ОБЪЕДИНЕННЫЙ
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# HARMONIC ANALYSIS ON THE LORENTZ GROUP, QUASIPOTENTIAL APPROACH AND PROTON-PROTON ELASTIC SCATTERING AT HIGH ENERGIES

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ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

Let us consider a two particle system. The nonrelativistic covariance (the Galilei principle of relativity) means that in the space of the wave functions of the system a unitary representation of the Galilei group is realized.

We shall give a group theoretical interpretation of certain properties of the non-relativistic wave functions admitting a relativistic analogy. From our point of view this analogy is essential when studying relativistic quasipotential amplitudes and wave functions. Because of the complete exploitation of the kinematics an essential simplification of formulae and possibly their interpretation can be obtained.

The reduction of the representation of the translation group /1/ leads to the usual harmonic (Fourier) analysis with four-dimensional plane-waves. So this reduction stands for group theoretical definition of the momentum representation if the coordinate representation is given.

The wave function of the system when a separation of the variables into center-of-mass and relative ones is performed, is factorized into an exponential and an effective wave function describing the relative motion. The effective wave function is transformed according to the unit representation of the Galilei group (up to rotations).

The group of the boosts of the relative momenta is isomorphic to the Galilei boost-group, but is not an invariance group of the Schrödinger effective-motion equation. The reduction of its unitary representation to irreducible ones leads to Fourier-analysis with three-dimensional plane waves and stands for a group-theoretical definition of the relative coordinate.

In the relativistic case the effective wave function cannot be defined ambiguously. The development of the quasipotential approach /2-7/ * has shown that this is

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* In papers /5,6/ one can find a full list of references of the considered subject.

Мавродиев С.Ш., Караджов Д. E2 - 8488

Гармонический анализ на группе Лоренца, квазипотенциальный подход и протон-протонное упругое рассеяние при высоких энергиях

Проведен численный анализ, показывающий экспоненциально-степенное поведение дифференциальных сечений упругого протон-протонного рассеяния. Амплитуда рассеяния получена в борновском приближении в предположении существования простого квазипотенциала в релятивистском относительном координатном пространстве.

Препринт Объединенного института ядерных исследований.
Дубна, 1974

Mavrodiev S.Cht., Karadjov D. E2 - 8488

Harmonic Analysis on the Lorentz Group, Quasipotential Approach and Proton-Proton Elastic Scattering at High Energies

The exponential-power behaviour of the proton-proton elastic differential cross section is proved by numerical analysis. The scattering amplitude in Born approximation is obtained under the assumption for a simple quasipotential to exist in relativistic relative coordinate space.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1974

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connected with the nonuniqueness of the off-mass-shell continuation.

From the view point of the analogy between nonrelativistic and relativistic cases it is natural to define the effective wave-function in Lobachevsky space. This, for example, is realized in the Kadyshevsky's variant of the quasipotential approach /3,4,7/ . Since the motion group of the Lobachevsky space is the Lorentz group, in the space of the effective wave functions one can define its unitary representation. The reduction of this representation to irreducible ones leads to a 'relativistic'' Fourier analysis and therefore determines the relativistic relative coordinate /7/.

In paper /8/ this harmonic analysis was studied and some physical consequances as mentioned below were derived.

For example, in this apparatus the rapidity

$$\chi = \frac{1}{2} \ell n \frac{E-P}{E+P}$$

replaces naturally the momentum.

Assuming that the elastic scattering amplitude can be calculated in Born approximation a formula suitable for the phenomenological description of the scattering processes was obtained. Generally speaking, in order to obtain such type of formulae it is enough in the Quantum Mechanical Born approximation formula to replace the transferred momentum by transferred rapidity.

It was shown, under certain assumptions about the analytic properties of the quasipotential in the relativistic $r$ -plane, that, when $t \ll 1$ ( $t$ is the squared transferred momenta) *, the amplitude has an exponential behaviour, while at $t \gtrsim 1$ it has a power behaviour. This result performs a qualitative description of the experimental data /9,10/ on elastic proton-proton scattering at high energies.

