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RENORMALIZATION OF SUPERSYMMETRIC GAUGE THEORIES

II. NON-ABELIAN CASE

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RENORMALIZATION OF SUPERSYMMETRIC GAUGE THEORIES
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Перенормировка суперсимметричных калибровочных теорий.
II. Неабелев случай

Для неабелевых калибровочных теорий сформулирована явным образом суперсимметричная процедура перенормировки. Показано, что все ультрафиолетовые расходимости устраняются общей перенормировкой волновой функции и заряда.

Препринт Объединенного института ядерных исследований.
Дубна, 1974

Slavnov A.A.

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Renormalization of Supersymmetric Gauge Theories.
II. Non-Abelian Case

A manifestly invariant renormalization procedure is formulated for the supersymmetric non-abelian gauge theories. All ultraviolet infinities are shown to be eliminated by a common wave function and charge renormalization.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1974

In a recent paper [1] we have suggested a manifestly invariant renormalization procedure for sypersymmetric (ss) quantum electrodynamics. All ultraviolet infinities were shown to be eliminated by a common wave function and mass renormalization of matter fields and wave function renormalization of a gauge multiplet. Now we extend this procedure to the s s non-abelian gauge theories [2, 3]. As in the previous case, we work in a manifestly s s gauge and show that in spite of the essentially nonlinear character of the Lagrangian only the finite number of s s counterterms is needed.

1. Supersymmetric Feynman Rules

In this section we briefly describe the s s generalization of Yang Mills theory following the paper [2] and derive s s Feynman rules. The matter fields are combined in chiral supermultiplets Φ_+, Φ_-. The Yang-Mills field A_μ is included in a supermultiplet described by the pseudoscalar hermitian matrix superfield $\Psi(x,\theta)$

$$\Psi(x,\theta) = A(x) + \bar\theta\gamma_5\chi(x) + \frac{1}{4}\{\bar\theta\theta F(x) + \bar\theta\gamma_5\theta G(x) + \bar\theta i\gamma_\nu\gamma_5\theta A_\nu(x)\} + \frac{1}{4}\bar\theta\theta\bar\theta\gamma_5\lambda(x) + \frac{1}{32}(\bar\theta\theta)^2 D(x) . \tag{1}$$

The generalized gauge transformation is

$$\Phi_\pm(x,\theta)\to\Omega_\mp(x,\theta)\Phi_\pm(x,\theta); \quad \exp\{g\Psi\}\to\Omega_-\exp\{g\Psi\}\Omega_+^{-1}, \tag{2}$$

$\Omega_\pm$ are chiral superfields

$$\det\Omega_\pm = 1 , \quad \Omega_+^* = \Omega_-^{-1}.$$

The infinitesimal transformation (2) can be written explicitly as

$$\Psi^a \to \Psi^a - \epsilon^{abc}\,\mathrm{Re}\,\delta\Omega_+^b\,\Psi^c + \mathrm{Im}\,\delta\Omega_+^a\,|\Psi|\,\mathrm{cth}\,g|\Psi| +$$

$$+\Psi^a|\Psi|^{-1}(\mathrm{Re}\,\delta\Omega_+^b\,\Psi^b)(g^{-1}|\Psi|^{-1} - \mathrm{cth}\,g|\Psi|)\,;\quad \delta\Omega = i/2\tau^a\,\delta\Omega^a\,. \qquad (3)$$

$$\delta\Omega_+^a = \exp\{-\frac{1}{4}\theta\,\hat{\partial}\,\gamma_5\,\theta\}[(u^a_{+1} + i\,u^a_{+4}) + \bar\theta(u^a_{+2} + i u^a_{+5}) + \frac{1}{4}\bar\theta(1+i\gamma_5)\,\theta(u_{+3} + i u_{+6})],$$

the components u^a_{+i} are hermitian.

