(1-x)^{\beta_p(M)+1}/B(\alpha_p, \beta_p + 2),$$

$$S^p = \{1 - g_p - 2\cdot\alpha_p/(\alpha_p + \beta_p + 1) - \alpha_p/(\alpha_p + \beta_p + 2)\}\cdot(1+\gamma_p)\cdot(1-x)^{\gamma_p(M)}/5$$

$$V^\pi = x^{\alpha_\pi(M)}\cdot(1-x)^{\beta_\pi(M)}/B(\alpha_\pi, \beta_\pi + 1)$$

and

$$S^\pi = \{1 - g_\pi - 2\cdot\alpha_\pi/(\alpha_\pi + \beta_\pi + 1)\}\cdot(1+\gamma_\pi)\cdot(1-x)^{\gamma_\pi(M)}/6,$$

where $B(a,b)$ is the Euler Beta function, g_p and g_π are the fractional momenta carried by the gluons in the nucleon and the pion, respectively.

The scaling violation of the structure functions is taken into account in the form suggested in [8]:

$$\alpha(M) = \alpha_0(M_0) + \alpha_1\cdot\bar{s},\quad \beta(M) = \beta_0(M_0) + \beta_1\cdot\bar{s},\quad \gamma(M) = \gamma_0(M_0) + \gamma_1\cdot\bar{s}.$$

with $\bar{s}$ defined as $\bar{s} = \ln\{\ln(M^2/\Lambda)/\ln(M_0^2/\Lambda^2)\}$ where Λ is the QCD scale factor.

In the formalism described above, the predicted cross-section is fully determined by 16 parameters: α_0^π, α_1^π, β_0^π, β_1^π, γ_0^π, γ_1^π, g_π, α_0^p, α_1^p, β_0^p, β_1^p, γ_0^p, γ_1^p, g_p, Λ, M_0 and additionally by the unknown function $K(x_f, M)$. All of this parameters cannot be determined from the existing data of muon pair production because of their limited kinematical region or/and correlation problems.

3. DETERMINATION OF NUCLEON AND PION STRUCTURE FUNCTION PARAMETERS

It is clear from the model description that the simultaneous fit to the measured in [3,4] double differential cross-section could improve the accuracy and allows a determination of the nucleon and pion structure functions totally independent of D.I.S measurements. However to perform such a determination an additional assumptions are needed for the K-function which very likely may correlate with the structure function parameters. In order to avoid this delicate point, i.e., to perform an estimations which are nearly independent of the K-function the minimization (in our case) will be carried out for normalized cross-section in a short mass intervals (0.4 GeV/c mass step for NA3 experiment and 0.57 GeV/c - for NA10 experiment) i.e.,

$$\sigma(x_{f_i}, M_j) = \frac{\dfrac{d\sigma}{\partial x_{f_i}\partial M_j}}{\sum_i \dfrac{\partial\sigma}{\partial x_{f_i}\partial M_j}}, \quad j - \text{fixed}.$$

We may assume that within such a mass intervals the K-function, if it exists, is mass- and very likely x_f-independent and will cancel. Such a shape fit in a short mass intervals will be applied for a model predictions which include a scaling violation for the nucleon as well for the pion structure function.

The minimization was performed using the autoregularized iteration processes by Gauss-Newton type [9].

Due to the short kinematical region of the available data all of the parameters mentioned above cannot be determined. We will restrict ourselves only to α_0^π, β_0^π, γ_0^π, α_0^p, β_0^p, γ_0^p.

The evolution of the nucleon structure functions was taken from neutrino measurement on iron [6] in the parametrization proposed in [10]: $\alpha_1^p = -0.16$, $\beta_1^p = 0.77$ and $\gamma_1^p = 1.5$. The pion evolution was taken as in [2]:

$$\alpha_1^\pi = 0.0579 - 0.237 \cdot \alpha_0^\pi - 0.0328\,\beta_0^\pi, \quad \beta_1^\pi = 0.5150 - 0.0114 \cdot \beta_0^\pi + 0.0544 \cdot \beta_1^\pi,$$

$$\gamma_1 = 0.$$

Due to a very strong correlation of the fractional momenta g_π and g_p with α_0^π and α_0^p, respectively, they are also fixed. We have used values $g_p = 0.5$ according to CDHS [6] measurement and $g_\pi = 0.47$ according to NA3 measurements [5]. The results of the fit together with the correlation coefficients between the parameters and the values of the χ^2 in the mass subintervals for which the cross-section was normalized are presented in Table 1.