In particular the quasipotential of the proton-proton interaction was chosen to be of the type

$$V(r,s) = \frac{\lambda(s)}{R^2(s) + r^2} ,$$

where $s$ is the square of the invariant energy, $\lambda(s)$ is the ''complex'' interaction constant, $R(s)$ is the interaction ''radius''. In Born approximation this led (in terms of the ''relativistic'' plave waves) to the following expression for the amplitude

$$T(s,t) = \frac{2\pi^2}{\sqrt{-t(1-t/4)}} \frac{\lambda(s)}{(1-t/2+\sqrt{-t(1-t/4)})^{R(s)}} . \quad (1)$$

We would like to stress that in terms of transferred rapidity

$$\chi_t = \ell n \, (1 - t/2 + \sqrt{-t(1-t/4)})$$

the formula (1) has an extreme simplicity

$$T(s, \chi_t) = 2\pi^2 \lambda(s) \frac{e^{-R(s)\chi_t}}{sh \, \chi_t} . \quad (2)$$

The normalization of the amplitude (1) is chosen in such a manner that the differential cross section is

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{16\pi \, s(s-4)} |T(s,t)|^2 . \quad (3)$$

Later on we can use formulae (1) and (3) for the description of the proton-proton elastic scattering experiments. As long as we could not find published full experimental data in a table form, we used the figure N. 13 from paper /10/ to extract the data for $d\sigma/dt(s,t)$.

The data thus obtained were used to solve an overdetermined nonlinear system of equations

$$\frac{d\sigma}{dt}(s,t) - \frac{d\sigma^{exp\,t}}{dt}(s,t) = 0 \quad (4)$$

for $s = 8, 12, 16, 25, 50, 2000$ ($GeV^2$), and $.2 \leq -t \leq 7$. ($GeV^2$), while $d\sigma/dt(s,t)$ is determined from formula (3). The preliminary numerical analysis led to a parametrization of the unknown functions $\lambda(s)$ and $R(s)$ in the following form

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*We are working in the atomic system of units $m = h = c = 1$.

$$\lambda(s) = A_1 + is\, A_2 (\ell n\, s)^{A_3} ,$$

$$R(s) = A_4 (\ell n\, s)^{A_5}.$$

System (4) has been solved by the least squares method both for all the single curves (with respect to s ) and for the case of simultaneous description of all curves. The numerical analysis was held on by the method of the regularized iterational procedures of the Gauss-Newton type /11/. This method performs low dependence on the initial approximations and stability with respect to the fluctuations of the nonlinear operator. A standard program COMPIL /12/ (JINR standard program library) was used. The values of the parameters $A_1, A_2, A_5$ turned out to be stable for all energies under consideration $A_1 = -1.25$, $A_2 = 0.87$, $A_5 = 0.82$.

This gave a possibility of writing the interaction "constant" and "radius" in the form

$$\lambda(s) = -1.25 + is\, 0.87 (\ell n\, s)^{a}$$

$$R(s) = R\, (\ell n\, s)^{0.82} ,$$

where $a = A_3$ and $R = A_4$.

In table 1 are given the solutions for the parameters $a$ and $R_0$.

Table 1

| s | 8 | 12 | 16 | 26 | 50 | 3000 | all curves |
|---|---|---|---|---|---|---|---|
| $a$ | 1.34 | 1.11 | 1.28 | .98 | 1.22 | 82 | 97 |
| $R_0$ | .54 | .67 | .88 | .94 | 1.09 | .77 | .86 |

It can be seen from the table, that the maximal deviations from the mean value of the parameters $a$ and $R_0$ (for all curves) do not exceed 35%, which is within the range of the errors of these parameters *.