The Lagrangian can be written in terms of a non-linear vector superfield V_μ

$$V_\mu = -g^{-1}[C^{-1}\gamma_\mu \frac{1+i\gamma_5}{2}]^{\alpha\beta} D_\alpha [\exp\{-g\Psi\} D_\beta \exp\{g\Psi\}]\,. \qquad (4)$$

C is a charge conjugation matrix and D_α is a covariant derivative

$$D_\alpha = \partial/\partial\bar\theta_\alpha - i/2\,(\gamma_\mu\theta)_\alpha\,\partial/\partial x^\mu\,.$$

Under the transformation (1)

$$V_\mu \to \Omega_+ V_\mu \Omega_+^{-1} + 2i\,g^{-1}\,\Omega_+\,\partial_\mu\,\Omega_+^{-1}\,,$$

and a gauge invariant Lagrangian looks as follows

$$\mathfrak{L} = \frac{1}{128}(\bar D D)^2\,\mathrm{Tr}(V_\mu V_\mu + V^*_\mu V^*_\mu) + \frac{1}{2}M(\bar D D)[\Phi^*_-\Phi^*_+ + \Phi^*_+\Phi^*_-] +$$

$$+\frac{1}{8}(\bar D D)^2\,[\Phi^*_+\exp\{g\Psi\}\Phi_+ + \Phi^*_-\exp\{-g\Psi\}\Phi_-]\,. \qquad (5)$$

This Lagrangian is degenerate, and a subsidiary condition should be imposed to quantize it. It was pointed out in the papers [2, 3] that one can choose a gauge

$$A = \chi = F = G = 0\,. \qquad (6)$$

In this gauge the Lagrangian (5) assumes a polynomial form and is nothing but the usual Yang-Mills Lagrangian plus some additional renormalizable interaction of spinor and scalar particles. (Its explicit form may be found in the papers [2, 3]). Unfortunately the gauge condition (6) destroys explicit supersymmetry of the theory and it is by no means self-evident that renormalization can be performed in a supersymmetric way, in particular, that the renormalized coupling constants and masses are equal.

For this reason we abandon noninvariant condition (6) and use the manifestly s s gauge.

We start from the gauge (6) in which the theory may be quantized in a standard way [4, 5] and the Green function generating functional can be written as follows

$$Z = N^{-1}\int \exp\{i\int \mathfrak{L}(x)\,dx + \mathrm{s.t.}\}\prod_x \delta(A)\,\delta(\chi)\,\delta(F)\,\delta(G)\,\delta(\partial_\mu A_\mu)\,\Delta_{F-P}\,d\mu\,. \qquad (7)$$

Here Δ_{F-P} is the Faddeev-Popov determinant, the measure $d\mu$ is the product of field differentials and s.t. means source terms.

Note that

$$\prod_x \delta(A)\,\delta(\chi)\,\delta(F)\,\delta(G)\,\delta(\partial_\mu A_\mu)\,[\,\Delta_{F-P}(A_\mu) - \bar\Delta(\Psi)] = 0\,,$$

where $\bar\Delta(\Psi)$ is a gauge invariant functional defined by the condition

$$\bar\Delta(\Psi)\int\prod_x \delta(A^\Omega)\,\delta(\chi^\Omega)\,\delta(F^\Omega)\,\delta(G^\Omega)\,\delta(\partial_\mu A^\Omega_\mu)\,d\Omega = 1\,. \qquad (8)$$

One can pass to the other guages using G. t' Hooft's trick.

