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M.Bordag, F.-M.Dittes,* D.Robaschik*

CASIMIR EFFECT WITH UNIFORMLY MOVING MIRRORS

* Sektion Physik der Karl-Marx-Universität, Leipzig, DDR

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1. INTRODUCTION

It is well known that the ground-state energy of a quantum mechanical system depends in a nontrivial manner on external parameters. This leads to observable effects, especially, to the Casimir effect in QED [1]. It has been studied extensively in several situations, mostly for static boundary conditions of different geometry [2,3]. Furthermore, there were several attempts to treat the problem of one or two moving mirrors in (1+1) dimensions [2,4].

Here we consider a comparatively simple example of Quantum Field Theory with nonstatic boundary conditions in (3+1) dimensions. We assume two neutral ideal-conducting infinite parallel plates to move relatively to each other with a constant velocity and calculate the Casimir energy as well as the vacuum expectation values of the corresponding energy-momentum tensor. This generalize our previous result for the scalar theory [5]. In that case it was possible to construct the scalar Green functions using the reflection principle. As a physical result, it turned out that the plates attract each other with a velocity-dependent force leading to the classical Casimir force in the nonrelativistic (quasi-static) limit.

Here we derive an analogous result for a physically more interesting case of the electromagnetic field. Again, we construct the Green functions by applying the reflection principle. This can be done in two independent ways, both presented here. In the first one, we construct the Green functions for the electromagnetic field strength directly and calculate the physically interesting quantities.

The second way consists in reducing our problem to the scalar case: We introduce potentials which allow separation of the boundary conditions so that one component of the potential has to satisfy the Dirichlet boundary condition, whereas the other - the Neumann one. This is the mathematical reason for the fact that the Casimir force is not twice the scalar one (as it was the case for static plates).

Our main result is explicit expression for the vacuum expectation values of the energy-momentum tensor. Surprisingly, the expressions obtained are much simpler than those for the scalar case.

Similarly to the two-dimensional case we observe a nontrivial distance and velocity dependence as well as the appea-

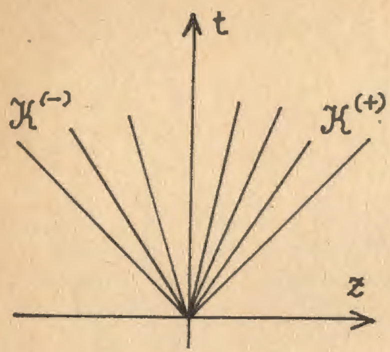

Fig.1. Straight world lines of the points between the two mirrors.

rance of an energy flux parallel to the direction of motion of the plates.

A clear physical interpretation can be obtained if one considers the motion of each point in between the two mirror system as described by a straight world line (see fig.1).

Then it turns out that in each local rest frame locally at the considered point the energy-momentum tensor has the same structure as for the standard Casimir problem with static plates.

2. GREEN FUNCTIONS FOR THE ELECTROMAGNETIC FIELD STRENGTH

The most direct way to solve our problem is to construct the Green functions $\overset{a}{D}_{\alpha\beta\lambda\rho}(x,y)$ of the electromagnetic field strength in the presence of two conducting plates moving with the velocity v relative to each other, so that the momentary distance is $a = vt$. The Green functions defined as

$$\overset{a}{D}_{\alpha\beta\lambda\rho}(x,y) = {}_a\langle 0|TF_{\alpha\beta}(x)F_{\lambda\rho}(y)|0\rangle_a \qquad (2.1)$$

depend on the parameter a through the distance-dependent vacuum state $|0\rangle_a$. From eq.(2.1) it follows immediately that they obey the symmetry relations

$$\overset{a}{D}_{\alpha\beta\lambda\rho}(x,y) = \overset{a}{D}_{\lambda\rho\alpha\beta}(y,x), \qquad (2.2)$$

$$\overset{a}{D}_{\alpha\beta\lambda\rho}(x,y) = -\overset{a}{D}_{\beta\alpha\lambda\rho}(x,y). \qquad (2.3)$$