As can be seen from Table the model gives a good description of the shape of x_f-distributions of the normalized cross-section in all mass subintervals of both experiments, exept for the one mass bin above the χ-resonance for NA10 data. In this region an anomalous scaling violation has been observed [2].

4. MASS DEPENDENCE OF THE K-FUNCTION

Next we shall determine the K-function in all mass subintervals using the same model and the extracted values of the pion and nucleon structure function parameters. To reproduce the absolute cross-section we need a K-function shown in Table 3. The mean value of this function for NA3 proton data is $K = 1.9+0.3$ and for NA10 pion data is $K = 2.2+0.3$ (the error bars include the statistical errors of the structure function parameters, the correlations between them and the systematical uncertainties). We note that within the error bars the K-function is mass independent in agreement with the calculations in [11].

It has been pointed out in [2] that in the next-to-leading logarithm approximation the value of fractional momenta carried by the gluons in the pion may change essentially the estimation of the K-factor in muon pair production. In the framework of the model applied here we tried to find new structure function parameters constraining the mean K-factor of both experiments equal to 1. It is worth stressing that this approach gives an

Table 1

Results of the fit for model with scaling violation in the nucleon and in the pion. (Fixed: proton $g_p = 0.50$, $\Lambda = 0.55$, $M_0^2 = 25$ GeV^2/c^2; pion $g_\pi = 0.47$, $\Lambda = 0.3$, $M_0^2 = 25$ GeV^2/c^2)

	Parameters	Correlation coefficients	χ^2- values/N.F. NA3	NA10
p	$\alpha_0^\pi = 0.37+0.05$		16/13	2.2/6
i	$\beta_0^\pi = 0.95+0.07$	.25	16/13	3.5/6
o	$\gamma_0^\pi = 4.3+1.0$	.28 .83	14/13	5.4/7
n			6/12	6.6/7
			11/12	2.2/8
p	$\alpha_0^p = 0.63+0.07$	.84 -.20 -.10	7/12	8.6/9
r	$\beta_0^p = 3.6+0.1$	.20 .13 .06 .29	10/12	9.1/9
o	$\gamma_0^p = 9.4+0.7$	-.75 .26 .13 -.70 .44	11/11	21/8
t			19/11	7/8
o			4/6	
n				
			114/115	66/68
		Total	=180 for 176 D.O.F.	

Table 2

Results of the fit for model with scaling violation in the nucleon and in the pion. (Fixed: proton $g_p = 0.365$, $\Lambda = 0.55$, $M_0^2 = 25$ GeV^2/c^2; pion $g_\pi = 0.05$, $\Lambda = 0.3$, $M_0^2 = 25$ GeV^2/c^2), $K_{NA3} = K_{NA10} = 1$.

	Parameters	Correlation coefficients	χ^2- values/N.F. NA3	NA10
p	$\alpha_0^\pi = 0.55+0.07$		16/13	2.3/6
i	$\beta_0^\pi = 0.94+0.09$	.49	16/13	3.7/6
o	$\gamma_0^\pi = 3.5+0.7$	.22 .86	14/13	5.8/7
n			6/12	6.5/7
			11/12	2.8/8
p	$\alpha_0^p = 0.73+0.08$	.67 -.23 -.46	7/12	9.3/9
r	$\beta_0^p = 3.7+0.1$	.60 .09 .30 .34	11/12	9.7/9
o	$\gamma_0^p = 9.3+0.7$	-.60 .28 .46 -.69 .40	11/11	20/8
t			19/11	7/8
o			4/6	
n				
			115/115	67/68
		Total	=182 for 176 D.O.F.	

estimations of $g_\pi = 0.05$ and $g_p = 0.37$, and values of the structure function parameters as given in Table 2. We note that the fit in this case is equally good as in the case shown in Table 1.