Let us introduce a gauge invariant and s s functional Δ defined by

$$\Delta(\Psi)\int\prod_x \delta((\Box\Psi^\Omega)_+ - c_+)\,d\Omega = 1\,. \qquad (9)$$

Here $(\Psi^\Omega)_+$ means the left-handed component of Ψ^Ω and c_+ is an arbitrary chiral superfield.

$$\Psi_+ = \exp\{-\frac{1}{4}\bar\theta\hat\partial\gamma_5\theta\}[(\Psi_{+4}+i\Psi_{+1})+\bar\theta(\Psi_{+5}+i\Psi_{+2})+\frac{1}{4}\bar\theta(1+i\gamma_5)\theta(\Psi_{+6}+i\Psi_{+3})]$$

and components Ψ_{+i} are hermitian.

$$\delta[\Box(\Psi^\Omega_+)-c_+] \equiv \Pi^6_{i=1}\,\delta[\Box(\Psi^\Omega)_{+i}-c_{+i}].$$

Mutliplying the generating functional (7) by the constant factor (9) and performing the change of variables $\Psi^\Omega \to \Psi$ one obtaines

$$Z = N^{-1}\int \exp\{i\int \mathcal{L}(x)\,dx + \text{s.t.}\}\prod_x \delta(\Box\Psi_+ - c_+)\,\Delta(\Psi)\,d\mu\,.$$

Finally, using the fact that Z does not depend on c_+ one can integrate it over c_+ with the measure

$$\exp\{i\,\mathrm{Tr}\int\frac{1}{4}\beta^{-1}(\bar D D)^2\, c^*_+ c_+\,dx\}.$$

The resulting functional is

$$Z = N^{-1}\int\exp\{i\int[\mathcal{L}(x)+\frac{1}{4}\beta^{-1}\mathrm{Tr}(\bar D D)^2\Box\Psi_+\Box\Psi^*_+ + \text{s.t.}]\,dx\}\,\Delta(\Psi)\,d\mu\,. \qquad (10)$$

In terms of the components the gauge term looks exactly as in quantum electrodynamics

$$\frac{1}{4}\beta^{-1}\mathrm{Tr}\int(\bar D D)^2[\Box\Psi_+\Box\Psi^*_+] = \beta^{-1}\{\frac{1}{4}[\partial_\mu(\Box A - D)]^2 + \qquad (11)$$

$$+[\partial_\mu(\partial_\nu A_\nu)]^2 + [\Box F]^2 + [\Box G]^2 + \frac{1}{8}(\Box\bar\chi + i\hat\partial\bar\lambda)i\hat\partial(\Box\chi + i\hat\partial\lambda).$$

To calculate $\Delta(\Psi)$ at the surface $\Psi_+ = c_+$ it is sufficient to integrate in the formula (9) in the vicinity of a unit element

$$\Delta^{-1}(\Psi)|_{\Psi_+ = c_+} = \int\delta(M u_+)\,du_+. \qquad (12)$$

As before, du_+ means $\Pi^6_{i=1} du_{+i}$. The operator M is defined by

$$\Box^{-1}Mu_+ = \{(\Psi^{1+\delta\Omega})_+ - \Psi_+\} \equiv (\Psi^u)_+. \qquad (13)$$

It follows from (12) that $\Delta(\Psi)$ can be represented as a gaussian integral

$$\Delta(\Psi) = \int\exp\{-\int\Sigma^6_{i=1}\bar u_{+i}\,\Box(\Psi^u)_{+i}\,dx\}\,du_+\,d\bar u_+, \qquad (14)$$

where the integration variables $\bar u_+$ and u_+ have commutation properties opposite to the usual one: scalars $\bar u_{1,3,4,6}$ and $u_{1,3,4,6}$ anticommute and spinors $u_{2,5}$ and $\bar u_{2,5}$ commute. So, in addition to the usual fermion Faddeev-Popov ghosts, here the boson ghosts are present.

The integral (14) up to the constant factor may be written in a s s form:

$$\Delta(\Psi) = \int\exp\{-\frac{1}{32}\mathrm{Tr}\int(\bar D D)^2[\bar u_+(\Psi^u)^*_+ + (\bar u_+)^*(\Psi^u)]dx\}d\bar u_+\,du_+\,. \qquad (15)$$

The formulae (10) and (15) define the manifestly s s Green function generating functional. At first sight, this functional corresponds to nonrenormalizable theory due to highly nonlinear character of the Lagrangian. However although the Lagrangian (5) contains terms proportional to Ψ^n with arbitrary n due to the anticommutativity of θ only Λ component may enter in an arbitrary degree. But the propagator $\overline{\Lambda\Lambda}$ defined by eqs. (10) and (15) decreases for $k\to\infty$ as k^{-4}. Therefore the addition of internal Λ-lines does not spoil the convergence of the integrals. Below we shall show that in the theory defined by the functional (10) there exists a finite number of "basic" primitively divergent diagrams. All other divergent diagrams can be expressed in terms of the basic ones with the help of the generalized Ward identities which will be derived in Section 3.

