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GAUGE-INVARIANT VARIABLES AND INFRARED CONFINEMENT IN QCD

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1. Introduction

The physical idea of "confinement" was based [1,2] on two experimental facts: (i) the "observation" of quarks as partons [3] in the regime of deep-inelastic scattering and (ii) their "nonobservation" in the free states.

In QCD [4] the fact (i) is explained by the asymptotic freedom phenomenon [5] which takes place in the Euclidean region, and the fact (ii) - by a linearly rising quark interaction potential, or by the validity of the Wilson criterion [1].

Generally speaking, the asymptotic freedom in QCD differs from the regime of quarks in the naive parton model [3]. In the latter the "observation" means the existence of imaginary parts of the quark diagrams with quarks on the mass shell (i.e., in the Minkowski space). This difference can be partially removed by the hypothesis of the probability equal to unity of the quarks hadronization and by the duality principle which is fruitfully used for the sum-rule construction [6].

However, the realization of the point (ii) in QCD is more problematic.

Recently it has been reported [7] that the Wilson criterion and the linearly rising potential do not give a correct mathematical definition of the confinement as far as they allow a pole to exist in the quark Green function. This means that there is a particle with quantum numbers of the quark in the physical spectrum of elementary excitations. (Recall that in quantum field theory and statistical physics the calculation of the Green function poles is a standard way to obtain the excitation spectrum). The absence of poles of the Green function of colour particles may be considered a mathematical definition of "nonobservation" of quarks and gluons [8].

However, this way of the foundation of confinement involves principal difficulties: the dependence of colour Green functions on the choice of the gauge and the infrared divergences. (The equivalence of the S-matrix for different gauges is proved [9] up to infrared divergences).

One of the examples of such a gauge dependence is the Gribov ambiguity [10] removed by the choice of the gauge, but this way has no sufficient physical basis yet.

In the present paper we study the infrared behaviour of non-Abelian fields and the Green functions taking for the physical criteria of the choice of an infrared regularization and a gauge the following postulates:

i) Finite space-time

$$S[A,\psi] \mapsto S_{R,T}[A,\psi] = \int_{-T/2}^{T/2} dt \int_{|\vec{x}|\le R} d^3x \, \mathcal{L}(A,\psi),$$

where S is the action, R is the radius of a three-dimensional sphere, T is a time interval. (Recall that the physical quantities in quantum field theory - cross-sections, decay probabilities, etc., - are normalized onto the finite space-time, independently of the way of regularization.)

ii) Quantization of the theory in terms of gauge-invariant dynamical variables

$$S^{inv}_{R,T}[A,\psi] \mapsto S_{R,T}(A^{inv}, \psi^{inv}).$$

To illustrate the gauge-invariant method of quantization we consider the Abelian theory (QED) in section 2. The non-Abelian theory is constructed on a formal level in terms of gauge-invariant variables in section 3. In section 4 we consider the phenomenon of topological vacuum degeneration in the theory i), ii). In section 5 the infrared regulized generating functional of the Green functions is constructed.

2. Gauge-Invariant Variables in the Abelian Theory

We treat the gauge-invariant method of quantization of the Abelian theory (QED) with the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + i\bar{\psi}\gamma_\mu(\partial_\mu + \hat{A}_\mu)\psi \tag{1}$$

$$(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \;;\; \hat{A}_\mu = ieA_\mu)$$

invariant under the gauge transformation

$$\hat{A}_\mu^g = g(\hat{A}_\mu + \partial_\mu)g^{-1} \;;\; \psi^g = g\psi \;;\; (g = \exp ie\lambda(\vec{x},t)). \tag{2}$$

The construction of the Hamiltonian of theory (1) is based, as a rule, on different interpretation of the field component A_0 and A_i. As the time derivative of A_0 is absent in the Lagrangian, the Gauss classical equation

$$\delta S/\delta A_0 = 0 \;\Rightarrow\; \partial_i^2 A_0 = \partial_0 \partial_i A_i + j_0 \tag{3}$$

$$(j_\mu = e\bar{\psi}\gamma_\mu\psi)$$

is used as a constraint equation that expresses A_0 in terms of the dynamical variables A_i, ψ :

$$A_0(\vec{x},t) = \frac{1}{\partial^2}(\partial_0\partial_i A_i + j_0) = -\frac{1}{4\pi}\int \frac{d^3y}{|\vec{x}-\vec{y}|}\left[\partial_0 \frac{\partial}{\partial y_i} A_i(\vec{y},t) + j_0(\vec{y},t)\right]. \tag{4}$$