Our aim is to construct the Green functions in the kinematical situation where the two plates $K^{(-)}$ and $K^{(+)}$ have the normal vectors $n^{(-)}_\mu = (0,0,0,-1)$ and $n^{(+)}_\mu = \frac{1}{\sqrt{1-v^2}}(v,0,0,-1)$ (The velocity of light is assumed to be equal to 1). The boundary conditions for the field strength $n^{(\pm)}_\mu \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}|_{K^{(\pm)}} = 0$ lead to the conditions

$$n^{(\pm)}_\mu \epsilon^{\mu\nu\alpha\beta} \overset{a}{D}_{\alpha\beta\lambda\rho}(x,y)|_{x\in K^{(\pm)}} = 0 \qquad (2.4)$$

for the Green functions. Moreover, they have to respect the classical equations of motion

$$\partial^\alpha_x \overset{a}{D}_{\alpha\beta\lambda\rho}(x,y) = 0 \quad (x \neq y) \qquad (2.5)$$

and

$$\partial^x_\mu \epsilon^{\mu\nu\alpha\beta} \overset{a}{D}_{\alpha\beta\lambda\rho}(x,y) = 0; \qquad x,y \notin K^{(\pm)}. \qquad (2.6)$$

In what follows, we show that the Green functions satisfying eqs. (2.2)-(2.6) can be constructed with the help of the reflection principle and give explicit expressions for them. For a given point $x_\mu = (t, -x_\perp, -z)$ we construct the images

$$x_{\pm m} = \overbrace{S \ldots S_\mp S_\pm}^{m\ \text{factors}} x = S_{\pm m} x \quad (m = 0,1,\ldots),$$

where the matrices

$$S_- = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \qquad (2.7)$$

and

$$S_+ = L^{-1} S_- L = \begin{pmatrix} \operatorname{ch} s & 0 & \operatorname{sh} s \\ 0 & 1 & 0 \\ -\operatorname{sh} s & 0 & -\operatorname{ch} s \end{pmatrix} \qquad (2.8)$$

with $s = \ln\frac{1+v}{1-v}$ describe the reflections at the plane $K^{(-)}$ and $K^{(+)}$ respectively. (The matrix L stands for a Lorentz boost from $K^{(-)}$ to $K^{(+)}$; we assume that all matrices transform covariant vectors into covariant ones). Exploiting the property $(S_\pm)^2 = 1$ and the addition theorems for hyperbolic functions, the matrices S_m can be represented in the form

$$S_{\substack{2n \\ 2n-1}} = \begin{pmatrix} \operatorname{ch} ns & 0 & \operatorname{sh} ns \\ 0 & 1 & 0 \\ \pm\operatorname{sh} ns & 0 & \pm\operatorname{ch} ns \end{pmatrix} \qquad (n = 0, \pm 1, \ldots). \qquad (2.9)$$

Note that $S^{-1}_{2n-1} = S_{2n-1}$ but $S^{-1}_{2n} = S_{-2n}$.

Before giving the explicit expression for the Green functions $\overset{a}{D}_{\alpha\beta\lambda\rho}$, we introduce the standard boundary free Green functions

$$\overset{\infty}{D}_{\alpha\beta\lambda\rho}(x,y) = \langle 0|TF_{\alpha\beta}(x)F_{\lambda\rho}(y)|0\rangle = \tag{2.10}$$

$$= (g_{\alpha\lambda}\partial^x_\beta\partial^y_\rho + g_{\beta\rho}\partial^x_\alpha\partial^y_\lambda - g_{\alpha\rho}\partial^x_\beta\partial^y_\lambda - g_{\beta\lambda}\partial^x_\alpha\partial^y_\rho)\overset{\infty}{D}(x,y)$$