2. Analysis of Divergencies

Let us calculate the degree of divergency of an arbitrary diagram. For brevity we consider a pure Yang-Mills field. The analysis of matter field diagrams is completely identical with the case of quantum electrodynamics /1/ and changes nothing in the result.

The free propagators defined by (5) and (11) have the following asymptotic behaviour:

$$\overline{DD} \sim 1,\ \overline{\lambda\lambda} \sim k^{-1},\ \overline{A_\mu A_\nu} \sim \overline{AD} \sim \overline{\lambda\chi} \sim k^{-2},\ \overline{\chi\chi} \sim k^{-3},$$
$$\overline{AA} \sim \overline{FF} \sim \overline{GG} \sim k^{-4}. \tag{16}$$

(In fact, the propagators $\overline{FF}$ and $\overline{GG}$ can be made decreasing arbitrarily fast by introducing more derivatives in eq. (11)). The explicit form of the free ghost Lagrangian is

$$\Sigma_{i=1,4}\,\partial_\mu \bar u_{+i}\,\partial_\mu u_{+i} + i/2\,\Sigma_{i=2,5}\,\bar u_{+i}\,\hat\partial u_{+i} + \Sigma_{i=3,6}\,\bar u_{+i}\,u_{+i}$$

and therefore the ghost propagators have asymptotic behaviour

$$\overline{\bar u_{+1,4}u_{+1,4}} \sim k^{-2},\ \overline{\bar u_{+2,5}u_{+2,5}} \sim k^{-1},\ \overline{\bar u_{+3,6}u_{+3,6}} \sim 1. \tag{17}$$

The Yang-Mills interaction Lagrangian can be represented symbolically as a sum of terms

$$\mathcal{L}_I^{(n)} \sim \{A_\nu(\partial^2\chi^2 + A_\nu\partial\chi^2 + \lambda\chi^3 + D\chi^2 + \chi^6 + \chi\partial\lambda + \partial^3 + \partial^2 A_\nu +$$
$$+ \partial D + \partial\chi^4 + \partial A_\nu^2 + DA_\nu + \chi^2 A_\nu^2 + \chi^4 A_\nu + A_\nu\lambda\chi + \lambda^2 + A_\nu^3) +$$
$$+ \chi(\chi^2\partial\lambda + \chi\partial D + \partial^2\chi^3 + \chi\lambda^2 + \partial\chi^5 + \partial^2\lambda + D\lambda) + \partial^2 D + \partial\lambda^2 + D^2\}A^n. \tag{18}$$

(Being at present interested only in the degree of divergency we omit all tensor structures, constants and also factors F and G, as F and G propagators do not increase the degree of divergency).

The ghost Lagrangian may be written analogously

$$\mathcal{L}_g^{(n)} \simeq \{\bar u_{+i}u_{+i}(D + \lambda\chi + A_\nu^2 + \chi^4 + A_\nu\chi^2 + \partial A_\nu + \partial\chi^2 + \partial^2) +$$
$$+ \bar u_i u_k\chi^2 + \bar u_i u_j(\lambda + A_\nu\chi + \chi^3 + \partial\chi) + \bar u_k u_k + \tag{19}$$
$$+ \bar u_k u_j\chi + \bar u_j u_j(\chi^2 + A_\nu + \partial)\}A^n\ ;\ (i=1,4;\ j=2,5;\ k=3,6).$$

Summarizing eqs. (16)-(19) we see that the index of divergency of the diagram with ℓ_i external lines of i-sort is

$$m \leq 4 - 2\ell_D - 2\ell_{u_{3,6}} - 3/2\ell_\lambda - 3/2\ell_{u_{2,5}} - \ell_{u_{1,4}} - \ell_{A_\nu} \tag{20}$$

(One can give more precise estimation, but eq. (20) is sufficient for our purposes).