The substitution of solution (4) into eq. (1) leads to the Lagrangian

$$\mathcal{L} = \frac{1}{2}[(\delta_{ij} - \partial_i \frac{1}{\partial^2}\partial_j)\partial_0 A_j]^2 + \frac{1}{2} j_0 \frac{1}{\partial^2} j_0 - \frac{1}{4} F_{ij}^2 + \tag{5}$$

$$+ i\bar{\psi}\gamma_\mu\partial_\mu\psi - j_i A_i + j_0 \frac{1}{\partial^2}(\partial_0 \partial_j A_j)$$

which depends only on variables A_i, ψ and is invariant under the transformation

$$\hat{A}_i^g = g(\hat{A}_i + \partial_i)g^{-1} \;;\; \psi^g = g\psi. \tag{6}$$

To fix gauge of A_i one imposes, generally speaking, an arbitrary constraint $f(A_i) = 0$.

Instead, the starting point of the method suggested here is the choice of dynamical variables A_i^I, ψ^I which are invariant under gauge transformations (6)

$$(A_i^I)^g = A_i^I \quad ; \qquad (\psi^I)^g = \psi^I. \tag{7}$$

We may define these variables using solution (4) of the constraint equation (3)/11,12/

$$\hat{A}_i^I = v(A)(\hat{A}_i + \partial_i)v^{-1}(A) \equiv (\delta_{ij} - \partial_i \frac{1}{\partial^2}\partial_j)\hat{A}_j \tag{8}$$
$$\psi^I = v(A)\psi,$$

where the gauge factor

$$v(A) = exp\left\{\int^t dt' \frac{1}{\partial^2}\partial_j \partial_0 \hat{A}_j\right\} \equiv exp\left\{\frac{1}{\partial_i^2}\partial_j \hat{A}_j\right\} \tag{9}$$

according to eq. (3) transforms as

$$v(A^g) \equiv v^g = v g^{-1}. \tag{10}$$

The transformations of $v(A)$ in eq. (8) completely compensate the transformations of A and ψ, and we obtain eq. (7).

Due to the gauge invariance the variables (8) identically satisfy the transversality conditions

$$\partial_i \partial_0 A_i^I \equiv 0 \quad ; \qquad \partial_i A_i^I \equiv 0. \tag{11}$$

The Lagrangian (1) in terms of these variables coincides with that in the Coulomb gauge

$$\mathcal{L} = \frac{1}{2}(\partial_0 A_i^I)^2 - \frac{1}{4}F_{ij}^2(A^I) - j_i^I A_i^I + \frac{1}{2}j_0^I \frac{1}{\partial_i^2} j_0^I + i\bar{\psi}^I \partial_\mu \gamma_\mu \psi^I. \tag{12}$$

We can construct the quantum theory only in the physical sector of transverse fields. Relativistic covariance of transverse fields in this approach is proved in ref./12,13/. The difference from the usual gauge approach consists in fixing the gauge from the very dynamics defined by eqs. (1), (3).

3. Gauge-Invariant Variables in the Non-Abelian Theory

We consider the Yang-Mills theory in a finite volume and for a finite time interval

$$S_{R,T} = \int_{-T/2}^{T/2} dt \int_{|\vec{x}|\le R} d^3x\, \mathcal{L} \tag{13}$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + i\bar{\psi}\gamma_\mu(\partial_\mu + \hat{A}_\mu)\psi \tag{14}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\epsilon^{abc}A_\mu^b A_\nu^c \; ; \; \hat{A}_\mu = e\frac{A_\mu^a \tau^a}{2i}. \tag{15}$$

The theory (13)-(15) on the solution of the constraint equation

$$\delta S/\delta A = 0 \quad \Rightarrow \quad \nabla_i^2(A)A_0^a = \nabla_i(A)\partial_0 A_i + j_0^a \tag{16}$$

$$j_\mu^a = e\bar{\psi}\gamma_\mu \frac{\tau^a}{2}\psi \quad ; \quad \nabla_i^{ab}(A) = \delta^{ab}\partial_i + e\epsilon^{acb}A_i^c \tag{17}$$

is invariant under gauge transformations of the dynamical variables A_i, ψ

$$\hat{A}_i^g = g(\hat{A}_i + \partial_i)g^{-1} \quad ; \quad \psi^g = g\psi. \tag{18}$$

