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MODEL-INDEPENDENT TESTS FOR $\Delta I = 1/2$ RULE IN Σ DECAYS

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1. Introduction

It is well known that the most remarkable regularity of the hyperon non-leptonic decays is the $\Delta I = 1/2$ rule which is experimentally satisfied to a surprising accuracy. The nonvanishing departures from this rule are usually attributed /1/ to the contribution of the $\Delta I = 3/2$ part of the weak Hamiltonian. Moreover, Gavroglu /2/ has presented recently model dependent indications that the electromagnetic interaction and CP violating effects may account for these departures.

Unfortunately, the usual tests for the $\Delta I = 1/2$ rule in Σ decay are based on the determination of the parity violating (s-wave) and parity-conserving (p-wave) decay amplitudes from the experimental data using some conventional hypotheses. Such that the decay amplitudes are taken to be real (CP invariance is assumed and final state interactions are neglected) and their absolute signs are assigned according to a convention (see ref. /1/) which involves for the s-wave amplitudes an approximate fit to the $\Delta I = 1/2$ rule and Lee-Sugawara relation. In consequence, the departure from the exact $\Delta I = 1/2$ rule in Σ decay, resulting from this analysis, depend mainly on these additional assumptions and exclude some of the very interesting conjectures concerning the origin of these deviations. Therefore, it is certainly of great interest to obtain reliable tests of the $\Delta I = 1/2$ rule for Σ decay and to determine the departures from this rule in a model independent way.

The purposes of this paper are to investigate in more detail the experimental consequences implied by a general triangular relationship (Sect. 2) in order to obtain (Sect. 3) certain tests of the $\Delta I = 1/2$ rule for Σ decay and to derive (Sect. 4) the supplementary constraints imposed on the decay experimental data when the decay amplitudes are taken to be real.

2. Experimental Consequences of Triangular Relationships

In order to have a unified treatment of the experimental consequences resulting from different triangular relationships, such as those derived from the $\Delta I = 1/2$ rule for Σ decay modes or Lee-Sugawara relations, we start with the following definitions. Let a_{sk} and a_{pk} , $k = 1,2,3$ be the usual /3/ parity-violating (s-wave) and parity-conserving (p-wave) decay amplitudes, respectively, for three hyperon non-leptonic decays which satisfy the sum rules:

$$\sum_{k=1}^{3} c_k (a_{s,p})_k = 0, \tag{1}$$

where c_k are real numbers.

Let us define

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, \tag{2a}$$

$$K_{ij} = c_i^2 c_j^2 \, \frac{1}{2} (1 - \vec{\xi}_i \cdot \vec{\xi}_j) \sigma_i \sigma_j , \quad i \neq j = 1,2,3, \quad \xi_k = (a_k, \beta_k, \gamma_k), \tag{2b}$$

$$\bar{K} = \frac{1}{3} (K_{12} + K_{23} + K_{13}), \tag{2c}$$

and denote by $\lambda(\sigma)$, $\lambda(\xi\sigma)$ and $\lambda_\xi^{(\pm)}$ the following functions

$$\lambda(\sigma) = \lambda(c_1^2 \sigma_1, c_2^2 \sigma_2, c_3^2 \sigma_3), \tag{2d}$$

$$\lambda(\xi\sigma) = \lambda(c_1^2 \xi_1 \sigma_1, \ c_2^2 \xi_2 \sigma_2, \ c_3^2 \xi_3 \sigma_3), \tag{2e}$$

$$\lambda_\xi^{(\pm)} = \lambda[c_1^2 (1 \pm \xi_1) \sigma_1 , \ c_2^2 (1 \pm \xi_2) \sigma_2 , \ c_3^2 (1 \pm \xi_3) \sigma_3], \tag{2f}$$

where

$$\sigma_k = |a_{sk}|^2 + |a_{pk}|^2, \tag{2g}$$

$$a_k \sigma_k = 2 \mathrm{Re} \, (a_{sk}^* a_{pk}), \quad \beta_k \sigma_k = 2 \mathrm{Im} \, (a_{sk}^* a_{pk}),$$

$$\gamma_k \sigma_k = |a_{sk}|^2 - |a_{pk}|^2,$$

and

$$\xi_k \equiv a_k , \beta_k , \gamma_k , \quad k = 1,2,3. \tag{2h}$$

Then, the sum rules (1) alone imply the following set of equalities:

$$\bar{K} = K_{12} = K_{13} = K_{23} , \tag{3a}$$

$$\frac{1}{4} | \lambda_\xi^{(+)} - \lambda_\xi^{(-)} | = [- 4 \bar{K} - \lambda(\sigma)]^{1/2} [4\bar{K} - \lambda(\xi\sigma)]^{1/2}, \tag{3b}$$

$$| 2\bar{K} + \frac{1}{4} \lambda(\sigma) - \frac{1}{4} \lambda(\xi\sigma) | = [- \frac{1}{4} \lambda_\xi^{(+)}]^{1/2} [- \frac{1}{4} \lambda_\xi^{(-)}]^{1/2}, \tag{3c}$$