Only diagrams with at most two D,λ or $u_{3(6)}, {}_{2(5)}$ external lines or four A_ν, $u_{1(4)}$ lines are superficially divergent. The number of "gauge" external lines A, F, G, χ is not fixed by eq. (20) and there are primitively divergent diagrams with arbitrary number of external gauge lines, but as we shall see all those diagrams are expressed in terms of the "basic" diagrams (without gauge external lines) and need not independent renormalization.

3. Generalized Ward Identities

We suppose to prove that all ultraviolet infinities can be eliminated by a finite number of supersymmetric and gauge invariant counterterms, namely

$$\mathcal{L}_R^{ef} = \mathrm{Tr}(\bar D D)^2\{\frac{1}{128}z_2 V_\mu(\tilde g)V_\mu(\tilde g) - \frac{1}{32}\tilde z_2\bar u_+(\Psi^4(\tilde g))^*_+ +$$
$$+ h.c. + \frac{1}{4}\beta^{-1}\Box\Psi_+^*\Box\Psi_+\}. \tag{21}$$

Here $\tilde g = z_1 z_2^{-1} g = \tilde z_1\tilde z_2^{-1} g$ (matter field Lagrangian can be treated in the same way). The counterterms z_2, z_1 and $\tilde z_2$ may be fixed, for example, by demanding the transverse part of A_μ propagator, 3-point A_μ^3 vertex, and u_{+1} propagator be finite. All other Green functions will be shown to need not independent renormalization.

The generalized Ward identities for the Green functions defined by the Lagrangian (21) can be derived in the same way as it was done in our paper[6] for the usual Yang-Mills theory. This derivation includes, however, explicit calculation of the Jacobian of a nonlocal gauge transformation and in the case of supergauge transformations becomes rather tedious. To avoid this complication we present in Appendix I a new simplified derivation.

The identity which allows one to express the Green functions with the gauge (i.e., A, χ, F, G, $\partial_\mu A_\mu$) external lines in terms of other Green functions looks as follows

$$\frac{\delta}{\delta\chi_{+i}(z)}\int \exp\{i\int[\mathcal{L}_R(x)+\frac{1}{4\beta}\mathrm{Tr}(\bar{D}D)^2(\Box\Psi_+\Box\Psi_+^*)+\eta_i(x)\,\Psi_i(x)]\,dx\}\Delta\times$$

$$\times\{\int[-\frac{1}{4\beta}\mathrm{Tr}(\bar{D}D)^2(\Box\Psi_+(y)\,\Box\chi_+^*(y)+\Box\chi_+(y)\Box\Psi_+^*(y)+$$

$$+\eta_i(y)\,\Psi_i^{\delta\Omega(v_+)}(y)\,]\,dy\}\,d\mu=0\,, \tag{22}$$

where $v_+=M^{-1}\chi_+$ . . The quantity χ_+ is as arbitrary chiral superfield parametrized by the components $\{\chi_4+i\chi_1,\ \chi_5+i\chi_2,\ \chi_6+i\chi_3\}$. .