To fix gauge we choose the dynamical variables of the type of (8), (9)*)

$$\hat{A}_i^I(A) = v(A)(\hat{A}_i + \partial_i)v^{-1}(A)$$
$$\psi^I(A,\psi) = v(A)\psi, \tag{19}$$

where the matrix $v(A)$ satisfies the equations

$$\partial_0 v(A) = v(A)\left\{\frac{1}{\nabla_i^2(A)}\left(\nabla_j(A)\partial_0 \hat{A}_j\right)\right\} \tag{20}$$

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*) Such variables of the non-Abelian theory first were considered in ref. /14/ (see also ref. /15/).

the general solution of which has the form of a time-ordered exponent

$$v(A) = T\exp\left\{\int^{t} dt\, \nabla_i^{-2}(A)(\nabla_j(A)\partial_0 \hat{A}_j)\right\} \tag{21}$$

(where $\nabla_i^{-2}(A)$ is the reverse operator which is calculated by perturbation theory).

Let us show that the variables (19) are invariant under the gauge transformations (18). The transformation properties of the matrix $v(A)$

$$v(A^g) \equiv v^g = v g^{-1} \tag{22}$$

follow from eqs. (16), (20) and are consistent with the transformation of the expression

$$\frac{1}{\nabla_i^2(A)} \nabla_j(A)\partial_0 \hat{A}_j = \hat{a}_0(A)$$

under (18)

$$\hat{a}_0^g = g(\hat{a}_0 + \partial_0)g^{-1}. \tag{23}$$

Really, substituting into the left- and the right-hand of eq. (20) the transformation (22), we get

$$1)\ \partial_0 v^g = \partial_0(v g^{-1}) = (\partial_0 v)g^{-1} + v(\partial_0 g^{-1}) = v\hat{a}_0 g^{-1} + v(\partial_0 g^{-1})$$

$$2)\ v^g \hat{a}_0^g = v^{-1} g^{-1} g(\hat{a}_0 + \partial_0)g^{-1} = v\hat{a}_0 g^{-1} + v(\partial_0 g^{-1}). \tag{24}$$

From (22) it is easy to show the invariance of the variables (19)

$$\hat{A}_i^I(A^g) = v g^{-1} g(\hat{A}_i + \partial_i)g^{-1} g v^{-1} = \hat{A}_i^I(A)$$

$$\psi^I(A^g, \psi^g) = v g^{-1} g \psi = v\psi = \psi^I(A, \psi).$$

As a result, the variables (19) satisfy equalities

$$\nabla_i(A^I)\partial_0 A_i^I \equiv 0 \quad ; \quad \int^{t} dt' \nabla_i(A^I)\partial_0 A_i^I \equiv 0 \tag{25}$$

that will be called the "covariant Coulomb gauge".

The Lagrangian (14) in terms of gauge-invariant fields A_i^I ψ^I has the form

$$\mathcal{L}^I = \frac{1}{2}(\partial_0 A_i^a)^2 - \frac{1}{4}F_{ij}^2(A) - j_i^a A_i^a + \frac{1}{2} j_0 \frac{1}{\nabla_i^2(A)} j_0 + i\bar{\psi}\gamma_\mu \partial_\mu \psi, \tag{26}$$

where

$$A_i = A_i^I \ ; \ \psi = \psi^I$$

The quantization of the dynamical system (26) with constraints (25) in the framework of the Hamilton approach has been made in the recent paper /16/, where the generating functional for the Green function was obtained.

The result of ref./16/ is easy to reproduce by using the Faddeev-Popov (F-P) method, the validity of which for gauges depending on time has been proved in ref./17/.

