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GENERALIZED COHERENT STATES FOR A RELATIVISTIC MODEL OF THE LINEAR OSCILLATOR

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A relativistic $\vec{r}$ -representation [1], introduced in the framework of quasipotential approach (QPA)[2,3] to the two-body problem, became the basis for constructing a formalism, possessing many important features of the nonrelativistic quantum mechanics. The essential difference from the quantum mechanics consists in the fact that Hamiltonian in the scheme considered is a differential-difference operator. In reference to this formalism the technique of difference differentiation was developed and analogues of the important functions of the continuous analysis were obtained[4,5]. Relativistic generalizations of the exactly solvable problems (the potential well, the Coulomb potential, the harmonic oscillator) of the quantum mechanics were considered [5-9](see also review [10], where a more extensive list of references is given). From the mathematical point of view power series, which define the solutions of these problems, are obtained from the corresponding nonrelativistic solutions by substituting in the latter the usual powers by the so-called generalized powers (see[1]). Bearing in mind the known special functions of mathematical physics (for instance, hypergeometric ones), such substitution is equivalent to transferring the dependence on the coordinate from the argument to the index.

On the other hand, the introduction of the generalized coherent states (CS), associated with the unitary representations of an arbitrary Lie group[11,12], has led to the possibility of applying CS formalism to a wider range of physical problems[11-16]. In particular, a path integral for the transition amplitude in the SU(2) CS (or spin CS) representation was contructed, which leads to classical equations of motion in a curved phase space in the form of a two-dimensional sphere [13]. In ref.[14] the SU(1,1) CS (or the three-dimensional Lorentz group CS) have been used and by means of the path integral the corresponding classical problem in a generalized phase space, namely the Lobachevskii plane, was formulated. Following the examples of the harmonic oscillator, the hydrogen atom, the Morse potential and a simple model of superfluid helium it was shown that in those cases when SU(1,1) is the dynamical symmetry group, the classical motion in a phase space will be oscillator-like.

In this paper the technique of contructing a path integral representation for the transition amplitude (propagator) between SU(1,1) CS, developed in[12-16], is applied to a model of relativistic linear oscillator, formulated in the framework of the quasipotential approach in quantum field theory[1-3]. In §§1-4 the model itself[8,9] is briefly characterized, the wave functions and the explicit realization of the dynamical symmetry group SU(1,1) generators in the momentum, as well as in the relativistic configurational x-representation[1], are given. In §§5,6 the explicit form of SU(1,1) CS for this problem is obtained and the corresponding partition function is evaluated. It is shown that the use of the semiclassical Bohr-Sommerfeld quantization rule yields the exact expression for the energy levels of the considered relativistic oscillator[17].

1. In the momentum representation, which is considered as one-dimensional Lobachevskii space realized on the hyperboloid $p_0^2 - p^2 = m^2c^2$, $p_0 > 0$ in the (p_0, p)-plane, a quasipotential model of the linear oscillator is described by the differential operator[8,9]

$$H(p) = c\left[p_0 - \frac{1}{2}\left(\frac{\hbar\omega}{mc^2}\right)^2 (p_0 - p)\, p_0 \nabla_p \left(p_0 \nabla_p - 1\right)\right], \qquad (1)$$

where $p_0 = E_p/c = (m^2c^2 + p^2)^{1/2}$ and $\nabla_p = \frac{d}{dp}$. The eigenfunctions of the Hamiltonian $H(p)$, satisfying the orthogonality relation

$$\int_{-\infty}^{\infty} d\Omega_p\, \psi_n^*(p)\, \psi_{n'}(p) = \delta_{nn'}, \qquad d\Omega_p = mc\,\frac{dp}{p_0},$$

are most simply expressed through the light-cone variable $p^+ = p_0 + p$ and have the form

$$\psi_n(p) = i^n C_n (2\eta)^\nu L_n^{2\nu-1}(2\eta)\, e^{-\eta}, \qquad \eta = \frac{cp^+}{\hbar\omega}. \qquad (2)$$

The normalization constants $C_n = [n!/mc\,\Gamma(n+2\nu)]^{1/2}$, where $2\nu = 1 + (1+4\mu^2)^{1/2}$, $\mu = mc^2/\hbar\omega$ and $L_n^\alpha(\eta)$ is the generalized Laguerre polynomials. The eigenvalues of the Hamiltonian

$$H(p) = \frac{\hbar\omega}{2}\left[\eta(1 - \nabla_\eta^2) + \mu^2/\eta\right] \qquad (1')$$

corresponding to the wave functions $\psi_n(p)$, are $E_n = \hbar\omega(n+\nu)$, $n = 0, 1, 2, \ldots$.

