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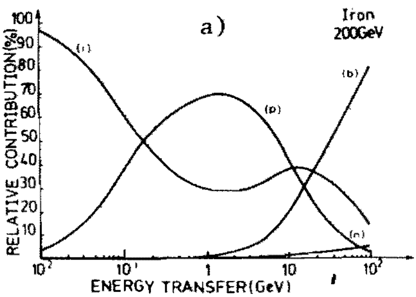
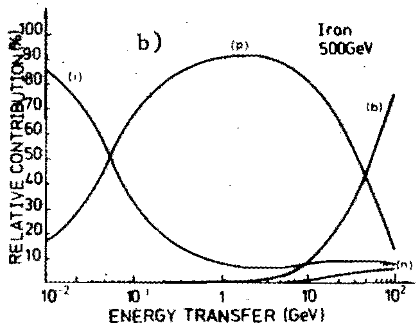
G.Todorova

MONTE-CARLO CALCULATIONS OF MUON ENERGY LOSSES DUE TO PAIR PRODUCTION

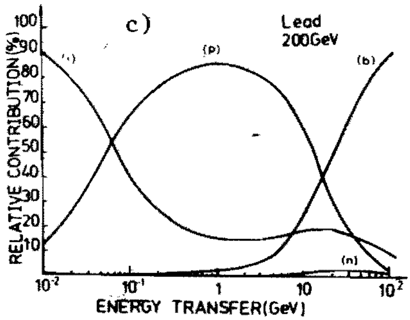
1985

When the high energy muons traverse the matter they loss energy because of ionization, bremsstrahlung, direct pair production, and nuclear interaction. The relative contributions of different processes to the total interaction probability for a given energy transfer v (GeV) are plotted in Figure a) for 200 GeV muons in iron. Figure a) shows that the pair production process dominates over the other ones for the energy transfer v from a few hundred MeV to 10 GeV and its contribution above 10 GeV continues to be important. This effect becomes more appreciable with increasing initial muon energy or atomic number of the matter (Figs. b),c)). To take into account any systematic effects due to a large energy loss in a single collision, the so-called "catastrophic" energy losses, the rigorous MC procedure of pair production losses is needed; this is the aim of the present paper.





Relative contribution in per cent of the ionization (i) [7], bremsstrahlung (b) [8], pair production (p) [1] and nuclear interaction (n) [9] to the total interaction probability for energy transfer (GeV): a) for 200 GeV muons in iron, b) for 500 GeV muons in iron, c) for 200 GeV muons in lead.



The Kokoulin and Petrukhin formulas [1] for pair production losses have been used as they are regarded to be the best approximation available now to the rigorous Kel'ner and Kotov calculations[2] (with the accuracy better than 2% in the relativistic region). The other approximation formulas[3] especially the often used Kobayakawa one[4] which is also presented in the CERN Yellow Report[5] underestimate the pair production losses up to 40%. The Kokoulin and Petrukhin formula[1] for the differential probability is written as follows:

$$\sigma(E,v,\rho)\,dv\,d\rho = \frac{2}{3\pi} Z(Z+1)\alpha^2 \frac{N}{A} r_e^2 \frac{1-v}{v}\left[\Phi_e + \frac{m^2}{\mu^2}\Phi_\mu\right] dv\,d\rho; \qquad (1)$$

$$\Phi_e = \left\{\left[(2+\rho^2)(1+\beta) + \zeta(3+\rho^2)\right]\ln\left(1+\frac{1}{\zeta}\right) + \frac{1-\rho^2-\beta}{1+\zeta} - (3+\rho^2)\right\} L_e;$$

$$\Phi_\mu = \left\{\left[(1+\rho^2)\left(1+\frac{3}{2}\beta\right) - \frac{1}{\zeta}(1+2\beta)(1-\rho^2)\right]\ln(1+\zeta) + \frac{\zeta(1-\rho^2-\beta)}{1+\zeta} + (1+2\beta)(1-\rho^2)\right\} L_\mu;$$

$$L_e = \ln \frac{189 Z^{-1/3}\sqrt{(1+\zeta)(1+J_e)}}{1 + \dfrac{2m\sqrt{e}\,189 Z^{-1/3}(1+\zeta)(1+J_e)}{Ev(1-\rho^2)}};$$

$$L_\mu = \ln \frac{189 Z^{-1/3}\dfrac{\mu}{m}\sqrt{(1+1/\zeta)(1+J_\mu)}}{1 + \dfrac{2m\sqrt{e}\,189 Z^{-1/3}(1+\zeta)(1+J_\mu)}{Ev(1-\rho^2)}};$$

$$J_e = \frac{5-\rho^2+4\beta(1+\rho^2)}{2(1+3\beta)\ln(3+1/\zeta) - \rho^2 - 2\beta(2-\rho^2)};$$

$$J_\mu = \frac{4+\rho^2+3\beta(1+\rho^2)}{(1+\rho^2)(3/2+2\beta)\ln(3+\zeta) + 1 - \frac{3}{2}\rho^2};$$

where $\beta = v^2/2(1-v)$ and $\zeta = \left(\frac{v\mu}{2m}\right)^2 \frac{1-\rho^2}{1-v}$, E is the primary muon energy, $v = \epsilon/E$ is the energy fraction transferred to the pair, $\rho = (\epsilon_+ - \epsilon_-)/\epsilon$ is the asymmetry coefficient of the energy distribution of the pair, μ, m are the muon and electron masses, respectively, Z, A are the atomic number and atomic weight of the target material, r_e is the classical electron radius, $\alpha = 1/137$ is the fine structure constant, N is Avogadro's number.