$$\frac{1}{2} | 2\bar{K} + \frac{1}{4} \lambda(\sigma) - \frac{1}{4} \lambda(\xi\sigma) + \frac{1}{4} \lambda_\xi^{(\pm)} | = [- \bar{K} - \frac{1}{4} \lambda(\sigma)]^{1/2} \times$$

$$\times [- \frac{1}{4} \lambda_\xi^{(\pm)}]^{1/2}, \tag{3d}$$

$$\frac{1}{2}\left|2\bar{K}+\frac{1}{4}\lambda(\sigma)-\frac{1}{4}\lambda(\xi\sigma)-\frac{1}{4}\lambda_{\xi}^{(\pm)}\right|=\left[\bar{K}-\frac{1}{4}\lambda(\xi\sigma)\right]^{1/2}\times$$

$$\times\left[-\frac{1}{4}\lambda_{\xi}^{(\pm)}\right]^{1/2} \quad (3e)$$

and the following set of inequalities

$$0\leq-\frac{1}{4}\lambda_{\xi}^{(\pm)}\leq\min_{(ij)}\{c_i^2c_j^2(1\pm\xi_i)(1\pm\xi_j)\sigma_i\sigma_j\}, \quad (4a)$$

$$\max_{(ij)}\{-c_i^2c_j^2\xi_i\xi_j\sigma_i\sigma_j\}\leq\frac{1}{4}\lambda(\xi\sigma)\leq\bar{K}, \quad (4b)$$

$$\bar{K}\leq-\frac{1}{4}\lambda(\sigma)\leq\min_{(ij)}\{c_i^2c_j^2\sigma_i\sigma_j\}, \quad (4c)$$

valid for any $\xi_k\equiv\alpha_k, \beta_k, \gamma_k$, $k=1,2,3$.

In order to prove the equalities (3a,b,c,d,e) and the inequalities (4a,b,c) we define the following bilinear forms:

$$M_{ij}^{(\pm\xi)}=[H_i^{(\pm\xi)}]^*H_j^{(\pm\xi)}, \quad (5a)$$

$$Z_{ij}^{(0)}=\frac{1}{2}[M_{ij}^{(+\xi)}+M_{ij}^{(-\xi)}],\ Z_{ij}^{(\xi)}=\frac{1}{2}[M_{ij}^{(+\xi)}-M_{ij}^{(-\xi)}], \quad (5b)$$

$$Y_{ij}=a_{si}a_{pj}-a_{pi}a_{sj}, \quad (5c)$$

where

$$H_k^{(\pm\xi)}\equiv\{a_{sk}\pm a_{pk};\ a_{sk}\pm ia_{pk};\ \sqrt{2}(a_{sk},\ a_{pk})\}, \quad (5d)$$

for $\xi=(\alpha,\beta,\gamma)$ respectively.

The bilinear forms $Z_{ij}^{(0)}$ are independent of the upper indices ξ since from the definitions (5a,b,d) we have

$$M_{ij}^{(+\alpha)}+M_{ij}^{(-\alpha)}=M_{ij}^{(+\beta)}+M_{ij}^{(-\beta)}=M_{ij}^{(+\gamma)}+M_{ij}^{(-\gamma)}. \quad (5c)$$

Now, by straightforward calculus we obtain:

$$|M_{ij}^{(\pm\xi)}|^2=(1\pm\xi_i)\sigma_i(1\pm\xi_j)\sigma_j, \quad (6a)$$

$$|Z_{ij}^{(\xi)}|^2=\frac{1}{2}[1-\vec{\xi}_i\cdot\vec{\xi}_j]\sigma_i\sigma_j+\xi_i\sigma_i\xi_j\sigma_j, \quad (6b)$$

$$|Z_{ij}^{(0)}|^2=\frac{1}{2}[1+\vec{\xi}_i\cdot\vec{\xi}_j]\sigma_i\sigma_j, \quad (6c)$$

$$|Y_{ij}|^2=\frac{1}{2}[1-\vec{\xi}_i\cdot\vec{\xi}_j]\sigma_i\sigma_j. \quad (6d)$$