2. The transition to the relativistic configurational x-representation[1]

$$\psi_n(x) = (2\pi\hbar)^{-1/2} \int d\Omega_p\, \xi(p,x)\, \psi_n(p)$$

is performed by the aid of decomposition over the matrix elements of representations of the one-dimensional Lobachevskii space motion group

$$\xi(p,x) = \left(\frac{p_0 - p}{mc}\right)^{-i\tilde{x}} = \left(\frac{\eta}{\mu}\right)^{i\tilde{x}},$$

where $\tilde{x} = x/\lambda$ is the dimensionless variable and $\lambda = \hbar/mc$ is the Compton wave length. To the Hamiltonian $H(p)$ the difference operator

$$H(x) = mc^2\left[\mathrm{ch}(i\nabla_{\tilde{x}}) + \frac{\tilde{x}(\tilde{x}+i)}{2\mu^2}\exp(i\nabla_{\tilde{x}})\right]$$

corresponds in the x-representation, where by definition $\exp(\beta\nabla_x) f(x) = f(x+\beta)$. The eigenfunctions of $H(x)$

$$\psi_n(x) = \frac{2C_n}{(2\pi\hbar)^{1/2}}\, mc\, \mu^{-i\tilde{x}}\, \Gamma(\nu + i\tilde{x})\, P_n^\nu(\tilde{x}; \pi/2) \qquad (3)$$

are expressed through the Pollaczek polynomials $P_n^\nu(x; \pi/2)$. Their explicit form (3) can easily be obtained with the help of the integral representation for the gamma-function

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt, \quad \mathrm{Re}\, z > 0,$$

upon making use of the formula (see[17])

$$P_n^\nu\left(i\eta\nabla_\eta; \frac{\pi}{2}\right)\eta^\nu e^{-\eta} = i^n L_n^{2\nu-1}(2\eta)\,\eta^\nu e^{-\eta}. \qquad (4)$$

The wave functions $\psi_n(x)$ satisfy the orthogonality relation

$$\int_{-\infty}^{\infty} dx\, \psi_n^*(x)\, \psi_{n'}(x) = \delta_{nn'}.$$

3. A dynamical symmetry group for the model of the linear oscillator under consideration is the SU(1,1) group (or isomorphic to it groups SO(2,1) $\simeq$ Sp(2,R) $\simeq$ Sl(2,R)). The corresponding Lie algebra is formed by the generators K_0 and $K_\pm$ with the commutation relations

$$[K_-, K_+] = 2K_0, \qquad [K_0, K_\pm] = \pm K_\pm. \qquad (5)$$

In the momentum representation these generators are realized by the differential operators

$$K_0 = \frac{1}{\hbar\omega} H(p), \qquad K_\pm = \pm i(K_0 - \eta) + i\eta\nabla_\eta,$$

whereas in the configurational x-representation - by the difference operators

$$K_0 = \frac{1}{\hbar\omega} H(x), \quad K_\pm = \pm i\left(K_0 - \mu e^{-i\nabla_{\tilde{x}}}\right) + \tilde{x}.$$