with

$$\overset{\infty}{D}(x,y) = \frac{1}{4\pi^2}\,\frac{1}{(x-y)^2} \tag{2.11}$$

being the free scalar Green function. The free Green functions obviously obey the relations (2.2), (2.3), (2.5) and (2.6) (with "a" substituted by "∞"). Furthermore, as a consequence of translational invariance and homogeneity, they have the property

$$\overset{\infty}{D}_{\alpha\beta\lambda\rho}(x,y) = \overset{\infty}{D}_{\alpha\beta\lambda\rho}(y,x) = \overset{\infty}{D}_{\alpha\beta\lambda\rho}(x-y,\ 0).$$

Then it can be shown that the Green functions we are interested in look as follows

$$\overset{a}{D}_{\alpha\beta\lambda\rho}(x,y) = \begin{cases} \overset{\infty}{D}_{\alpha\beta\lambda\rho}(x,y) - (S_{\pm1})^{\alpha'}_{\alpha}(S_{\pm1})^{\beta'}_{\beta}\overset{\infty}{D}_{\alpha'\beta'\lambda\rho}(S_{\pm1}x,y) & (2.12) \\ \text{outside the plate } K^{(\pm)} & \\ \sum\limits_{n=-\infty}^{+\infty}[(S_{2n})^{\alpha'}_{\alpha}(S_{2n})^{\beta'}_{\beta}\overset{\infty}{D}_{\alpha'\beta'\lambda\rho}(S^{-1}_{2n}x,y) - (2n\to 2n-1)] & \\ \text{between the two mirrors} & (2.13) \end{cases}$$

Here the action of the reflection matrices S_n on the arguments of the functions $\overset{a}{D}_{\alpha\beta\lambda\rho}$ is in analogy with the scalar case whereas the action on the indices is necessary to guarantee the symmetry relations and the fulfillment of the boundary conditions. The proof of these properties can be done using the explicit expression (2.9).

3. CASIMIR FORCE AND ENERGY MOMENTUM TENSOR

Now it is possible to calculate physical quantities. To begin with, let us consider the energy density $\overset{a}{\mathcal{E}}(x)$ at a given point x defined as

$$\overset{a}{\mathcal{E}}(x) = \frac{1}{4}\sum_{\alpha,\beta}\overset{a}{D}_{\alpha\beta\alpha\beta}(x,x). \tag{3.1}$$

Using the eqs.(2.9)-(2.13), we obtain (some details of the calculation are presented in the appendix)

$$\overset{a}{\mathcal{E}}(x) = \begin{cases} \overset{\infty}{\mathcal{E}} & \text{outside the 2-mirror-system} \\ \overset{\infty}{\mathcal{E}} + \dfrac{3z^2+t^2}{8\pi^2(z^2-t^2)^3}\Sigma(v) & \text{between the mirrors} \\ \left(\Sigma(v) = \sum\limits_{n=1}^{\infty}\left(\mathrm{sh}\dfrac{ns}{2}\right)^{-4}\right). & \end{cases} \tag{3.2}$$

Here $\overset{\infty}{\mathcal{E}}$ is the infinite energy density of the free vacuum resulting from the Green functions $\overset{a}{D}_{\alpha\beta\lambda\rho}(x,x)$.

Amusingly the obtained result is much simpler than the scalar one /5/. After subtracting the free vacuum energy density the obtained expression is finite. Furthermore, outside the two mirror system the vacuum density does not feel the existence of the mirrors at all. Both properties are in opposite to the scalar case.

Let us analyse the obtained result in more detail. Introducing $\lambda = \frac{z}{a} \in [0,1]$ and $t = \frac{a}{v}$ with $v \in [0,1)$ (we suppose from now on $a > 0$ and $v \geq 0$) we get for the energy density between the two mirrors

$$\overset{a}{\mathcal{E}}(x) - \overset{\infty}{\mathcal{E}} = -\frac{1+3(v\lambda)^2}{8\pi^3a^4(1-(v\lambda)^2)^3}v^4\Sigma(v). \tag{3.3}$$

From (3.3) we see that the energy shift is always negative and reaches its minimum value at the moving plate. For a given nonzero velocity v and distance a it behaves as shown in fig.2.