The equalities (3a) are obtained from (6d) and

$$c_1c_2Y_{12}=c_1c_3Y_{13}=c_2c_3Y_{23}, \quad (7a)$$

which follow directly from the sum rules (1). Next, it is easy to see that the sum rules (1) alone imply:

$$c_i^2c_j^2[(\mathrm{Re}N_{ij})^2-N_{ii}N_{jj}]=\frac{1}{4}\lambda(c_1^2N_{11},c_2^2N_{22},c_3^2N_{33}), \quad (7b)$$

for any $N_{ij}\equiv M_{ij}^{(\pm\xi)}, Z_{ij}^{(c)}, Z_{ij}^{(\xi)}$ and

$$c_i^2c_j^2[\mathrm{Im}\,M_{ij}^{(\pm\xi)}]^2=-\frac{1}{4}\lambda_{\xi}^{(\pm)}=\{[-\bar{K}-\frac{1}{4}\lambda(\sigma)]^{1/2}\pm$$

$$\pm\eta_{\xi}[\bar{K}-\frac{1}{4}\lambda(\xi\sigma)]^{1/2}\}^2, \quad (7c)$$

$$c_i^2 c_j^2 [\,\mathrm{Im} Z_{ij}^{(0)}]^2 = -\bar{K} - \frac{1}{4}\lambda(\sigma) = \frac{1}{16}\{[-\lambda_\xi^{(+)}]^{1/2} + \eta_\xi [-\lambda_\xi^{(-)}]^{1/2}\}^2, \tag{7d}$$

$$c_i^2 c_j^2 [\,\mathrm{Im} Z_{ij}^{(\xi)}]^2 = \bar{K} - \frac{1}{4}\lambda(\xi\sigma) = \frac{1}{16}\{[-\lambda_\xi^{(+)}]^{1/2} - \epsilon_\xi [-\lambda_\xi^{(-)}]^{1/2}\}^2, \tag{7e}$$

where

$$\eta_\xi \equiv \mathrm{sign}\{-\lambda_\xi^{(+)} + \lambda_\xi^{(-)}\}, \epsilon_\xi = \mathrm{sign}\{-8\bar{K} - \lambda(\sigma) + \lambda(\xi\sigma)\}. \tag{7f}$$

We note of course that in derivation of (7c,d,e) we have used the equalities (3a).

Now, using the equalities (7c,d,e,f) and the positivity conditions:

$$[\,\mathrm{Re} N_{ij}]^2 \geq 0,\ [\,\mathrm{Im} N_{ij}]^2 \geq 0,\quad N_{ij} \equiv M_{ij}^{(\pm\xi)},\ Z_{ij}^{(0)},\ Z_{ij}^{(\xi)}$$

we obtain the equalities (3b,c,d,e) and the inequalities (4a,b,c).

3. Tests of $\Delta I = 1/2$ Rule for Σ Decays

Let us consider the non-leptonic Σ decay modes

$$\Sigma_+^+ \to n + \pi^+,\quad \Sigma_0^+ \to p + \pi^\circ,\quad \Sigma_-^- \to n + \pi^-, \tag{8}$$

where the subscript on the hyperon refers to the sign of the decaying pion. The $\Delta I = 1/2$ rule for the hyperon non-leptonic decays implies the following relation between the Σ decay amplitudes:

$$a_{s,p}(\Sigma_+^+) - a_{s,p}(\Sigma_-^-) + \sqrt{2}\ a_{s,p}(\Sigma_0^+) = 0. \tag{9}$$

The quantities of interest for making tests of the theoretical predictions of $\Delta I = 1/2$ rule for Σ decay are

$$\lambda(\sigma) \equiv \lambda(\sigma_+, \sigma_-, 2\sigma_+),\ \lambda(\xi\sigma) \equiv \lambda(\xi_+\sigma_+, \xi_-\sigma_-, 2\xi_0\sigma_0), \tag{10a}$$

$$\lambda_\xi^{(\pm)} \equiv \lambda[(1 \pm \xi_+)\sigma_+,\ (1 \pm \xi_-)\sigma_-,\ 2(1 \pm \xi_0)\sigma_0], \tag{10b}$$

$$K_{+-} = \frac{1}{2}(1 - \vec{\xi}_+ \cdot \vec{\xi}_-)\sigma_+\sigma_-,\ K_{i0} = (1 - \vec{\xi}_i \cdot \vec{\xi}_0)\sigma_0\sigma_i,\ i = +,- \tag{10c}$$

and

$$\bar{K} = \frac{1}{3}(K_{+-} + K_{+0} + K_{-0}) \tag{10d}$$

where the λ -function is defined by the relation (2a).

The quantities σ_i, K_{ij} which are presented in table I, have been calculated using the experimental data for mean lives, branching ratios and the decay asymmetry parameters given in ref. /1/ . Next, using these results we have estimated $\bar{K}$ and all the λ -functions given in table II. The quantities $\bar{K}_{th}(\xi)$ are calculated according to

$$\bar{K}_{th}(\xi) = -\frac{1}{8}\lambda(\sigma) + \frac{1}{8}\lambda(\xi\sigma) - \frac{1}{2}\epsilon_\xi [-\frac{1}{4}\lambda_\xi^{(+)}]^{1/2} [-\frac{1}{4}\lambda_\xi^{(-)}]^{1/2}, \tag{11}$$

where [see definition (7f)] $\epsilon_\alpha = \epsilon_\gamma = +1$, $\epsilon_\beta = -1$. These results should be compared with the values of $\bar{K}$ since the $\Delta I = 1/2$ rule implies: $\bar{K}_{th}(\xi) = \bar{K}$ for any $\xi = \alpha, \beta, \gamma$. These predictions are derived from (7d) or (7e).