According to the F-P method the generating functional in gauge (25) has the form of the continual integral

$$Z^I_{R,T}(\text{sources}) = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi}\, \Delta(A)\, \delta[\nabla_i(A)\partial_0 A_i] \exp\{iS_{RT} + \text{sources}\}, \tag{27}$$

where $\Delta(A)$ is defined by the relation

$$\Delta(A)\int \mathcal{D}u(x)\, \delta[\nabla_i(A_i^u)\partial_0 A_i^u] = 1 \ ; \ \hat{A}_i^u = u(\hat{A}_i + \partial_i)u^{-1}.$$

Using the transformation properties

$$\partial_0 \hat{A}_i^u = u[\partial_0 \hat{A}_i + \nabla_i(A)(\partial_0 u^{-1})u]u^{-1}$$

$$\nabla_i(A^u)\partial_0 \hat{A}_i^u = u[\nabla_i(A)\partial_0 \hat{A}_i + \nabla_j^2(A)(\partial_0 u^{-1})u]u^{-1}$$

for the F-P determinant we get the expression

$$\Delta(A) = (\mathcal{D}et\, \nabla_i^2(A)).$$

Integrating in the functional integral (27) over A_0 (that corresponds to the use of the constraint equation (16)) we obtain the final result

$$Z^{I}_{R,T}(\eta,\bar\eta,J)=\int \mathcal{D}A_i\mathcal{D}\psi\mathcal{D}\bar\psi\,(\mathrm{Det}\,\nabla_i^2)^{1/2}\,\delta[\nabla_i(A)\partial_0 A_i]\times$$

$$\times\exp\Big\{i\int_{-T/2}^{T/2}dt\int_{|\vec x|\leq R} d^3x\,[\mathcal{L}^I(A,\psi)+\bar\psi\eta+\bar\eta\psi+J_i^aA_i^a]\Big\}, \qquad (28)$$

where $\mathcal{L}^I$ is defined in eq. (26). This result for gluodynamics coincides with the result of ref. /16/.

4. Topological Vacuum Degeneration in the Non-Abelian Theory

The integral (28) may be rigorously proved only for a quantum system with finite number of degrees of freedom /16/. In the theory of continual fields some problems arise, one of which is the problem of "zeroes" in the F-P determinant. As it has been shown in ref. /10/, this problem bears a relation to the fact that the gauge condition in the non-Abelian theory $f(A)=0$ does not fix fields uniquely, i.e., there are gauge transformations $A\mapsto A^g$ that do not change the gauge condition $f(A^g)=0$. The equation for the matrix g, $f(A^g)-f(A)=0$, called the Gribov equation, describes zero eigenvalue of the differential operator in the F-P determinant.

At present there are different opinions concerning possible solutions of this problem. Some of the authors (in particular, Gribov /10/) consider that the gauge ambiguity reflects long-wave (infrared) peculiarities of the non-Abelian fields, and it is necessary to take into account them by modification of the F-P integral. Other authors (for example, see /16/) incline to consider the ambiguity by an artifact and choose the gauge, where the Gribov equation has only the trivial solution $g=1$.

In our opinion, the very fact of the dependence of S-matrix in the infrared region on the choice of gauge raises the question about physical criteria of such a choice.

In the preceding section we have shown that as a criterion of that type one may use the postulate of quantization of the non-Abelian theory in terms of the gauge-invariant dynamical variables, which leads to the covariant Coulomb gauge. In this gauge the Gribov equations have the form

$$\nabla_i^2(g^{-1}\partial_0 g)=0\quad;\quad \int^t dt'\,\nabla_i^2(g^{-1}\partial_{0'}g)=0. \qquad (29)$$

In the lowest order of perturbation theory instead of eqs. (29) we get the Laplace equation for the gauge phase

$$g(x,t)=e^{\hat\lambda(\vec x,t)} \qquad (30)$$

$$\partial_i^2\partial_0\hat\lambda(\vec x,t)=0\quad;\quad \partial_i^2\hat\lambda(\vec x,t)=0. \qquad (31)$$

(In the Coulomb gauge /10/ the Gribov equation has a more complex form $\partial_i(g\partial_i g^{-1})=0$).