Using eq.(3.2), it is now easy to find the attractive force $\overset{a}{F}$ (per unit area) between the two plates

$$\overset{a}{F}(v) = -\frac{d}{da}\int_0^a(\overset{a}{\mathcal{E}}(x) - \overset{\infty}{\mathcal{E}})dz = -\frac{3}{8\pi^2a^4}v^4\frac{\Sigma(v)}{(1-v^2)^2}. \tag{3.4}$$

Expanding $\Sigma(v)$ in a power series in v, we recover the static Casimir force and find the first relativistic correction

$$\overset{a}{F}(v)\Big|_{v\approx 0} = -\frac{\pi^2}{240a^4}\left[1 + v^2\left(\frac{2}{3} - \frac{10}{\pi^2}\right) + O(v^4)\right]. \tag{3.5}$$

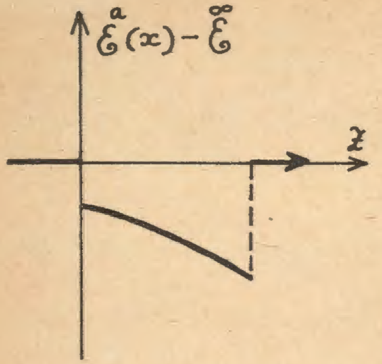

Fig.2. Behaviour of the energy density.

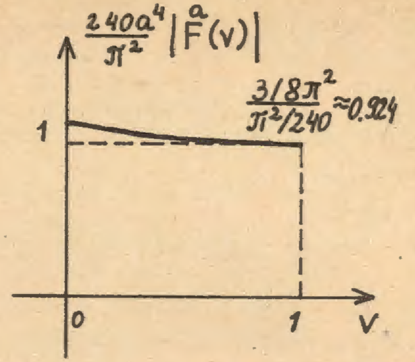

Fig.3. Velocity dependence of the Casimir force.

Similarly, it is possible to treat the ultrarelativistic limit

$$\overset{a}{F}(v)\Big|_{v=1} = -\frac{3}{8\pi^2 a^4}\left[1 + \frac{(1-v^2)^2}{16} + O((1-v^2)^4)\right]. \tag{3.6}$$

Moreover a numerical calculation gives the surprising result that the force $\overset{a}{F}(v)$ is within 10% accuracy velocity independent (see fig.3).

The vacuum expectation values of the other components of the energy momentum tensor can be considered analogously: we define

$$\overset{a}{T}{}^{\mu\nu}(x) = -\overset{a}{D}{}^{\mu\kappa\nu}{}_{\kappa}(x,x) + \frac{1}{4} g^{\mu\nu} \overset{a}{D}{}^{\kappa\sigma}{}_{\kappa\sigma}(x,x). \tag{3.7}$$

After subtracting the distance independent divergent $\overset{\infty}{T}{}^{\mu\nu}$ we find that outside the two mirror system all matrix elements are zero whereas in the in between region one has

$$\tilde{T}^{\mu\nu} \equiv \overset{a}{T}{}^{\mu\nu}(x) - \overset{\infty}{T}{}^{\mu\nu} = \frac{\Sigma(v)}{8\pi^2} \begin{pmatrix} \frac{3z^2+t^2}{(z^2-t^2)^3} & 0 & \frac{4zt}{(z^2-t^2)^3} \\ 0 & \frac{1}{(z^2-t^2)^2} & 0 \\ \frac{4zt}{(z^2-t^2)^3} & 0 & \frac{3t^2+z^2}{(z^2-t^2)^3} \end{pmatrix} \tag{3.8}$$