Moreover, the $\Delta I = 1/2$ rule for Σ decay predicts that the following test quantities are all equal to zero [see the equalities (3a,b,c,d,e)] :

$$T_1(ij) = \bar{K} - K_{ij},\quad i \neq j = +,-,0, \tag{12a}$$

$$T_2(\xi) = \frac{1}{4}|\lambda_\xi^{(+)} - \lambda_\xi^{(-)}| - [-4\bar{K} - \lambda(\sigma)]^{1/2}[4\bar{K} - \lambda(\xi\sigma)]^{1/2}, \tag{12b}$$

$$T_3(\xi) = |2\bar{K} + \frac{1}{4}\lambda(\sigma) - \frac{1}{4}\lambda(\xi\sigma)| - [-\frac{1}{4}\lambda_\xi^{(+)}]^{1/2}[-\frac{1}{4}\lambda_\xi^{(-)}]^{1/2}, \tag{12c}$$

Table I
The experimental values of σ_i , ξ_i and K_{ij} , for the Σ decay modes

Decay modes	σ_i *)	α_i	β_i	γ_i
$\Sigma^+_+ \longrightarrow n + \pi^+$	2.553	0.066 ± 0.016	0.234	−0.97
$\Sigma^+_0 \longrightarrow p + \pi^0$	2.657	− 0.979 ± 0.016	0.113	0.17
$\Sigma^-_- \longrightarrow n + \pi^-$	2.730	− 0.069 ± 0.008	0.158	0.98

K_{+-}	6.584
K_{+0}	8.160
K_{-0}	5.425

*) units: 10^8 (sec.MeV)$^{-1}$

Table II
The experimental values of $-\frac{1}{4}\lambda(\sigma)$, $\frac{1}{4}\lambda(\xi\sigma)$, $-\frac{1}{4}\lambda_\xi^{(\pm)}$ $\bar{K}$ and $\bar{K}_{th}(\xi)$, $\xi=\alpha,\beta,\gamma$ estimated using the definitions (2a) and (10a,b,c,d) and prediction (11).

ξ	$-\frac{1}{4}\lambda_\xi^{(+)}$	$-\frac{1}{4}\lambda_\xi^{(-)}$	$\frac{1}{4}\lambda(\xi\sigma)$	$\bar{K}_{th}(\xi)$	$\bar{K}$	$-\frac{1}{4}\lambda(\sigma)$
α	0.284	0.166	6.745	6.749		
γ	0.281	0.163	6.748	6.752	6.756	6.970
β	9.920	4.444	−0.212	6.702		

Table III
The experimental values of the test quantities (12a,b,c,d,e) for the $\Delta I = 1/2$ rule in the Σ decays

Test quantity		Test quantity	
$T_1(+-)$	+0.072	$T_4(+\gamma)$	−0.001
$T_1(+0)$	−1.404	$T_4(-\alpha)$	−0.003
$T_1(-0)$	+1.331	$T_4(-\beta)$	+0.175
$T_2(\alpha)$	−0.074	$T_4(-\gamma)$	−0.002
$T_2(\beta)$	+0.592	$T_5(+\alpha)$	−0.015
$T_2(\gamma)$	−0.047	$T_5(+\beta)$	+0.023
$T_3(\alpha)$	−0.014	$T_5(+\gamma)$	−0.010
$T_3(\beta)$	+0.105	$T_5(-\alpha)$	−0.023
$T_3(\gamma)$	−0.008	$T_5(-\beta)$	+0.024
$T_4(+\alpha)$	−0.002	$T_5(-\gamma)$	−0.014
$T_4(+\beta)$	+0.126		

$$[T_4(\pm\xi) = \frac{1}{2}\left| 2\bar{K} + \frac{1}{4}\lambda(\sigma) - \frac{1}{4}\lambda(\xi\sigma) + \frac{1}{4}\lambda_\xi^{(\pm)}\right| - [-\bar{K} - \frac{1}{4}\lambda(\sigma)]^{1/2} \times$$

$$\times [-\frac{1}{4}\lambda^{(\pm)}]^{1/2}, \tag{12d}$$

$$T_5(\pm\xi) = \frac{1}{2}\left| 2\bar{K} + \frac{1}{4}\lambda(\sigma) - \frac{1}{4}\lambda(\xi\sigma) - \frac{1}{4}\lambda_\xi^{(\pm)}\right| - [\bar{K} - \frac{1}{4}\lambda(\xi\sigma)]^{1/2} \times$$

$$\times [-\frac{1}{4}\lambda_\xi^{(\pm)}]^{1/2} \tag{12e}$$

for any $\xi = \alpha, \beta, \gamma$. We note that $T_3(\xi) = 0$ are all equivalent to $\bar{K}_{th}(\xi) = \bar{K}$.