Equations (30), (31) have nontrivial solutions in the finite space $R(3)\,(|\vec x|\leq R)$ in the class of continuous smooth matrices $g(\vec x,t)$. The condition of such continuity of g everywhere in $R(3)$ including the boundary (which physically means the absence of sources, i.e. an empty space) is the equality of the topological functional

$$n=\frac{1}{24\pi^2}\int_{|\vec x|\leq R} d^3x\,\epsilon_{ijk}\,\mathrm{tr}(\hat V_i\hat V_j\hat V_k) \qquad (32)$$

$$\hat V_i=g\partial_i g^{-1}=e^{\hat\lambda}\partial_i e^{-\hat\lambda} \qquad (33)$$

to integers

$$n=\pm(0,1,2,\dots). \qquad (34)$$

The general solution of eqs. (30)-(34) is

$$g(\vec x,t)=g_{n,\varphi_i}(x)=\exp\hat\lambda_{n,\varphi_i}(\vec x) \qquad (35)$$

$$\hat\lambda_{n,\varphi_i}(\vec x)=i\tau^a\,\Omega^{ab}(\varphi_i)\,\frac{x^b}{R}\,\pi n, \qquad (36)$$

where the matrix Ω is defined by

$$\tau^a\Omega^{ab}(\varphi_i)=U(\varphi_i)\tau^a U^{-1}(\varphi_i)\;;\quad U(\varphi_i)=e^{i\tau_1\frac{\varphi_1}{2}}e^{i\tau_2\frac{\varphi_2}{2}}e^{i\tau_1\frac{\varphi_3}{2}}, \qquad (37)$$

$\varphi_1,\varphi_2,\varphi_3$ are three Euler angles which describe an orientation of the $\vec x$-space coordinates with respect to colour coordinates.

We may give these solutions the unique physical interpretation if we recall the dynamical origin of the covariant Coulomb gauge.

It is unique gauge where the Gribov equation (29) coincides with the homogeneous Gauss equation (16)

$$\nabla_i^2(A) A_0^a = 0,$$

and the gauge arbirariness has the dynamical nature of the infrared zeroes of the operator ∇^2, which has not been taken into account in the construction of the gauge-invariant dynamical variables (19), (20), (21). In the absence of charges and transverse fields A_i the matrix v (21), with taking into account the zeroes of ∇^2, coincides with the matrix g (35) and describes the vacuum configuration of gluon fields (33) with zero energy. From a point of view of the dynamics of the system the fact of existence of topological solutions (33)-(36) means the vacuum degeneration, with the degeneration parameters n, φ_i (32). This phenomenon does not occur in the Abelian theory, where there is only a trivial solution of the Laplace equation described by a constant phase.

5. Infrared Topological Confinement

Taking into account the zero eigenvalues of the operator ∇_i^2 we get the following action

$$S_{R,T} = \int_{-T/2}^{T/2} dt \int_{|\vec{x}|\le R} d^3x \left\{ \mathcal{L}^I(A,\psi) + \bar{\eta}\psi + \bar{\psi}\eta + A_i^a J_i^a \right\}, \tag{38}$$

where $\mathcal{L}^I$ is defined by eq. (26) and

$$\hat{A}_i = g(\hat{A}_i^I + \partial_i) g^{-1}$$
$$\psi = g\psi^I \; ; \quad \bar{\psi} = \bar{\psi}^I g^{-1} \tag{39}$$

The matrix $g(\vec{x},t)$ satisfies the equations

$$\nabla_i(A^I)^2 (g^{-1}\partial_0 g) = 0 \; ; \quad \int^t dt' \nabla_i(A^I)^2 (g^{-1}\partial_0 g) = 0 \tag{40}$$

and the field A^I satisfies the gauge (25). As $g(\vec{x},t)$ is a smooth function, we get

$$\mathcal{L}^I(A,\psi) = \mathcal{L}^I(A^I,\psi^I)$$

The action (38) differs from that in integral (28) by phases of the colour particles, which depend on the infrared degeneration parameters n, φ_i :

$$S_{R,T} = \int_{-T/2}^{T/2} dt \int_{|\vec{x}|\le R} d^3x \left\{ \mathcal{L}^I(A^I,\psi^I) + \bar{\eta}_{n,\varphi}\psi^I + \bar{\psi}^I \eta_{n,\varphi} + (J_{n,\varphi})_i^a A_i^{Ia} \right\} \tag{41}$$

$$\eta_{n,\varphi}(\vec{x},t) = g_{n,\varphi}^{-1}(\vec{x})\eta(\vec{x},t) \; ; \quad \bar{\eta}_{n,\varphi}(\vec{x},t) = \bar{\eta}(\vec{x},t) g_{n,\varphi}(\vec{x})$$
$$\hat{J}_{n,\varphi}(\vec{x},t) = g_{n,\varphi}(\vec{x})^{-1} \hat{J} g_{n,\varphi}(\vec{x}) \tag{42}$$
$$g_{n,\varphi}(\vec{x}) = \exp\left\{ -i \frac{x^a \Omega^{ab}(\varphi) \tau^b}{R} \pi n \right\}.$$