The Casimir invariant (for both the realizations)

$$K^2 = K_0^2 - \frac{1}{2}\left(K_+K_- + K_-K_+\right)$$

turns out to be equal in this case $\nu(\nu-1)\hat{I}$, where $\hat{I}$ is the identity operator. Thus, all states of the relativistic linear oscillator belong to an infinite-dimensional irreducible unitary representation $D^+(\nu)$ (the positive discrete series) of the universal covering group $\widetilde{SU}(1,1)$. The generators K_0 and $K_\pm$ act on the wave functions ψ_n, defined by the formulas (2) and (3), in the following way:

$$K_0\psi_n = (n+\nu)\psi_n, \quad K_+\psi_n = \varkappa_{n+1}\psi_{n+1}, \quad K_-\psi_n = \varkappa_{n-1}\psi_{n-1}, \qquad (6)$$
$$\varkappa_n = \left[n(n+2\nu-1)\right]^{1/2}.$$

As follows from (6), an arbitrary state ψ_n can be obtained by the n-fold action of the raising operator K_+ on the ground state ψ_0, i.e.,

$$\psi_n = \left[n!(2\nu)_n\right]^{-1/2} K_+^n \psi_0, \qquad (7)$$

where $(\nu)_z = \Gamma(\nu+z)/\Gamma(\nu)$ is Pochhammer's symbol.

4. In a limiting case when the velocity of light $c\to\infty$ the functions ψ_n coincide with the wave functions for the nonrelativistic linear oscillator. From a group-theoretical point of view it means that the contraction [18,19] of the algebra (5) leads to generators of the Heisenberg-Weyl group. In fact, if one introduces the operators $K_0' = \varepsilon K_0$ and $K_\pm' = (\varepsilon/2)^{1/2}K_\pm$, where $\varepsilon = \mu^{-1} = \hbar\omega/mc^2$, then it can easily be verified that in the limiting case when $c\to\infty$ (upon that $\varepsilon\to 0$ and $\nu\simeq\varepsilon^{-1}\to\infty$, but $\varepsilon\nu\to 1$ [19]) they satisfy the commutation relations

$$[K_-', K_+'] = K_0', \quad [K_0', K_\pm'] = 0$$

of the Heisenberg-Weyl algebra, formed by the creation a^+ and annihilation a operators together with the identity operator $\hat{I}$.

5. In the Hilbert space of states ψ_n of the unitary irreducible representation $D^+(\nu)$ of the dynamical group SU(1,1) the coherent states $|\zeta,\nu\rangle$ are defined by acting with the operator [12-16]

$$D(\alpha) = \exp\left(\alpha K_+ - \alpha^* K_-\right)$$

on the fixed vector ψ_0, i.e.,

$$|\zeta,\nu\rangle = D(\alpha)\psi_0 = \left(1-|\zeta|^2\right)^\nu e^{\zeta K_+}\psi_0, \qquad (8)$$

where $\alpha = -\frac{\tau}{2}e^{-i\varphi}$ and $\zeta = -\mathrm{th}\frac{\tau}{2}e^{-i\varphi}$, τ and φ being group parameters. From (7) and (8) it follows that the decomposition of $|\zeta,\nu\rangle$ over the basis vectors ψ_n has the form

$$|\zeta,\nu\rangle = \left(1-|\zeta|^2\right)^\nu \sum_{n=0}^{\infty}\left[\frac{(2\nu)_n}{n!}\right]^{1/2}\zeta^n\psi_n. \qquad (9)$$

Using the formula (2) and the generating function for the Laguerre polynomials

$$\sum_{n=0}^{\infty} z^n L_n^\alpha(y) = (1-z)^{-(\alpha+1)}\exp\left(\frac{zy}{1-z}\right), \quad |z|<1$$

it is easy to show that in the momentum representation

$$\langle p|\zeta,\nu\rangle = \frac{(2\eta)^\nu}{[mc\Gamma(2\nu)]^{1/2}}\,\frac{\left(1-|\zeta|^2\right)^\nu}{(1-i\zeta)^{2\nu}}\exp\left(-\eta\frac{1+i\zeta}{1-i\zeta}\right).$$