The values of all test quantities determined from the experimental data (table II), are displayed in table III.

Now, from tables II and III, we see that the values of all test quantities, except for $T_1(+0)$, $T_1(-0)$ and $T_2(\beta)$ are consistent with the theoretical predictions of the $\Delta I = 1/2$ rule for Σ decays. The breaking effects seem to manifest only in Σ_0^+ decay mode since a significant departure from the $\Delta I = 1/2$ predictions is observed only for the test quantities $T_1(+0)$ and $T_2(-0)$. Therefore, for the determination of the breaking effects we can choose

$$a_s(\Sigma_-^-) - a_s(\Sigma_+^+) = \sqrt{2}\,(1-\epsilon)\,a_s(\Sigma_0^+) = \sqrt{2}\,a_s'(\Sigma_0^+), \tag{13a}$$

$$a_p(\Sigma_-^-) - a_p(\Sigma_+^+) = \sqrt{2}\,(1+\epsilon)\,a_p(\Sigma_0^+) = \sqrt{2}\,a_p'(\Sigma_0^+) \tag{13b}$$

as the most economical parametrization for these effects. Then, using the expressions

$$\sigma_0' = (1 + |\epsilon|^2 - 2\gamma_0 \,\mathrm{Re}\,\epsilon)\,\sigma_0 , \tag{14a}$$

$$\alpha_0'\sigma_0' = (1 - |\epsilon|^2)\alpha_0 - 2\beta_0 \,\mathrm{Im}\,\epsilon , \tag{14b}$$

$$\beta_0'\sigma_0' = (1 - |\epsilon|^2)\beta_0 + 2\alpha_0 \,\mathrm{Im}\,\epsilon , \tag{14c}$$

$$\gamma_0'\sigma_0' = (1 + |\epsilon|^2)\gamma_0 - 2\,\mathrm{Re}\,\epsilon , \tag{14d}$$

we define $\lambda(\sigma')$, $\lambda(\xi'\sigma')$, $\lambda^{(\pm)}_{\xi'}$ and K'_{ij}, $\vec{\bar{K}}'$, $i, j = +, -, 0$ by (10a,b,c) and the substitutions $\vec{\xi}_0 \to \vec{\xi}_0' \equiv (\alpha_0', \beta_0', \gamma_0')$, $\sigma_0 \to \sigma_0'$. Then, for K'_{j0} we obtain

$$K'_{j0} = (1 - \vec{\xi}_0' \cdot \vec{\xi}_j)\,\sigma_0'\sigma_j = K_{j0} + \tag{15}$$

$$+\sigma_j\sigma_0\{|\epsilon|^2(1 + \vec{\xi}_j \cdot \vec{\xi}_0 - 2\gamma_j\gamma_0) + 2(\gamma_j - \gamma_0)\,\mathrm{Re}\,\epsilon + 2(\alpha_j\beta_0 - \alpha_0\beta_j)\,\mathrm{Im}\,\epsilon\}.$$

Therefore, since the sum rules (13a,b) alone imply that the test quantities: $T_1(i,j)$, $i \neq j = 0', +, -$, $T_2(\xi')$, $T_3(\xi')$, $T_4(\pm\xi')$, $T_5(\pm\xi')$ [obtained from (12a,b c,d,e) by the substitution $\sigma \to \sigma_0'$, $\vec{\xi}_0 \to \vec{\xi}_0'$] are all equal to zero, the best solution for the breaking parameters can be obtained optimizing all the absolute values of these test functions. Here, for the determination of ϵ we have used only the equations implied by $T_1(i,0')$, $i = +, -$, which are equivalent to

$$K_{+-} = K'_{i0} , \quad i = +, -, \tag{16}$$

In this way we get two solutions:

$$\mathrm{Re}\,\epsilon_1 = 0.101, \quad \mathrm{Im}\,\epsilon_1 = 0.002, \tag{17a}$$

$$\mathrm{Re}\,\epsilon_2 = 0.091, \quad \mathrm{Im}\,\epsilon_2 = -0.42. \tag{17b}$$

Then, from (14a,b,c,d) we obtain

$$\sigma_{01}' = 2.593, \quad \alpha_{01}' = -0.993, \quad \beta_{01}' = 0.110, \quad \gamma_{01}' = -0.031, \tag{18a}$$

$$\sigma_{02}' = 3.066, \quad \alpha_{02}' = -0.610, \quad \beta_{02}' = 0.793, \quad \gamma_{02}' = 0.019. \tag{18b}$$

Now, evaluating the λ-functions and all the test quantities (12a,b,c,d,e) and (11) using (18a,b) instead of σ_0 and ξ_0 in (10a,b) we obtain the results presented in tables IV and V. These results enable us to understand that the solution (17a) is a good candidate for the breaking parameter ϵ. However, the solution (17b) can be rejected since it predicts unreasonable CP violating effects in the Σ decays.

