сообщения
объединенного
института
ядерных
исследований
дубна

E2-85-104

V.P.Gerdt, A.S.Ilchev*, V.K.Mitrjushkin,
A.M.Zadorozhny

ON THE PHASE STRUCTURE OF LATTICE SU(2) GAUGE-HIGGS THEORY

* Sofia University, Department of Physics

1985

INTRODUCTION

The formulation of gauge-theories on an (Euclidean) space-time lattice [1] opens the door to powerful analytic and numerical methods for studying such theories beyond the framework of perturbation theory. A straightforward argument based on the renormalization group points to the fact that a continuum limit can be reached at a critical point where the system undergoes a second order phase transition characterized by an infinite correlation length. Therefore, given a lattice formulation of some field theory, it is important to understand its phase structure. Gauge-Higgs models on a lattice have recently been the object of considerable interest because they have been shown to possess a rich phase structure, especially when the size of the Higgs field (the Higgs radial model) is allowed to vary [2-5]. The SU(2) gauge theory coupled to radially active scalar (Higgs) fields in the fundamental representation of the gauge group is the simplest non-Abelian model of the above type. Recently Kuhnelt, Lang and Vones [5] have published the results of their Monte-Carlo study of this model. We have been investigating the same model [3] along with the Abelian case [4] and have observed a much reacher phase structure than the one reported by the authors of [5]. The apparent disagreement of the two sets of results concerns even the type of phase transitions: we have observed first order phase transitions and second order phase transitions as end-points of lines of first order phase transitions, whereas Kuhnelt et al. have seen only second order phase transitions. This situation calls for a more detailed analysis of the results as well as of the methods of investigating the phase structure of models with many parameters. This is precisely what we intend to do on the present paper which is organized as follows: first we recall the definition of the model and give the conversion formulae between our parametrization and that of Kuhnelt et al., next we discuss the problems connected with describing the phase structure of models with many parameters (of which our is an example); then we summarize our Monte-Carlo results and support them with the analysis of the approximate effective potential, further on we comment on the results of Kuhnelt et al. and make conclusions.

1. THE MODEL AND CONVERSION FORMULAE

The continuum Euclidean action of a SU(2) gauge-Higgs theory with fundamental representation scalars

$$S_{cont.} = \frac{1}{4} \int d^4x\,(F_{\mu\nu}(x))^2 + \frac{1}{2} \int d^4x\,|(\partial_\mu + igA_\mu(x))\Phi(x)|^2 + \tag{1.1}$$

$$+ \int d^4x\,[\frac{m^2}{2}\Phi^*\Phi + \lambda(\Phi^*\Phi)^2] ,$$

can be written in various forms on a hypercubic lattice, one of which is

$$S_{Latt} = \beta \sum_{\square} S_\square + \sum_{\ell} S_\ell , \tag{1.2}$$

where the plaquette part of the action is

$$S_\square = 1 - \frac{1}{2}\,\mathrm{Tr}\,U_\square , \qquad U_\square = U_{ij}\,U_{jk}\,U_{k\ell}\,U_{\ell i} , \qquad U_{ij} = U_{ji}^{-1} = U_\ell \in SU(2), \tag{1.3}$$

and the link part of the action is

$$S_\ell \equiv S_{ij} = \frac{1}{4}(\frac{m^2}{2}R_i^2 + \lambda R_i^4) + R_i^2 - R_i R_j \frac{1}{2}\,\mathrm{Tr}\,(\phi_i U_{ij}\,\phi_j^*) , \tag{1.4}$$

where $0 \leq R_i < \infty$ is the radial mode of the Higgs field at site i and $\phi_i \in SU(2)$ is its angular part. The model (1.2) has bare (cutoff dependent) parameters $\beta \equiv \frac{4}{g^2(a)}$, $m^2 \equiv m^2(a)$ and $\lambda \equiv \lambda(a)$; where a is the lattice spacing.