As expected, the trace condition is fulfilled. Of course the energy momentum tensor is form invariant

$$\tilde{T}^{\mu\nu}(x) = L^{\mu}{}_{\mu'} L^{\nu}{}_{\nu'} \tilde{T}^{\mu'\nu'}(x') = \tilde{T}^{\mu\nu}(x')$$

under transformations which respect the symmetry of the problem (for example rotations in the x,y plane), surprisingly this is true for Lorentz transformations in the z-direction too:

$$z \to z' = \frac{z - wt}{\sqrt{1-w^2}}, \quad t \to t' = \frac{t - wz}{\sqrt{1-w^2}}. \tag{3.9}$$

For further considerations it is convenient to introduce the parametrization $z = \xi t$, $0 \le \xi \le v$. In this way all points between the mirrors are parametrized. Note that points having the same value of ξ lay on one and the same straight world line (see fig.1).

From the form invariance of $\tilde{T}^{\mu\nu}$ it is clear now that, in terms of ξ, the energy-momentum tensor has the structure (3.8) even in a more general situation where the mirrors move with constant velocities v_+ and v_-:

$$\tilde{T}^{\mu\nu}(x) = -\frac{\Sigma(v)}{8\pi^2 t^4} \begin{pmatrix} \frac{1+3\xi^2}{(1-\xi^2)^3} & 0 & \frac{4\xi}{(1-\xi^2)^3} \\ 0 & -\frac{1}{(1-\xi^2)^2} & 0 \\ \frac{4\xi}{(1-\xi^2)^3} & 0 & \frac{3+\xi^2}{(1-\xi^2)^3} \end{pmatrix} \tag{3.10}$$

with $v = (v_+ - v_-)/(1 - v_+ v_-)$ and $\xi = z/t$ running now from v_- to v_+. In fact, this situation is just our initial problem ($v_- = 0, v_+ = v$), as it is seen in an arbitrary Lorentz frame.

As interesting result we obtain: At the point $\xi = 0$ the tensor is proportional to the well-known energy momentum tensor of the classical Casimir effect with static plates, the non-diagonal matrix elements vanish, there is no energy flux. The equation $z = \xi t$ describes the world lines of the points in between the two mirrors in the choosen Lorentz frame. $\xi = 0$ corresponds to the point at rest. In other words: From. eq. (3.10) it follows that in the local rest frame of an arbitrary point inside the two mirror system locally at this point: $\tilde{T}^{\mu\nu}$ is proportional to $\overset{a}{T}{}^{\mu\nu}_{v=0}$:

$$\tilde{T}^{\mu\nu}(t, x_\perp, 0) = -\Sigma(v)/8\pi^2 t^4 \cdot \mathrm{diag}(1, -1, -1, 3). \tag{3.11}$$

(And vice versa: From eq.(3.11), valid in all local rest frames, eq.(3.10) can be reconstructed).

4. FORMULATION WITH THE HELP OF THE ELECTROMAGNETIC POTENTIAL

In this last section we will offer another possibility of treating the generalized Casimir problem based on the electromagnetic potential A_μ. Again, here we restrict our considerations to the free electromagnetic field with boundary conditions. In classical electrodynamics it has been shown that the source free electro-magnetic field with boundary conditions on cylindric surfaces can be completely described with two specially chosen Hertz vectors, an electric one and a magnetic one /6/. This means that the potential can be described by the following two modes (corresponding essentially to the two independent Hertz vectors).

$$A_\mu = \frac{1}{\sqrt{-\Delta_2}} \begin{pmatrix} 0 \\ -\partial/\partial x^2 \\ \partial/\partial x^1 \\ 0 \end{pmatrix} a_1 + \frac{1}{\sqrt{-\Delta_0}} \begin{pmatrix} \partial/\partial x^3 \\ 0 \\ 0 \\ \partial/\partial x^0 \end{pmatrix} a_2, \tag{4.1}$$

$$\Delta_0 \equiv \left(\frac{\partial}{\partial x^0}\right)^2 - \left(\frac{\partial}{\partial x^3}\right)^2, \quad \Delta_2 \equiv \left(\frac{\partial}{\partial x^1}\right)^2 + \left(\frac{\partial}{\partial x^2}\right)^2 .$$