Similarly, with the aid of (3) and the generating function for the Pollaczek polynomials

$$\sum_{n=0}^{\infty} z^n P_n^\nu\left(x;\frac{\pi}{2}\right) = (1-iz)^{ix-\nu}(1+iz)^{-ix-\nu}, \quad |z|<1$$

it is easy to see that the SU(1,1) CS in the configurational x-representation have the form

$$\langle x|\zeta,\nu\rangle = \left[2\pi\lambda\Gamma(2\nu)\right]^{-1/2}\Gamma(\nu+i\tilde{x})\left(2\frac{1-|\zeta|^2}{1+\zeta^2}\right)^\nu\left[\frac{1-i\zeta}{\mu(1+i\zeta)}\right]^{i\tilde{x}}.$$

The SU(1,1) CS $|\zeta,\nu\rangle$ are nonorthogonal and the overlap of two states (or the reproducing kernel) is given as

$$\langle\zeta',\nu|\zeta,\nu\rangle = \left[\left(1-|\zeta'|^2\right)\left(1-|\zeta|^2\right)\right]^\nu\left(1-\zeta\zeta'^*\right)^{-2\nu}. \qquad (10)$$

The important property of these states is the resolution of unity

$$\int d\rho_\nu(\zeta)\,|\zeta,\nu\rangle\langle\zeta,\nu| = \hat{I}\,, \tag{11}$$

where the invariant measure on the coset space SU(1,1)/SO(2)

$$d\rho_\nu(\zeta) = \frac{2\nu-1}{\pi}\left(1-|\zeta|^2\right)^{-2} d^2\zeta\,.$$

The matrix elements of the infinitesimal operators in the SU(1,1) CS representation have the form*)

$$\langle\zeta',\nu|K_0|\zeta,\nu\rangle = \nu\,\frac{1+\zeta\zeta'^*}{1-\zeta\zeta'^*}\,\langle\zeta',\nu|\zeta,\nu\rangle\,, \tag{12}$$

$$\langle\zeta',\nu|K_+|\zeta,\nu\rangle = \frac{2i\nu\zeta'^*}{1-\zeta\zeta'^*}\langle\zeta',\nu|\zeta,\nu\rangle\,,\qquad \langle\zeta',\nu|K_-|\zeta,\nu\rangle = -\frac{2i\nu\zeta}{1-\zeta\zeta'^*}\langle\zeta',\nu|\zeta,\nu\rangle\,.$$

6. The transition amplitude (propagator) between the SU(1,1) CS is defined as

$$K(\zeta',\zeta;T) = \langle\zeta',\nu|\exp\left(-\tfrac{i}{\hbar}TH\right)|\zeta,\nu\rangle = \langle\zeta',\nu|\exp\left(-i\omega T K_0\right)|\zeta,\nu\rangle\,. \tag{13}$$

Using (9) and (10) it can be shown that

$$K(\zeta',\zeta;T) = e^{-i\nu\omega T}\langle\zeta',\nu|\zeta e^{-i\omega T},\nu\rangle = e^{-i\nu\omega T}\left[(1-|\zeta|^2)(1-|\zeta'|^2)\right]^{\nu}\left(1-\zeta\zeta'^* e^{-i\omega T}\right)^{-2\nu}.$$

The partition function for the model of the relativistic oscillator under consideration

$$Z = \mathrm{Tr}\,K(\zeta',\zeta;-i\hbar\beta) = e^{-\beta\nu\hbar\omega}\left(1-e^{-\beta\hbar\omega}\right)^{-1} = e^{-(\nu-\frac{1}{2})\beta\hbar\omega}\,Z_{NR}\,,$$

where Z_{NR} is the partition function for the nonrelativistic linear oscillator.

7. To derive a path integral for the propagator (13) let $\zeta_0=\zeta$, $\zeta_1=\zeta_0+\Delta\zeta_1,\dots,\zeta_N=\zeta_{N-1}+\Delta\zeta_N=\zeta'$. Making use of the completeness relation (11) in the points $\zeta_1,\dots,\zeta_{N-1}$, it is possible to represent (13) as /20/

*) The generators K_+ and K_- are related to the one's, employed in /14/, as $K_+=iK_+^{GS}$ and $K_-=-iK_-^{GS}$, i.e., they differ by a rotation through the angle $\pi/2$ in the (1,2) plane, which is irrelevant for the subsequent considerations.