Kuhnelt et al. have chosen to rewrite (1.4) in a somewhat different form in order to simplify the study of various extreme cases and in particular to the $R_i = 1$ (frosen to unity radial degree of freedom) case. Their parameterization is

$$S_\ell' = \frac{1}{4}[\lambda_L(\rho_i^2 - 1)^2 + \rho_i^2] - \kappa\rho_i\rho_j\,\mathrm{Tr}\,\phi_i U_{ij}\,\phi_j^* . \tag{1.5}$$

Now if we rescale $\sqrt{2\kappa}\,\rho_i \equiv R_i$ and compare (1.5) with (1.4) we shall get the following conversion formulae:

$$\lambda = \lambda_L / 4\kappa^2 , \qquad m^2 = (1 - 2\lambda_L - 8\kappa)/\kappa . \tag{1.6}$$

2. THE PHASE STRUCTURE OF MODELS WITH MANY PARAMETERS

When a lattice system depends on many parameters, the analysis of its structure and the representation of the results can be a very difficult task. Even for a modest number of parameters (three in our case) the problem is quite formidable: the phase transitions occur on surfaces which divide the different phases. The boundaries of these surfaces can themselves be of (higher order) phase-transition points. In addition the order parameters of the theory under study display what is called cross-over behaviour near such boundaries and this can sometimes easily be mistaken for a phase transition. Normally it requires certain effort in order to localize a single phase transition point and to determine its type. That is why the phase structure of models with many parameters is usually described by means of sets of two-dimensional phase diagrams (i.e., all but two of the parameters are kept fixed). It is essential to remember, though that such a description is incomplete and that, in principle, it is possible to overlook a portion of the parameter space where the model possesses nontrivial phase structure. Further on in this paper we shall argue that this was precisely the case with ref. /5/, but first let us briefly review our results concerning the phase diagrams of the model (1.3-4).

3. PHASE DIAGRAMS OF THE MODEL IN THE (β, m^2, λ) SPACE

We have considered (β, m^2)-planes for fixed, not very large values of λ ($\lambda \leq 0.7$). For small values of β ($|\beta| \leq 0.3$) we have employed an approximate effective potential

$$V_{eff}(x;\beta;m^2;\lambda) = V^{(0)}(x;m^2;\lambda) + \beta V^{(1)}(x) + \beta^2 V^{(2)}(x) + O(\beta^3) , \tag{3.1}$$

where

$$V^{(0)} = (1 + \frac{m^2}{8})x + \frac{\lambda}{4}x^2 - \ln[\frac{2I_1(x)}{x}] - \frac{3}{8}\ln x , \qquad V^{(1)} = \frac{3}{2}[1 - \frac{I_2^4(x)}{I_1^4(x)}] ,$$

$$V^{(2)} = -\frac{3}{2}\{\frac{21}{2}[1 - \frac{I_2^8(x)}{I_1^8(x)}] - 36\frac{I_2(x)}{xI_1(x)} - 36[\frac{I_2(x)}{xI_1(x)}]^2 - \tag{3.2}$$

$$- 96[\frac{I_2(x)}{xI_1(x)}]^3 + 96[\frac{I_2(x)}{xI_1(x)}]^4\} .$$

The value of the order parameter $\bar{R}^2$ for the current β, λ, and m^2 is determined at the minimum of the effective potential: $\bar{R}^2 \equiv x_{min}$. If V_{eff} happens to develope a second local minimum, it

is convenient to interpret it as a metastable phase. For certain values of λ ($\lambda \leq 0.1$) and $|\beta| \leq 0.3$ we observe the formation of a second minimum of the effective potential as $|m^2|$ increases and at some point $m^2 = m_c^2(\beta;\lambda)$ the two minima of the effective potential become equal. This is the phase transition point for these values of β and λ. Carrying the system across such a point we observe that the order parameter is discontinued and hence this is a first order phase transition. The values of $m_c^2(\beta;\lambda)$ obtained in this way have been conformed by independent Monte-Carlo calculations. Again by Monte-Carlo calculations we have confirmed that the phase transitions are of the first order.
A more detailed analysis of the phase transition of the model (1.2-3) is to be found in [3]. Here we would like to underline the fact that the above-mentioned first order phase transition lines have been shown to terminate at end-points which themselves are points of second order phase transition [3,4]. Our finding are summarized on fig.1.