With the help of this representation the boundary conditions $n_\mu^{(\pm)} \partial_\nu \epsilon^{\mu\nu\alpha\beta} A_\beta|_{K^{(\pm)}} = 0$ reduce in standard way to boundary conditions for the independent field modes

$$a_1(x) = 0, \quad x \in K^{(\pm)} \tag{4.2}$$

$$\left(n_\mu^{(\pm)} \frac{\partial}{\partial x_\mu}\right) \frac{a_2(x)}{\sqrt{-\Delta_0}} = 0, \; x \in K^{(\pm)} \quad . \tag{4.3}$$

Taking into account the representation (4.1) the standard action of the classical electromagnetic field

$$S = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu} \tag{4.4}$$

can be rewritten as

$$S = -\frac{1}{2} \int d^4x (a_1 \Box a_1 + a_2 \Box a_2). \tag{4.5}$$

So we end up with a Lagrangian density which consists of two independent free massless scalar fields satisfying different boundary conditions (4.2), (4.3).

An independent quantization of these two free fields solves our problem. For our purposes we need the two point Green functions of the field operators only. Defining

$$\overset{a}{D}_j(x,y) = {}_a\langle 0|T a_j(x) a_j(y)|0\rangle_a, \quad j = 1,2 \tag{4.6}$$

these Green functions satisfy

$$\Box \overset{a}{D}_1(x,y) = -i\delta(x-y), \quad \overset{a}{D}_1(x,y)|_{x \text{ or } y \in K^{(\pm)}} = 0 \tag{4.7}$$

$$\Box \overset{a}{D}_2(x,y) = -i\delta(x-y), \quad (n_\mu^{(\pm)} \partial^\mu) \frac{1}{\sqrt{-\Delta_2}} \overset{a}{D}_2(x,y)|_{x \text{ or } y \in K^{(\pm)}} = 0 . \tag{4.8}$$

Both Green functions can be constructed with the help of the reflection principle. For example, let us discuss the case when both points x and y are inside of the two mirror system. For the Green functions we find

$$\overset{a}{D}_1(x,y) = \sum_{n=-\infty}^{+\infty} (-1)^n \overset{\infty}{D}(S_n x - y), \tag{4.9}$$

$$\overset{a}{D}_2(x,y) = \sum_{n=-\infty}^{+\infty} \overset{\infty}{D}(S_n x - y). \tag{4.10}$$

Obviously, $\overset{a}{D}_1$ satisfies the relations (4.7), whereas the second Green function satisfies the boundary condition

$$(n_\mu^{(\pm)} \partial_x^\mu) \overset{a}{D}_2(x,y)|_{x \in K^{(\pm)}} = (n_\mu^{(\pm)} \partial_y^\mu) \overset{a}{D}_2(x,y)|_{y \in K^{(\pm)}} = 0 \tag{4.11}$$

which is not exactly the boundary condition (4.8).

However in the sum (4.10) each Green function satisfies the Klein-Gordon equation. Therefore for all points $x \neq y$ the action of the operator Δ_0 on solutions of the Klein Gordon equation is equal to that one of the operator Δ_2. On the other hand the operator Δ_2 applied to functions on the boundary acts only inside the boundary, so that it commutes with $(n_\mu^{(\pm)} \partial^\mu)$ and the boundary conditions (4.8) and (4.11) are equivalent.