Let us consider, for example, the identity $\delta Z/\delta\chi_{+1}=0$ setting $\eta_i=0$, $\eta_i\neq\eta_{A_\mu}\equiv J_\mu$.

$$\int\exp\{i\int[\mathcal{L}_R(x)+\frac{1}{4\beta}\mathrm{Tr}(\bar{D}D)^2(\Box\Psi_+\Box\Psi_+^*)+J_\mu^a(x)\,A_\mu^a(x)]\,dx\}\,\Delta(\Psi)\times$$

$$\times\{\beta^{-1}\Box^2\,\Psi^a_{+1}(z)+\int J_\mu^d(y)[\partial_\mu M_{11}^{-1da}(y,z)+f_\mu^{da}(y,z)]dy\}\,d\mu=0. \tag{23}$$

Here $\Box\Psi^a_{+1}=\partial_\mu A^a_\mu$. $M^{-1}_{1,1}(z,y)$ is the Green function $\overbrace{\bar u_1 u_1}$ in the external field Ψ. The explicit form of f_μ may be found from eq. (3). It is sufficient for our purpose to know that

$$\Box M^{-1da}_{1,1}(z,y)+\partial_\mu f_\mu^{da}(z,y)=\delta^{da}\,\delta(z-y)\,. \tag{24}$$

This equation follows from the equality

$$\Box(\Psi^{\delta\Omega(v)})_{+1}=\partial_\mu(A_\mu^{\delta\Omega(v)})=\chi_{+1}\,.$$

Differentiating (23) with respect to J_ν one obtains

$$i\beta^{-1}\langle T\Box\partial_\mu A^d_\mu(x)\,A^a_\nu(y)\rangle=-\langle T\partial_\nu M^{-1da}_{1,1}(z,y)\rangle-\langle T f^{da}_\nu(z,y)\rangle. \tag{25}$$

Differentiating eq. (25) with respect to y_ν and using eq. (24) we get the well-known relation

$$i\beta^{-1}\langle T\Box\partial_\mu A^d_\mu(x)\,\partial_\nu A^a_\nu(y)\rangle=-\delta^{ad}\delta(x-y)\,. \tag{26}$$

It means that only the transverse part of the Green function needs renormalization. The constant z_2 can therefore be chosen in such a way that the Green function $\overbrace{A_\mu A_\nu}$ is finite. The constants z_1 and $\tilde{z}_2$ may be fixed by demanding the 3-point vertex A^3_μ and the propagator $\overbrace{\bar u_1 u_1}$ be finite. No more independent renormalization constants are necessary. All other diagrams are automatically finite.

Let us demonstrate it for the $A_\mu\bar u_1 u_1$ vertex and the four-point A^4_μ vertex. Differentiating eq. (23) with respect to J_μ twice we have

$$i\beta^{-1}\langle TA^a_\mu(x)\,A^b_\nu(y)\,\partial_\rho A^c_\rho(z)\rangle=\langle T\partial^x_\mu M^{-1ac}_{1,1}(x,z)\,A^b_\nu(y)\rangle+$$

$$+\langle T f^{ac}_\mu(x,z)\,A^b_\nu(y)\rangle+(x\longleftrightarrow y,\ a\longleftrightarrow b,\ \mu\longleftrightarrow\nu)\,. \tag{27}$$

Separating the structure transverse with respect to ∂^x_μ and differentiating with respect to y_ν one gets for the Fourier transformation

$$D^{tr}_{\mu\nu}\,\frac{k^\nu}{k^2}\,\frac{(p+k)^\rho}{(p+k)^2}\,\Gamma^{abc}_{\lambda\nu\rho}(p,k)=\frac{k^\nu}{k^2}\,G(p+k)\,\gamma^{abc\ tr}_{\mu\nu}(p,k). \tag{28}$$

Here $D_{\mu\nu}$, G are the A_μ and u_1 propagators, $\Gamma_{\lambda\nu\rho}$ is the proper vertex part of $\langle TA_\mu A_\nu A_\rho\rangle$. Due to eq. (24)