We observe that for increasing λ the phase transition lines are moving upward and the end-points to the right. We expect a linear asymptotic of the end points (second-order phase transitions) which will eventually reach the critical point of the SU(2) Ising model [5] :

$$\beta = t,\quad \lambda = t/4\kappa_0^2,\quad m^2 = (1 - t/2\kappa_0^2 - 8\kappa_0)/\kappa_0,\quad \kappa_0 = 0.355 \pm 0.001,\quad t \to \infty. \quad (3.3)$$

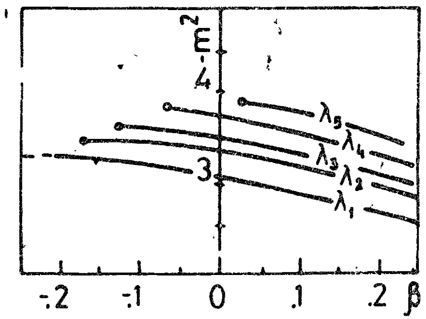


Fig. 1

Above and to the left of this line we enter into a portion of the parameters space which is more or less trivial as far as phase transitions are our concern. The order parameter $\bar{R}^2$ can show a crossover behaviour in that region, though - espectially near the line of end points (see the discussion of the effective potential in [3] and [4]).

4. DISCUSSION

Now we have come to the discrepancy between our results and those of Kuhnelt et al. Since the question of the order of the phase transitions is of paramount importance, the authors of [5] have given it due attention and have concluded that all the transitions are of the second order. In support of this Kuhnelt et al. supply their Monte-Carlo simulations at $\lambda_L = 0.5$. By looking at the conversion formulae (1.6) we observe that this means a fixed value of $m^2 = -8$ and thermal cycles in k between zero and 0.8 (0.6) at fixed β. This translates in terms of (β, m^2, λ) into thermal cycles in λ between infinity and 0.15 (0.21) for fixed β and $m^2 = -8$. For small values of β these thermal cycles remain entirely in the "trivial" part of the parameter space and with increasing β they approach the cross over band and eventually the surface of first order phase transitions, but this happens for rather large values of λ for which it is very difficult to observe hysteresis loops in the thermal cycles. The decisive test performed by Kuhnelt et al., though, is the observation of the relaxation behaviour of the order parameter for simulations from different initial configurations. They choose the point $\lambda_L = 0.5$, $\beta = 2.25$, and $\kappa = 0.27$ which, according to (1.6), translates into $\lambda = 1{,}715$, $\beta = 2.25$, and $m^2 = -8$. Notice a rather large value of λ. We have observed a rapid shrinking of the hysteresis loop at $\beta \simeq 2.2$ even for $\lambda = 0.7$:

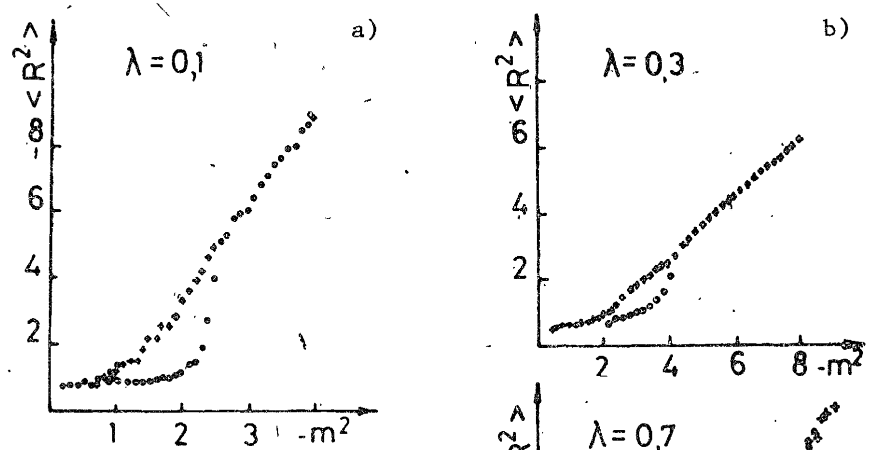
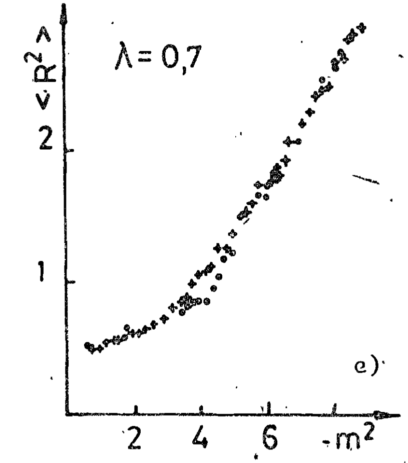


Fig. 2



Thermal cycles in m^2 at $\beta = 2.2$ and for different values of λ.