$$K(\zeta',\zeta;T) = \int\cdots\int\prod_{j=1}^{N-1} d\rho_\nu(\zeta_j)\,\langle\zeta',\nu|\exp\left(-\tfrac{iT}{\hbar N}H\right)|\zeta_{N-1},\nu\rangle\cdots\langle\zeta_1,\nu|\exp\left(-\tfrac{iT}{\hbar N}H\right)|\zeta,\nu\rangle\,. \tag{14}$$

With the help of (12) it is easy to see that for the large N each factor in the integrand (14)

$$\langle\zeta_j,\nu|\exp\left(-\tfrac{iT}{\hbar N}H\right)|\zeta_{j-1},\nu\rangle \simeq \exp\left\{\ln\langle\zeta_j,\nu|\zeta_{j-1},\nu\rangle - \frac{i\epsilon}{\hbar}\mathcal{H}(\zeta_j,\zeta_{j-1})\right\},$$

where $\epsilon = T/N$ and

$$\mathcal{H}(\zeta_j,\zeta_{j-1}) \equiv \frac{\langle\zeta_j,\nu|H|\zeta_{j-1},\nu\rangle}{\langle\zeta_j,\nu|\zeta_{j-1},\nu\rangle} = \nu\hbar\omega\cdot\frac{1+\zeta_j^*\zeta_{j-1}}{1-\zeta_j^*\zeta_{j-1}}\,.$$

Besides, as it follows from (10)

$$\ln\langle\zeta_j,\nu|\zeta_{j-1},\nu\rangle \simeq \nu\left(1-|\zeta_j|^2\right)^{-1}\left(\zeta_j\Delta\zeta_j^* - \zeta_j^*\Delta\zeta_j\right).$$

Thus, when $N\to\infty$ we arrive at the following path integral for the propagator (13)

$$K(\zeta',\zeta;T) = \int\mathcal{D}\rho_\nu(\zeta)\exp\left\{\frac{i}{\hbar}\int_0^T\mathcal{L}(\zeta,\dot\zeta;\zeta^*,\dot\zeta^*)\,dt\right\} \tag{15}$$

with the "Lagrangian"

$$\mathcal{L}(\zeta,\dot\zeta;\zeta^*,\dot\zeta^*) = \frac{i\nu\hbar}{1-|\zeta|^2}\left\{\dot\zeta(t)\zeta^*(t)-\zeta(t)\dot\zeta^*(t)\right\} - \mathcal{H}(\zeta,\zeta^*)\,. \tag{16}$$

8. The corresponding to (15) classical Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\zeta}\right) = \frac{\partial\mathcal{L}}{\partial\zeta}\,,\qquad \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\zeta^*}\right) = \frac{\partial\mathcal{L}}{\partial\zeta^*} \tag{17}$$

are obtained by variation of the action $S=\int_0^T\mathcal{L}\,dt$. Using (16) it is possible to represent (17) in the form of Hamilton's equations

$$\dot\zeta^* = i\,\frac{(1-|\zeta|^2)^2}{2\nu\hbar}\frac{\partial\mathcal{H}}{\partial\zeta}\,,\qquad \dot\zeta = -i\,\frac{(1-|\zeta|^2)^2}{2\nu\hbar}\frac{\partial\mathcal{H}}{\partial\zeta^*}\,. \tag{18}$$

Defining the Poisson brackets as

$$\left\{ A,B \right\} = \frac{(1-|\zeta|^2)^2}{2i\nu\hbar}\left(\frac{\partial A}{\partial \zeta}\frac{\partial B}{\partial \zeta^*} - \frac{\partial A}{\partial \zeta^*}\frac{\partial B}{\partial \zeta} \right), \tag{19}$$

one can write (18) in a more compact form

$$\dot{\zeta}^* = \left\{ \zeta^*, \mathcal{H} \right\}, \qquad \dot{\zeta} = \left\{ \zeta, \mathcal{H} \right\}. \tag{18'}$$

As follows from the Poisson brackets (19), the equations (18') describe the classical motion in a curved phase space in the form of a Lobachevskii plane (see /21,14/).