4. Consequences of Additional Constraints on Hyperon Decay Amplitudes

As we have mentioned in introduction, the traditional tests of the $\Delta I = 1/2$ rule (in Σ decay) and Lee-Sugawara relation are based on the determination of decay amplitudes

Table IV

The values of $-\frac{1}{4}\lambda(\sigma')$, $\frac{1}{4}\lambda(\xi'\sigma')$, $-\frac{1}{4}\lambda^{(\pm)}_{\xi'}$, $\bar{K}'$ and $\bar{K}'_{th}(\xi')$, estimated from (10a,b,c) and (11) with the substitution $\sigma_0 \to \sigma'_{0i}$, $\xi_0 \to \xi'_{0i}$, $i = 1,2$ (18a,b) respectively

ξ'	$-\frac{1}{4}\lambda^{(+)}_{\xi}$	$-\frac{1}{4}\lambda^{(-)}_{\xi}$	$\frac{1}{4}\lambda(\xi'\sigma')$	$\bar{K}'_{th}(\xi)$	$\bar{K}'$	$-\frac{1}{4}\lambda(\sigma)$
α'_1	0.087	0.628	6.610	6.672		
γ'_1	0.155	0.468	6.656	6.677	6.684	6.967
β'_1	9.880	4.461	−0.205	6.700		
α'_2	4.485	1.741	3.492	6.594		
γ'_2	0.268	0.060	6.625	6.644	6.684	6.789
β'_2	4.477	2.268	3.417	6.696		

from the experimental data using the following additional hypotheses: Time-invariance is assumed and final state interactions are neglected, so the decay amplitudes are taken to be real and $\beta = 0$. Therefore, it is of great interest to investigate in more detail the experimental consequences

Table V

The values of the test quantities: $T_2(\xi')$, $T_3(\xi')$, $T_4(\pm\xi')$ and $T_5(\pm\xi')$ calculated from the experimental data and (12a,b,c,d,e) by the substitution $\sigma_0 \to \sigma'_{01}$, $\xi_0 \to \xi'_{01}$ using (18a)

Test quantity		Test quantity	
$T_2(\alpha'_1)$	−0.066	$T_4(-\alpha'_1)$	−0.003
$T_2(\beta'_1)$	+0.175	$T_4(-\beta'_1)$	+0.041
$T_2(\gamma'_1)$	−0.043	$T_4(-\gamma'_1)$	−0.002
$T_3(\alpha'_1)$	−0.026	$T_5(+\alpha'_1)$	−0.019
$T_3(\beta'_1)$	+0.033	$T_5(+\beta'_1)$	+0.007
$T_3(\gamma'_1)$	−0.014	$T_5(+\gamma'_1)$	−0.016
$T_4(+\alpha'_1)$	−0.005	$T_5(-\alpha'_1)$	−0.001
$T_4(+\beta'_1)$	+0.035	$T_5(-\beta'_1)$	+0.009
$T_4(+\gamma'_1)$	−0.004	$T_5(-\gamma'_1)$	−0.008

of the sum rules (1) when the non-leptonic decay amplitudes satisfy the constraints:

$$\mathrm{Im}\, a_{sk} = \mathrm{Im}\, a_{pk} = 0. \tag{19}$$

For these purposes, using the definitions (5d) and (5a,b), we rewrite $Z^{(0)}_{ij}$, $Z^{(\xi)}_{ij}$ and $M^{(\pm\xi)}_{ij}$ in the explicit form:

$$Z_{ij}^{(0)} = a_{si}^* a_{sj} + a_{pi}^* a_{pj} \quad , \tag{20a}$$

$$Z_{ij}^{(\alpha)} = a_{si}^* a_{pj} + a_{pi}^* a_{sj} \quad , \tag{20b}$$

$$Z_{ij}^{(\beta)} = i\,(a_{si}^* a_{pj} - a_{pi}^* a_{sj}), \tag{20c}$$

$$Z_{ij}^{(\gamma)} = a_{si}^* a_{sj} - a_{pi}^* a_{pj} \quad , \tag{20d}$$

$$M_{ij}^{(\pm\xi)} = Z_{ij}^{(0)} \pm Z_{ij}^{(\xi)} \quad . \tag{20e}$$

Now, it is easy to see that the constraints (19) alone imply

$$\mathrm{Im}\, Z_{ij}^{(0)} = 0, \tag{21a}$$

$$\mathrm{Im}\, Z_{ij}^{(\xi)} = 0, \quad \mathrm{Im}\, M_{ij}^{(\pm\xi)} = 0\,, \quad \xi = \alpha\,, \gamma\,, \tag{21b}$$

$$\mathrm{Re}\, Z_{ij}^{(\beta)} = 0, \quad \mathrm{Re}\, M_{ij}^{(\pm\beta)} = \mathrm{Re}\, Z_{ij}^{(0)} \quad , \mathrm{Im}\, M_{ij}^{(\pm\beta)} = \pm \mathrm{Im}\, Z_{ij}^{(\beta)} \tag{21c}$$