Expression (1) is valid for $E \geq 5$ GeV and

$$4m/E = v_{min} \leq v \leq v_{max} = 1 - \mu/E$$

$$0 \leq |\rho| \leq \rho_{max} = \sqrt{1 - \frac{4m}{Ev}}\left[1 - \frac{6\mu^2}{E^2(1-v)}\right].$$

There are three standard methods to generate random numbers according to the one-dimensional distribution function $f(x)$ $x_{low} \leq x \leq x_{high}$. The first, when the cumulative distribution function $\Phi(x) = \int_{x_{low}}^{x} f(y)\,dy \Big/ \int_{x_{low}}^{x_{high}} f(y)\,dy$ can be analytically inverted. The second is the so-called percentile method which consists in numerical inversion of the cumulative distribution function $\Phi(x)$ in 100 "percentile" points and use of a reasonable interpolation. (The CERN library program "FUNRAN" uses this method).

The third method is several variations of the so-called "rejection" method. For pair production energy losses none of the methods can be used. The cumulative distribution function

$$\Phi(x) = 2\int_{v_{min}}^{x}\int_{0}^{\rho_{max}} \sigma(E,v,\rho)\,d\rho\,dv \Big/ 2\int_{v_{min}}^{v_{max}}\int_{0}^{\rho_{max}} \sigma(E,v,\rho)\,d\rho\,dv$$

cannot be inverted analytically. The "rejection" method cannot work effectively because the r.h.s. of (1) is very cumbersome. Due to a rapidly decreasing cross section (about 7 orders) the second "FUNRAN" method works with errors from a few tens to a few hundreds per cent (for losses above 4.5%). The proposed Monte-Carlo method works accurately in all energy loss interval $[v_{min}, v_{max}]$. To make MC procedure fast enough the losses below some minimum energy loss - v_{cut} were treated as "restricted" average energy losses[6]

$$\frac{dE}{dx} = 2E\int_{v_{min}}^{v_{cut}} v \int_{0}^{\rho_{max}} \sigma(E,v,\rho)\,d\rho\,dv$$

(A reasonable choice for v_{cut} is 0.005). The tail above v_{cut}, where the so-called "catastrophic" losses appear, was treated by the following MC procedure. The cumulative distribution pair production function

$$\Phi(x) = \int_{v_{cut}}^{x}\int_{0}^{\rho_{max}} \sigma(E,v,\rho)\,d\rho\,dv \Big/ \int_{v_{cut}}^{v_{max}}\int_{0}^{\rho_{max}} \sigma(E,v,\rho)\,d\rho\,dv$$

in the region $v_{cut} - v_{max}$ was approximated by the sum $G(x)$ of Chebyshev polynomials

$$G(x) = \sum_{i=0}^{n} a_i T_i(x),$$

where $T_i(x) = \cos(i \arccos(x))$.

To achieve a better approximation accuracy with reasonably small number of coefficients, the region $v_{cut}-v_{max}$ was divided into eight subintervals (0.005, 0.01, 0.021, 0.045, 0.09, 0.19, 0.36, 0.64, v_{max}). For each subinterval the cumulative pair production function $\Phi(x)$ was calculated at the points corresponding to the zeros of the n-th Chebyshev polynomial $a_j = \cos((2j-1)\pi/2n)$; $j = 1,\ldots, n$ using the CERN library program RGAUSS [10]. It was found that the sum of Chebyshev polynimials up to sixth order (n = 6) achieves a fitting precision better than 10^{-4} of the pair production cumulative function $\Phi(x)$.

The coefficients a_i were obtained by using the orthogonality relationship

$$\sum_{k=1}^{n} T_i(a_k)\,T_j(a_k) = \begin{cases} \delta_{ij}\,\dfrac{n}{2} \\ n\,;\ i = j = 0 \end{cases} .$$

The equation

$$\Phi(x) = \sum_{i=0}^{6} a_i T_i(x) = z$$

was solved for Chebyshev reference points z_i ; $0 \le z_i \le 1$ using the CERN library program RZERO [11].

The above method for Chebyshev approximation gives the analytical expression for the inverse function $\Psi(z)$ of the cumulative pair production function $\Phi(x)$ as

$$\Psi(z) = \sum_{i=0}^{m} b_i T_i(z).$$

For m = 6 and 7 subintervals (0, 0.59, 0.937, 0.994, 0.999, 0.9998, 0.99998, 1) the fitting precision is better than 0.5%.

The inverse function $\Psi(z)$ of the cumulative distribution pair production function $\Phi(x)$ depends weakly on the initial muon energy. If the muon energy interval 50-280 GeV is desired to be covered, two subintervals, 50-120 GeV and 120-280 GeV, are enough for achieving good accuracy. In each of them the inverse function $\Psi_1(z)$ and $\Psi_2(z)$ corresponding to 80 and 200 GeV, respectively, can be used for MC calculations with accuracy better than 1%.

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Тодорова Г. E2-85-167

Монте-Карло вычисления потери энергии высокоэнергетических мюонов в процессе образования электрон-позитронных пар

Для потери энергии высокоэнергетических мюонов в процессе образования электрон-позитронных пар стандартные Монте-Карло методы не могут быть использованы для моделирования с хорошей точностью так называемых "катастрофических" потерь. Точный метод для Монте-Карло моделирования потери энергии при образовании пар представлен в работе.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Todorova G. E2-85-167

Monte-Carlo Calculations of Muon Energy Losses Due to Pair Production

For pair production energy losses of high energy muons none of the standard methods for simulation can be used to generate accurately the so-called "catastrophic" losses. The accurate and fast method for full Monte-Carlo generation of pair production energy losses is presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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