We conclude that the point $\lambda = 1.715$, $\beta = 2.25$, $m^2 = -8$ is either in the "trivial" region or the two minima of the true effective potential are very close to be distinguishable by Monte-Carlo methods. As a comparison, we draw ones attention to the region of much smaller λ and β, where we have observed a clear signal of a first order phase transition both from our Monte-Carlo calculations and from the effective potential in the approximation (3.1-2). Here is a typical example: $\lambda = 0.05$, $\beta = 0$, $m_c^2 = -2.86$. According to the conversion formulae (1.6) this corresponds to: ($\lambda_L = 0.0073$, $\beta = 0$, $\kappa = 0.1916$) or ($\lambda_L = 34.017$, $\beta = 0$, $\kappa = -13.04$) - both are far away from the point at which the type of the phase transition has been determined in [5].

5. CONCLUSION

The investigations of various lattice gauge-scalar models have revealed their rich phase structure whose adequate description, though, is not always straightforward. This also applies to the simplest non-Abelian gauge-Higgs model - the one with a SU(2) -symmetry. We have shown that a reparametrization of the model, while making it easier to see the connection with certain extreme cases, may also obscure the phase structure of the theory. As for the second order phase transition of the SU(2) - Ising model, we argue that it is the limit of the critical points at the end of the first order phase transition lines [$m_c^2(\lambda)$, $\beta_c(\lambda)$] for $\lambda \to \infty$.

REFERENCES

1. Wilson K.G. Phys.Rev., 1974, D10, p. 2445.
2. Munehisa T., Munehisa Y. Phys.Lett., 1982, 116B, p. 353; Nucl.Phys., 1983, B125, FS7, p. 508; Koutsoumbas G. Phys. Lett., 1984, 140B, p. 379; Munehisa Y. Phys.Rev., 1984, D20, p. 1310; Yamanashi Univ. Preprint 84-02, 1984.
3. Gerdt V.P., Ilchev A.S., Mitrjushkin V.K., Sobolev I.K., Zadorozhny A.M. JINR, E2-84-313, Dubna, 1984; Gerdt V.P., Ilchev A.S., Mitrjushkin V.K., Zadorozhny A.M. JINR, E2-85-103, Dubna, 1985.
4. Gerdt V.P., Ilchev A.S., Mitrjushkin V.K. JINR, E2-83-758, Dubna, 1983; Gerdt V.P., Ilchev A.S., Mitrjushkin V.K. Yad.Fiz., 1984, 40, p. 1097.
5. Kuhnelt H., Lang C.B., Vones G. Nucl.Phys., 1984, B230, FS10, p. 16.

Received by Publishing Department
on February 14, 1985.

SUBJECT CATEGORIES OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Гердт В.П. и др. E2-85-104

О фазовой структуре SU(2) -Хиггс-калибровочной теории на решетке

В настоящей работе мы обсудим наши результаты по исследованию фазовой структуры SU(2) калибровочной теории поля, связанной с хиггсовскими полями, радиальная мода которых "разморожена". Показано, что наши результаты не противоречат результатам работы [5]. Мы наблюдаем фазовые переходы первого рода, что подтверждается как вычислениями методом Монте-Карло, так и исследованием на основе приближенного построения эффективного потенциала.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Gerdt V.P. et al. E2-85-104

On the Phase Structure of Lattice SU(2) Gauge-Higgs Theory

In this paper we discuss our results on the phase structure of SU(2) gauge theory coupled to radially active Higgs fields. It is shown that our results are not in contradiction with those in [5]. The first order phase transitions observed by us are confirmed both by Monte-Carlo calculations and from the analysis of an approximate effective potential.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985