The generating functional for the Green functions in such a theory coincides with (28) up to the change (42) and the average over the generation parameters

$$Z_{conf}(\eta,\bar{\eta},J) = \lim_{R,T\to\infty} \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} Z_{R,T}^I(\eta_{n,\varphi}, \bar{\eta}_{n,\varphi}, J_{n,\varphi}), \tag{43}$$

where $Z_{R,T}^I$ is defined by eq. (28). The variation of this functional with respect to the sources is accompanied by the average over all the Euler angles describing the colour coordinates orientation with respect to space coordinates

$$\left(\prod_{\alpha=1}^{k} \int d\Omega(\varphi_\alpha) \frac{\delta}{\delta \eta_{n,\varphi_\alpha}(x_\alpha)} \right) \left(\prod_{\beta=1}^{\ell} \int d\Omega(\varphi_\beta) \frac{\delta}{\delta \bar{\eta}_{n,\varphi_\beta}(y_\beta)} \right) \left(\prod_{\gamma=1}^{m} \int d\Omega(\varphi_\gamma) \frac{\delta}{\delta (J_{i_\gamma}^{a_\gamma})_{n,\varphi_\gamma}(z_\gamma)} \right). \tag{44}$$

From expression (43) we can get the usual F-P integral, if the limit $R,T \to \infty$ is put under signs of the functional integration and averaging over the infrared parameters $\left(\lim_{R\to\infty} g_n(x) = 1\right)$.

However, as is known[18] in statistical physics and quantum field theory, the infrared regularization is removed after averaging over parameters and integration.

Let us calculate the Green function of a quark. In the lowest order of perturbation theory we get

$$G(\vec{x},\vec{y}) = \frac{\delta^2}{\delta\eta(x)\delta\bar{\eta}(y)} Z_{conf}(\eta,\bar{\eta}) \Big|_{\eta=\bar{\eta}=0} = G_0(x-y) f(\vec{x},\vec{y}), \tag{45}$$

where

$$f(\vec{x},\vec{y}) = \lim_{R\to\infty} \lim_{L\to\infty} (1/L) \sum_{n=-L/2}^{L/2} [\hat{\mathcal{O}}(\vec{x})\hat{\mathcal{O}}(\vec{y})]^n = \begin{cases} 1, & |\vec{x}| = |\vec{y}| \\ 0, & |\vec{x}| \neq |\vec{y}| \end{cases} \qquad (46)$$

$$\hat{\mathcal{O}}(\vec{x}) = \exp\left(i \frac{\tau^a x^a}{R} \pi\right). \qquad (x = (\vec{x}, x_0 = t))$$

It is easy to check that the propagator (45), (46) in the momentum representation is equal to zero

$$\int d^4x\, d^4y\, e^{i x_\mu p_\mu - i y_\mu q_\mu} G(x,y) = 0. \qquad (47)$$

We may obtain this result directly in the momentum space, if in the limit $R \to \infty$ we change the noncommuting matrices τ^a by the constant vectors

$$e^{i \frac{x^a \tau^a}{R} \pi n} \Longrightarrow e^{-i(t_\mu x^\mu) n} \; ; \; t_\mu = (t_0 = 0, t^a = \frac{\tau^a}{R}\pi). \qquad (48)$$

In the approximation the Green function in the momentum space

$$G(p) = \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{L/2} \frac{1}{(\hat{p} + \hat{t} n)} = 0 \; ; \quad (\hat{p} = p_\mu \gamma_\mu) \qquad (49)$$

disappears due to infinite large momenta of the quark interacting with the infrared vacuum.