for any $i,j = 1,2,3$. Then, the predictions (7b,c,d,e,f) of the sum rules (1) imply that (21a,b,c) are equivalent to

$$\bar{K} = -\frac{1}{4}\lambda(\sigma), \quad \lambda_\xi^{(+)} = \lambda_\xi^{(-)}, \quad \xi = \alpha\,,\beta\,,\gamma, \tag{22a}$$

$$\bar{K} = \frac{1}{4}\lambda(\xi\sigma), \quad \lambda_\xi^{(+)} = \lambda_\xi^{(-)} = 0\,, \quad \xi = \alpha\,, \gamma \tag{22b}$$

and

$$\frac{1}{4}\lambda(\beta\sigma) = -\{\beta_i c_i^2 \beta_j c_j^2 \sigma_i\, \sigma_j\}, \quad \lambda_\beta^{(+)} = \lambda_\beta^{(-)} = \lambda(\sigma) \tag{22c}$$

respectively. We note that the constraints (22c) are identically satisfied when $\beta_k = 0$ are required by (19).

In order to compare the predictions (21a,b,c) with the available experimental data for Σ decay we have estimated $\mathrm{Re}\, N_{+-}$ and $|\mathrm{Im}\, N_{+-}|$, $N_{+-} \equiv Z_{+-}^{(0)}$, $Z_{+-}^{(\xi)}$, for $\xi = \alpha, \beta, \gamma$ using eqs. (7d,e) and

$$\mathrm{Re}\, Z_{ij}^{(0)} = (2 c_i c_j)^{-1} (c_k^2 \sigma_k - c_i^2 \sigma_i - c_j^2 \sigma_j), \tag{23a}$$

$$\mathrm{Re}\, Z_{ij}^{(\xi)} = (2 c_i c_j)^{-1} (c_k^2 \xi_k \sigma_k - c_i^2 \xi_i \sigma_i - c_j^2 \xi_j \sigma_j), \tag{23b}$$

where the coefficients c_ℓ , $\ell = +,-,0$ are determined according to the sum rules (9). The values of these quantities, as well as the values of $\mathrm{Re}\, N'_{+-}$ and $|\mathrm{Im}\, N'_{+-}|$ corresponding to σ'_{01} , ξ'_{01} from (18a), are shown in table VI. We have chosen N_{+-} since the effects of the $\Delta I = 1/2$ breaking are expected to be small only in these quadratic forms. However, to estimate these effects we have calculated $|\mathrm{Im} N_{+-}|$ using different $\Delta I = 1/2$ predictions (see the last column of table VI).

Therefore, from tables II, III and VI we see that the deviations from the predictions (21a,b,c) or equivalently (22a,b,c) are in general higher than the values of $T_{2,3}\,(\xi)$ $T_{4,5}\,(\pm\xi)$ and are up to 50% from $T_1(+0)$ and $T_1\,(-0)$. Moreover, we observe that these deviations (see the values of $|\mathrm{Im}\, Z_{+-}^{(0)}|$ from table VI) are higher than: $|\mathrm{Re}\, Z_{+-}^{(0)}| = 0.016$, $|\mathrm{Re}\, Z_{+-}^{(\gamma)}| = 0.353$, $|\mathrm{Re}\, M_{+-}^{(+\gamma)}| = 0.369$, and $|\mathrm{Re}\, M_{+-}^{(-\gamma)}| = 0.337$. In consequence, if the decay amplitudes are taken to be real, then the quantities comparable to $\Delta I = 1/2$ breaking effects and parity-violating $(\mathrm{Re}\, a_{s+})$ parity-conserving $(\mathrm{Re}\, a_{p-})$ contributions are neglected.

Next, from table VI, we remark that it is more appropriately to consider the constraints:

$$\mathrm{Re}\, Z_{+-}^{(0)} = 0 \qquad \text{and} \qquad \mathrm{Im}\, Z_{+-}^{(\xi)} = 0 \qquad \text{for } \xi = \alpha, \gamma \tag{24}$$

as being in reasonable agreement to experimental data.

Table VI

The values of Re N_{+-} , Re N'_{+-} , $|\mathrm{Im}\, N_{+-}|$, $|\mathrm{Im}\, N'_{+-}|$ calculated from the experimental data and (18a) using the relations (23a,b) and the last column for $|\mathrm{Im}\, N_{+-}|$ ($|\mathrm{Im}\, N'_{+-}|$).