$ip_\mu \gamma_{\mu\nu}(p,k)$ is a proper vertex part of $<T\bar{u}_1(x)u_1(y)A_\mu(z)>$. According to eq. (20) $<T\bar{u}_1 u_1 A_\mu>$ diverges at most linearly therefore $\gamma_{\mu\nu}$ may diverge at most logarithmically, i.e., a possible divergent structure is proportional to $g^{\mu\nu}$. But it follows from eq. (28) that $k_\nu \gamma^{tr}_{\mu\nu}(p,k)$ is finite (because all other factors in this equation $D_{\mu\nu}$, G, $\Gamma_{\lambda\nu\rho}$ are finite by construction). Therefore the complete $\gamma_{\mu\nu}$ is finite and the same is true for the $<T\bar{u}_1 u_1 A_\mu>$ proper vertex part. Differentiating eq. (23) three times with respect to J_μ one obtains the relation that expresses the $<T\bar{u}_1 u_1 A_\mu A_\nu>$ vertex in terms of the four-point A^4_μ vertex and lower Green functions. The finiteness of the four-point A^4_μ vertex follows from the relation

$$\beta^{-1}<T\partial_\mu A^a_\mu(x)\partial_\nu A^b_\nu(y)\partial_\rho A^c_\rho(z)\partial_\lambda A^d_\lambda(u)>=\delta(x-u)\delta(y-z)\delta^{ad}\delta^{bc}+\text{sym}. \tag{29}$$

which is obtained by differentiating the identity (23) for the four-point vertex. This relation states that the connected part of the four-vertex is equal to zero and, therefore, expresses the proper four-vertex in terms of the proper three-vertex and the Green function which are finite by construction. It also follows from eq. (29) that if the local part of the three-vertex is

$$\Gamma^{abc}_{\lambda\nu\rho} = i\epsilon^{abc}\,[\,\delta^{\nu\lambda}(p-k)^\rho + \delta^{\nu\rho}(k-q)^\lambda + \delta^{\lambda\rho}(q-p)^\nu\,] \tag{30}$$

(this form is dictated by the symmetry properties of Γ) then the local part of the four-vertex is

$$\Delta^{abcd}_{\mu\nu\rho\lambda}(p,k,q) = P\{\epsilon_{eab}\epsilon_{ecd}\,\delta_{\mu\rho}\,\delta_{\nu\lambda}\}, \tag{31}$$

where P is the symmetrization operator with respect to the pairs $a\mu$, $b\nu$, $c\rho$, $d\lambda$.

Analogously, one can derive relations which express the Green functions with n external A, F, G, χ lines in terms of the Green functions with $(n-1)$ external gauge lines. It is simpler, however, to prove the finiteness of the corresponding diagrams using the supersymmetry of the effective Lagrangian rather than the identities (23).

For this purpose let us write the Green functions generating functional in a completely supersymmetric form

$$Z(\eta_i) = N^{-1}\int \exp\{i\int[\mathcal{L}_{ef}(x) + \frac{1}{16}\mathrm{Tr}(\bar{D}D)^2(\Psi\eta)]\,dx\}\,d\mu, \tag{32}$$

where the source η is a superfield with the components

$$\eta_i = \{2\eta_D, -\eta_\lambda, \eta_F, \eta_G, \eta_{A_v}, -\eta_\chi, 2\eta_A\}$$

and $\mathcal{L}_{ef}$ includes the ghost Lagrangian, $Z(\eta)$ is evidently invariant with respect to the s s transformation $\eta_i \to \eta_i'$. Due to the s s of the exponent one can compensate this transformation by the s s change of variables $\Psi_i \to \Psi_i'$.

Therefore the generating functional for the one-particle irreducible Green functions Γ defined by a Legendre transformation

$$\Gamma(R) = -i\,\ell n\, Z(\eta) - \frac{1}{8}\int(\bar{D}D)^2(\eta R)\,dx \tag{33}$$

is invariant with respect to the s s transformation of its arguments R_i. In particular, the local part of $\Gamma(R)$ is supersymmetric. Due to eqs. (26), (30) and (31) the local parts of the vertices with 2,3 and 4 external vector lines are finite and form a gauge invariant structure, i.e., the Yang-Mills Lagrangian. The complete local part of $\Gamma(R)$ should therefore be an s s generalization of the Yang-Mills Lagrangian with finite parameters. Such a generalization is given by the formula (5) where the arguments Ψ should be replaced by R. Therefore the counterterms introduced in eq. (21) really eliminate all the divergencies. The finiteness of the vertices including the ghost lines is proved analogously if one introduces in the functional (32) the source term for the ghosts.