Now let us consider the quark loop as the vacuum average of the current product

$$\langle j^\Gamma(x) j^\Gamma(y) \rangle \quad ; \quad j^\Gamma = \bar{\psi} \Gamma \psi .$$

In the lowest order of perturbation theory we get

$$\langle j^\Gamma(x) j^\Gamma(y) \rangle = \lim_{R,T\to\infty} \int d\Omega(\varphi_x) d\Omega(\varphi_y) \lim_{L\to\infty} \frac{1}{L} \sum_{-L/2}^{L/2} \bar{x}$$

$$tr\left[\left(g^{-1}_{n,\varphi_x}(\vec{x}) \Gamma g_{n,\varphi_x}(\vec{x})\right) G_0(x-y) \left(g^{-1}_{n,\varphi_y}(\vec{y}) \Gamma g_{n,\varphi_y}(\vec{y})\right) G_0(y-x)\right]. \qquad (50)$$

where $d\Omega$ is the integral over Euler angles. If the matrix Γ is a scalar under the colour group transformation: $g^{-1}\Gamma g = \Gamma$, we have the quark loop with the usual propagators

$$\langle j^\Gamma(x) j^\Gamma(y) \rangle = tr\, \Gamma G_0(x-y) \Gamma G_0(y-x), \qquad (51)$$

the imaginary part of which does not equal zero. If the matrix Γ is colour, we get an expression of the type of (45)-(47).

In the momentum space, in the approximation (48) the result (51) corresponds to the expression

$$\Pi(q) = \lim \frac{1}{L} \sum_{n=-L/2}^{L/2} \int d^4p\, tr\left[\Gamma G_0(p + tn) \Gamma G_0(q - (p + tn))\right] =$$

$$= \int d^4p\, tr\left[\Gamma G_0(p) \Gamma G_0(q-p)\right],$$

in which the total compensation of large vacuum moments occurs. A similar mechanism of the infrared confinement is described in ref. [19] (see also [20]).

After averaging over the infrared degeneration parameters all the Green functions which are not scalar under colour gauge transformations disappear. But the colourless Green functions of the type of correlators between electromagnetic and weak currents coincide with the usual QCD-perturbative Green functions. We get the colour confinement in a spirit of the naive parton model [3] with the zero norm of all physical coloured states [21].

Such topological confinement can be realized in any non-Abelian theory constructed on a semisimple group G if it contains the "minimal" subgroup SU(2). That means, the fundamental representation of G is an irreducible one of SU(2). For example, for SU(3) the generators of the minimal subgroup are the Gell-Mann matrices λ_2, λ_5, λ_7 which coincide with the vector SU(2) representation. The topological structures of the infrared vacuum for SU(N) and SU(2) groups coincide in accordance with the known formula for the homotopic group of mapping of the space $R(3)$ onto SU(N)

$$\pi_3(SU(N)) = \pi_3(SU(2)) = Z .$$

6. Conclusions

We have considered a possible mechanism of confinement realized in the framework of perturbation theory. This mechanism is based on the use i) of the physical infrared regularization by choosing a finite space and ii) of the local gauge-invariant variables which follow from the very dynamics and correspond to the choice of the covariant Coulomb gauge. (Schwinger[13] has insisted upon the application of that type of variables in the gauge theories). We have shown that in such a modified non-Abelian theory there is the topological vacuum degeneration and gauge phases of colour fields in the vacuum depend on infrared degeneration parameters. The confinement of colour fields is the result of the purely quantum interference of the vacuum phase factors.

Here we do not concern all other aspects and problems of perturbation theory, in particular, the asymptotical freedom which takes place in a deep-virtual regime of subgraphs of the Feynman diagrams, describing colourless processes in terms of the usual quark and gluon propagators.

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Азимов Р.И., Первушин В.Н. E2-85-203
Калибровочно-инвариантные переменные и инфракрасный конфайнмент в КХД

Рассматривается калибровочно-инвариантный подход к квантованию неабелевых теорий в конечном пространстве-времени. В такой теории имеет место топологическое вырождение вакуума, которое может быть физической причиной ненаблюдаемости свободных цветных частиц.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Azimov R.I., Pervushin V.N. E2-85-203
Gauge-Invariant Variables and Infrared Confinement in QCD

The gauge-invariant approach to quantization of the non-Abelian theory in a finite-volume space-time is considered. In this theory the topological vacuum degeneration takes place, and this degeneration may be a physical cause of non-observation of free colour particles.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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