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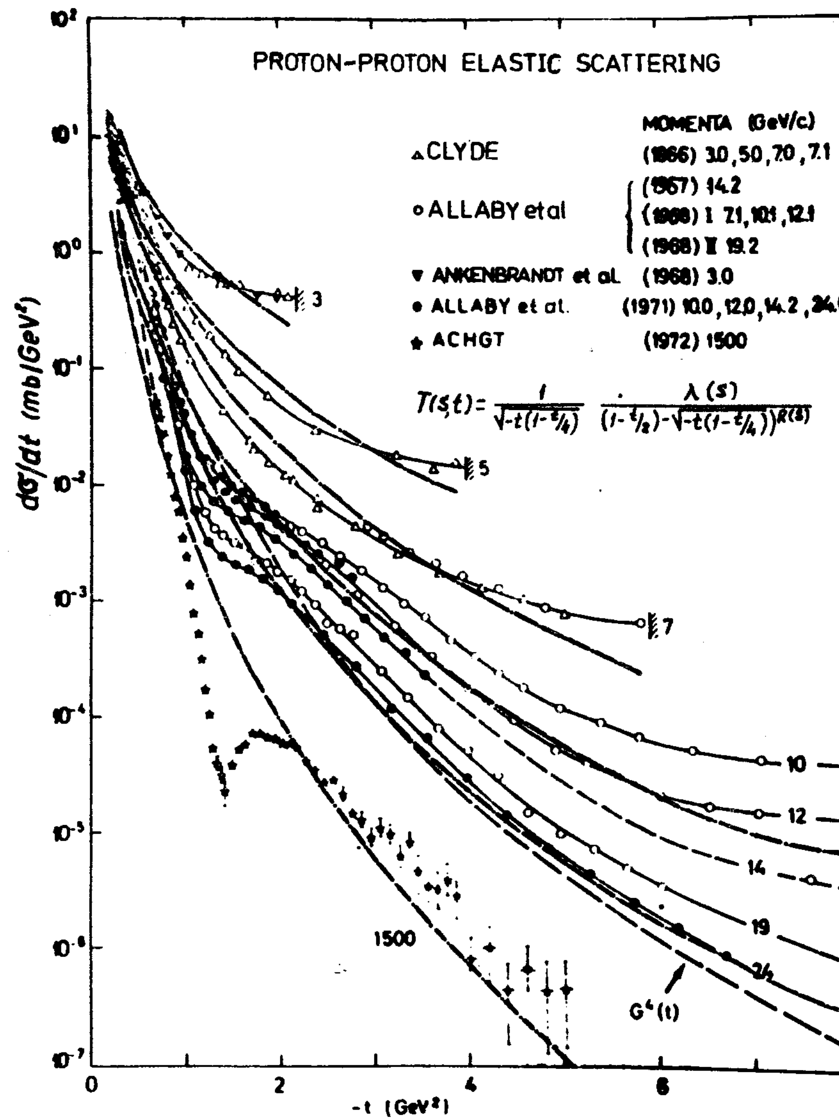
* Due to the unreliability of the available experimental data and the lack of the values of the input data errors, we do not discuss the uniqueness of the obtained solutions as well as the determination of their statistical errors.



The figure gives a comparison of the experimental and theoretical curves which are obtained with the solutions of the parameters obtained above. (The dashed lines are the theoretical ones).



It becomes clear from the figure that the chosen potential performs a description of the exponential power behaviour of the differential cross section in the given interval of squared four-momentum transfer.

The comparison of theoretical and experimental curves at different energies shows that there is a better agreement with the decreasing of the ratio $t_{max}/s$. Therefore we can consider the relativistic analogue of the condition of applicability of the Born approximation to be the condition $t_{max}/s \ll 1$. In table 2 are given the values of the interaction "radius" (which is an analogue of the diffraction peak slope-parameter) at different energies.

Table 2

| s | 8 | 12 | 16 | 26 | 50 | 3000 |
|---|---|---|---|---|---|---|
| $R(s)=R_0(\ell n\ s)^{\alpha}$ | 0.98 | 1.40 | 2.03 | 2.47 | 3.32 | 4.33 |

The description of the detailed structure of the experimental curves in our approach can be obtained by using quasipotentials with pole singularity on the real axes in the r -space. For example, a quasipotential

$$V(r,s) = \frac{\lambda(s)}{R^2(s)+r^2} + \frac{\lambda_1(s)}{R_1^2(s)-r^2}$$

gives the following formula for the amplitude

$$T(s,t) = \frac{2\pi^2}{\sqrt{-t(1-t/4}} \left[ \frac{\lambda(s)}{F(t)^{R(s)}} - \lambda_1(s)\cos(\ell n\,(F(t)_1^{R(s)})\right], \quad (5)$$

where $F(t) = 1 - t/2 + \sqrt{-t(1-r/4)}$ .

In terms of rapidity

$$\chi_t = \ell n \quad F(t)$$

one has

$$T(s,\chi_t) = \frac{2\pi}{sh\ \chi_t}\left[\lambda(s)e^{-R(s)\chi_t} - \lambda_1(s)\cos(R_1(s)\chi_t)\right].$$



We hope to compare formula (5) with experiment in a subsequent paper.

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