4. Discussion

Thus, we have shown that renormalization of the s s Yang-Mills theory can be performed preserving supersymmetry of the theory. Only the common wave function renormalization for the whole gauge multiplet and charge renormalization are needed. In the presence of a matter field one more wave function and mass renormalization appear.

Our result proves, in particular, that the s s Yang-Mills theory is asymptotically free, provided the number of the matter field multiplets is not too large.

Appendix I

The renormalized generating functional Z can be written as follows

$$Z = N^{-1} \int \exp\{ i \int [\mathcal{L}_R(x) + \Sigma_i \eta_i \Psi_i(x)]\, dx\} \Delta(\Psi) \exp\{ \frac{i}{4\beta} \int \mathrm{Tr}(\bar{D}D)^2\, \Box c_+ \Box c_+^* \} \times \tag{A.1}$$

$$\times \prod_x \delta(\Psi_+ - c_+)\, d\Psi_i\, dc_+ , \qquad \Psi_i = \{A, \chi, F, G, A_\nu, \lambda, D\}.$$

Introduce the gauge invariant functional $\tilde{\Delta}(\Psi)$

$$1 = \tilde{\Delta}(\Psi) \int \prod_x \delta[(\Psi^{\Omega^{-1}})_+ - c_+ - \chi_+]\, d\Omega , \tag{A.2}$$

where χ_+ is infinitesimal chiral superfield. Multiplying (A.1) by (A.2) and performing the change of variables $\Psi^{\Omega^{-1}} \to \Psi$ one obtains

$$Z = N^{-1} \int \exp\{ i \int [\mathcal{L}_R(x) + \Sigma_i \eta_i \Psi_i^{\Omega}(x)]\, dx\} \tilde{\Delta}(\Psi) \times$$

$$\times \exp\{ \frac{i}{4\beta} \int \mathrm{Tr}(\bar{D}D)^2\, \Box c_+ \Box c_+^* dx\} \prod_x \delta(\Psi_+ - c_+ - \chi_+)\, d\Psi_i\, dc_+ .$$

The functional $\tilde{\Delta}(\Psi)$ at the surface $\Psi_+ = c_+ + \chi_+$ is equal to $\Delta(\Psi)$ at the surface $\Psi_+ = c_+$. The quantity $\Omega(v_+)$ is defined by the equation

$$(\Psi^{\Omega(v_+)})_+ = c_+ \tag{A.3}$$

at the surface $\Psi_+ = c_+ + \chi_+$. At this surface eq. (A.3) becomes

$$M v_+ = \chi_+ . \tag{A.4}$$

Integrating over c_+ and setting the coefficients at χ_+ equal to zero we obtain the identity which is an exact analogue of eq. (23) in our paper /6/

$$\frac{\delta}{\delta \chi^+} \int \exp\{ i [\mathcal{L}_R(x) + \eta_i(x) \Psi_i(x) + \frac{1}{4\beta} \mathrm{Tr}(\bar{D}D)^2 (\Box\Psi_+ \Box\Psi_+^*)\} \Delta(\Psi) \times$$

$$\times \{ \int [-\frac{1}{4\beta} \mathrm{Tr}(\bar{D}D)^2 (\Box\Psi_+(x) \Box\chi_+^*(x) + \Box\chi_+(x) \Box\Psi_+^*(x)) +$$

$$+ \eta_i \Psi_i^{\delta\Omega(v_+)}(x)]\, dx\}\, d\mu = 0.$$

$$v_+ = M^{-1} \chi_+ , \quad (\Psi^{\delta\Omega(v_+)})_+ = \chi_+ . \tag{A.5}$$

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