Since in the case considered $\mathcal{H}(\tau) \equiv \mathcal{H}(\zeta,\zeta^*) = \nu\hbar\omega \operatorname{ch}\tau$, the equation (18') written in terms of the group parameters τ and φ is reduced to

$$\dot{\tau} = \left\{ \tau, \mathcal{H}(\tau) \right\} = 0, \qquad \dot{\varphi} = \left\{ \varphi, \mathcal{H}(\tau) \right\} = \omega. \tag{20}$$

Evidently, the solutions of (20) are τ =const and $\varphi = \omega t + \varphi_0$.

9. To find the possible values for the energy of a classical system, described by the equations (18'), let us express the Lagrangian (16) through the parameters τ and φ

$$\mathcal{L}(\tau,\varphi) = \hbar\nu\left[(\operatorname{ch}\tau - 1)\dot{\varphi} - \omega \operatorname{ch}\tau \right] = \nu\hbar\, \widetilde{\mathcal{L}}(\tau,\varphi). \tag{21}$$

Upon introducing the canonically conjugate to the "coordinate" φ momentum

$$\widetilde{p} = \frac{\partial \widetilde{\mathcal{L}}}{\partial \dot{\varphi}} = \operatorname{ch}\tau - 1,$$

the Lagrangian (21) can be written as

$$\widetilde{\mathcal{L}}(\tau,\varphi) = \widetilde{p}\,\dot{\varphi} - \omega(\widetilde{p} + 1). \tag{21'}$$

Since $\widetilde{\mathcal{L}}(\tau,\varphi)$ does not depend on the coordinate φ (i.e., φ is a cyclic one), then the canonically conjugate to φ momentum $\widetilde{p}$ is a constant (which is also evident from the first equation of the system (20)).

Now substituting (21) into (15) we arrive at the representation

$$K(\zeta',\zeta;T) = \int \mathcal{D}\rho_\nu(\zeta) \exp\left\{ i\nu \int_0^T \widetilde{\mathcal{L}}(\tau,\varphi)\,dt \right\}. \tag{22}$$

Since the parameter ν , characterizing an irreducible representation $D^+(\nu)$ of the dynamical symmetry group SU(1,1), behaves like $\nu \simeq mc^2/\hbar\omega$ as $\hbar \to 0$, then from (22) it follows that for ν sufficiently large the motion becomes quasi-classical (see /16/). Therefore for large values of ν we can make use of the Bohr-Sommerfeld quantization rule

$$\oint \widetilde{p}\, d\varphi = \frac{2\pi}{\nu} n, \qquad n = 0, 1, 2, \dots,$$

which leads to the following possible values of the momentum $\widetilde{p} = \frac{n}{\nu}$. Consequently, the energy of the considered system is equal to

$$E = \mathcal{H}(\tau) = \nu\hbar\omega \operatorname{ch}\tau = \nu\hbar\omega(\widetilde{p} + 1) = \hbar\omega(n + \nu). \tag{23}$$

Thus, as in the nonrelativistic case, the semiclassical Bohr-Sommerfeld quantization rule yields for the energy levels of the relativistic linear oscillator (1) the expression (23), which coincides with the exact one.

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Обобщенные когерентные состояния для релятивистской модели линейного осциллятора

Получен явный вид обобщенных когерентных состояний для релятивистской модели линейного осциллятора как в импульсном, так и в конфигурационном x-представлении. Вычислена соответствующая функция распределения. Показано, что использование квазиклассического правила квантования Бора-Зоммерфельда приводит к точному выражению для уровней энергии рассмотренного релятивистского осциллятора.

Работа выполнения в Лаборатории теоретической физики ОИЯИ.

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Atakishiyev N.M., Mir-Kasimov R.M. E2-85-214
Generalized Coherent States for a Relativistic Model of the Linear Oscillator

The explicit form of the SU(1,1) coherent states for a relativistic model of the linear oscillator is obtained both in momentum and in configurational x-representation. The corresponding partition function is evaluated. It is shown that the use of semiclassical Bohr-Sommerfeld quantization rules yields the exact expression for the energy levels of the relativistic oscillator considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985