As an example, we discuss now the calculation of the energy density of the electromagnetic field. This can be done directly, starting from the known energy density of the electromagnetic field, or starting from the energy density expression for the two scalar fields. In both cases we end up with the same expression

$$\overset{a}{\mathcal{E}} = \frac{1}{2}\left(\frac{\partial}{\partial x^0}\frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^1}\frac{\partial}{\partial y^1} + \frac{\partial}{\partial x^2}\frac{\partial}{\partial y^2} + \frac{\partial}{\partial x^3}\frac{\partial}{\partial y^3}\right)[\overset{a}{D}_1(x,y) + \overset{a}{D}_2(x,y)]|_{x \to y} . \tag{4.12}$$

Here we see: the energy density is the sum of the energy densities of the two scalar fields, which however satisfy different boundary conditions, one the Dirichlet boundary condition, the other the Neumann one. Taking into account the construction of the Green functions (4.9), (4.10), we see again that in the final result (4.12) the odd reflections drop out (in opposite to the case of one scalar field only).

APPENDIX

We give here some details of the calculation of the energy density $\overset{a}{\mathcal{E}}(x)$. Consider, at first, the free Green functions $\overset{\infty}{D}_{\alpha\beta\lambda\rho}$. Performing the differentiation in eq.(2.10) yields

$$\overset{\infty}{D}_{\alpha\beta\lambda\rho}(x,y)=\frac{1}{\pi^2}\Big\{\frac{g_{\alpha\lambda}g_{\beta\rho}-g_{\alpha\rho}g_{\beta\lambda}}{\Delta^4}+\frac{2}{\Delta^6}[g_{\alpha\rho}\Delta_\beta\Delta_\lambda + g_{\beta\lambda}\Delta_\alpha\Delta_\rho - g_{\alpha\lambda}\Delta_\beta\Delta_\rho - g_{\beta\rho}\Delta_\alpha\Delta_\lambda]\Big\},\quad \Delta=x-y. \tag{A.1}$$

To determine $\overset{a}{\mathcal{E}}(x)$ via eq.(3.1), we have to calculate $\overset{a}{D}_{\alpha\beta\lambda\rho}(x,x)$ which in turn (through eqs.(2.12) and (2.13)) are expressed in terms of $\overset{\infty}{D}_{\alpha\beta\lambda\rho}(S_m x-x,0)$. Using eq.(2.9), we find

$$(S_{\substack{2n\\2n-1}}x-x)_\mu\equiv(\Delta_{\substack{2n\\2n-1}})_\mu=(t(\mathrm{ch}\,ns-1)-z\,\mathrm{sh}\,ns,\,0,\,0,\,\mp t\,\mathrm{sh}\,ns+z(1\mp\mathrm{ch}\,ns)). \tag{A.2}$$

Combining eqs.(A.1) and (A.2), we find three types of contributing terms

$$A_m=\overset{\infty}{D}_{0303}(\Delta_m,0)=\overset{\infty}{D}_{1212}(\Delta_m,0)=\frac{1}{\pi^2}\frac{1}{(\Delta_m)^4},$$

$$B_m=\overset{\infty}{D}_{0101}(\Delta_m,0)=\overset{\infty}{D}_{0202}(\Delta_m,0)=\overset{\infty}{D}_{1313}(\Delta_m,0)=$$

$$=\overset{\infty}{D}_{2323}(\Delta_m,0)=\frac{1}{\pi^2}\frac{(\Delta_m)_0^2+(\Delta_m)_3^2}{(\Delta_m)^6},$$

$$C_m=\overset{\infty}{D}_{0131}(\Delta_m,0)=\overset{\infty}{D}_{0232}(\Delta_m,0)=\frac{1}{\pi^2}\frac{2(\Delta_m)_0(\Delta_m)_3}{(\Delta_m)^6}.$$

Using now again eqs.(2.9), (2.12), and (2.13), we find - for example, in the region between the plates - the following expressions for the Green functions:

$$\overset{a}{D}_{0101}(x,x)=\overset{a}{D}_{0202}(x,x)=\sum_{n=-\infty}^{+\infty}[(B_{2n}-B_{2n-1})\,\mathrm{ch}\,ns-(C_{2n}+C_{2n-1})\,\mathrm{sh}\,ns],$$

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