Table 3

	NA3	$K_{NA3} = 1.9 \pm 0.3$		NA10	$K_{NA10} = 2.2 \pm 0.3$	
N	M_i	K_i	$\pm \Delta K_i$	M_i	K_i	$\pm \Delta K_i$
1	4.5	0.98	0.03	4.01	1.14	0.09
2	4.9	0.98	0.03	4.59	1.09	0.13
3	5.3	0.98	0.05	5.16	1.15	0.17
4	5.7	1.03	0.06	5.73	1.19	0.21
5	6.1	1.06	0.09	6.31	1.25	0.25
6	6.5	1.07	0.11	6.88	1.25	0.28
7	6.9	1.13	0.13	7.45	1.30	0.32
8	7.3	1.18	0.17	10.32	1.62	0.53
9	7.7	1.22	0.19	11.45	1.14	0.42
10	8.1	1.47	0.27			

5. CONCLUSION

From the simultaneous fit of the double differential cross-section of NA3 proton data and NA10 pion data for high mass muon pair production, we may conclude:

i) We have determined the structure function parameters of the nucleon and the pion independently of D.I.S measurements and independently of the assumption of the K-factor or K-function. The values obtained support the assumption that the structure functions in the muon pair production processes and in the D.I.S. processes are very likely the same.

ii) The data are in favour of mass independent K-function.

iii) The absolute value of the so-called K-factor depends strongly on the momentum distribution of gluons in hadrons. There is a good description of the data with $K = 1$.

In order to precise conclusion of the differences between the model predictions and the data for the absolute cross section and the K-factor more precise measurements are needed for the gluon distributions in the hadrons.

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REFERENCES

1. Drell S., Yan T.M. Phys.Rev.Lett., 1970, 25, p.316; Ann.Phys., 1971, 66, p.578.
2. Betev B. et al. CERN Preprint EP/85-04, Geneva, 1875; submitted to Z.Phys.
3. Badier J. et al. CERN Preprint EP/84-16, Geneva, 1984; submitted to Phys.Lett.B.
4. Betev B. et al. CERN Preprint EP/85-03, Geneva, 1985; submitted to Z.Phys.
5. Badier J. et al. CERN Preprint EP/80-148, Geneva, 1980; Badier J. et al. Z.Phys.C, 1983, 18, p.281.
6. De Groot J.G.H. et al. Phys.Lett., 1979, 82B, p.456; De Groot J.G.H. Z.Phys., 1979, C1, p.143.
7. Gross D.J., Llewellyn Smith C.H. Nucl.Phys., 1969, B14, p.337.
8. Buras A.J., Gaemers J.F. Nucl.Phys., 1978, B132, p.249.
9. Alexandrov L. et al. PSR-165/REGN, RSIC, ORNL, USA, 1981; Alexandrov L. JINR, B1-11-82-767, Dubna, 1982.
10. Weisz S. Thesis. Universite de Paris-Sud, Orsay, France, 1982.
11. Kubar J. et al. Nucl.Phys., 1980, B175, p.251.

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Бетев Б., Бурилков Д., Мавродиев С.Щ. E2-85-312

Структурные функции пиона и нуклона из рождения тяжелых мюонных пар

Получены структурные функции распределения кварков в нуклоне и пионе на основе описания формы двойного дифференциального сечения $\frac{d\sigma}{\partial M \partial x_f}$, измеренного в NA3 эксперименте в реакции протон-платина при 400 ГэВ/с и NA10 эксперименте в реакции π^--вольфрам при 194 ГэВ/с.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Structure Functions of Pion and Nucleon Determined from High Mass Muon Pair Production

We present results on quark densities of nucleon and pion estimated from the shape of double differential cross-section $\frac{d\sigma}{\partial M \partial x_f}$ measured in NA3 experiment by 400 GeV/c protons on Platinum and NA10 experiment by 194 GeV/c negative pions on Tungsten.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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