N_{+-}	$\mathrm{Re}\, N_{+-}$	$\mathrm{Re}\, N'_{+-}$	$\|\mathrm{Im}\, N_{+-}\|$	$\|\mathrm{Im}\, N'_{+-}\|$	The relations for $\|\mathrm{Im}\, N_{+-}\|$
$Z_{+-}^{(\sigma)}$	−0.016	+0.049	0.535	0.532	$[-K_{+-}-\frac{1}{4}\lambda(\sigma)]^{1/2}$
			0.462		$[-\bar{K}-\frac{1}{4}\lambda(\sigma)]^{1/2}$
			0.480	0.543	$\frac{1}{2}[-\frac{1}{4}\lambda_\alpha^{(+)}]^{1/2}+\frac{1}{2}[-\frac{1}{4}\lambda_\alpha^{(-)}]^{1/2}$
			0.521	0.516	$\frac{1}{2}\|[-\frac{1}{4}\lambda_\beta^{(+)}]^{1/2}-[-\frac{1}{4}\lambda_\beta^{(-)}]^{1/2}\|$
			0.467	0.539	$\frac{1}{2}[-\frac{1}{4}\lambda_\gamma^{(+)}]^{1/2}+\frac{1}{2}[-\frac{1}{4}\lambda_\gamma^{(-)}]^{1/2}$
$Z_{+-}^{(\alpha)}$	2.590	2.565	0.105	0.272	$[\bar{K}-\frac{1}{4}\lambda(\alpha\sigma)]^{1/2}$
			0.062	0.249	$\frac{1}{2}\|[-\frac{1}{4}\lambda_\alpha^{(+)}]^{1/2}-[-\frac{1}{4}\lambda_\alpha^{(-)}]^{1/2}\|$
$Z_{+-}^{(\beta)}$	0.214	0.229	2.640	2.624	$[\bar{K}-\frac{1}{4}\lambda(\beta\sigma)]^{1/2}$
			2.626		$[K_{+-}-\frac{1}{4}\lambda(\beta\sigma)]^{1/2}$
			2.629	2.628	$\frac{1}{2}[-\frac{1}{4}\lambda_\beta^{(+)}]^{1/2}+\frac{1}{2}[-\frac{1}{4}\lambda_\beta^{(-)}]^{1/2}$
$Z_{+-}^{(\gamma)}$	−0.353	+0.225	0.090	0.167	$[\bar{K}-\frac{1}{4}\lambda(\gamma\sigma)]^{1/2}$
			0.063	0.145	$\frac{1}{2}\|[-\frac{1}{4}\lambda_\gamma^{(+)}]^{1/2}-[-\frac{1}{4}\lambda_\gamma^{(-)}]^{1/2}\|$

In this case, from

$$\frac{1}{4}\lambda_\xi^{(+)} - \frac{1}{4}\lambda_\xi^{(-)} = 2\sum_{i\neq j\neq k} c_i^2 c_j\, c_k \xi_i \sigma_i\, \mathrm{Re}\, Z_{jk}^{(0)} = 2\sum_{i\neq j\neq k} c_i^2 c_j c_k \sigma_i\, \mathrm{Re} Z_{jk}^{(\xi)}, \tag{25}$$

and (23a,b) it follows that constraints such as

$\mathrm{Re}\, Z_{ij}^{(0)} = 0$ (or $\mathrm{Re}\, Z_{ij}^{(\xi)} = 0$) and $\mathrm{Im} Z_{ij}^{(\xi)} = 0$

[equivalent to $\lambda_\xi^{(+)} - \lambda_\xi^{(-)} = 0$, see (7d,e)] imply the mirror symmetry

$$\xi_i = -\xi_j \ . \tag{26}$$

Therefore, using the sum rules (9) and the constraints (24) we prove the mirror symmetry

$$\alpha_+ = -\alpha_- \qquad \text{and} \qquad \gamma_+ = -\gamma_- \tag{27}$$

which is in remarkable agreement with the experimental data for $\Sigma_\pm^\pm$ decay modes (see table I).

5. Conclusions

In this paper we have proved (Sect. 2) that general triangular relationships (1) on the decay amplitudes imply the equalities (3a,b,c,d,e) in terms of experimental observables. These predictions are sufficient to obtain certain tests for these relations when complete and accurate experimental data for mean lives, branching ratios and decay asymmetry parameters (α,β,γ) are available. Then, numerical results on the test quantities (12a,b,c,d,e), derived from $\Delta I = 1/2$ rule for Σ decay, are presented in Sect. 3 (see tables II and III). We find that the $\Delta I = 1/2$ breaking effects are of 14.3% from $a_{s,p}(\Sigma_0^+)$ and equal but of oposite sign for both the parity-violating and the parity-conserving decay amplitudes. Unfortunately, with the information given in ref. /1/ it is impossible to obtain a consistent treatment of the errors on the test quantities because of the presence of off-diagonal terms in the error matrix.

The supplementary contstraints on the experimental data, when the decay amplitudes are taken to be real, are given by the relations (22a,b,c) from Sect. 4. Numerical results for a comparative study of these predictions are presented in table VI. We have found that it is more appropriate to consider $\mathrm{Re}\, Z_{+-}^{(0)} = 0$ and $\mathrm{Im}\, Z_{+-}^{(\xi)} = 0$,

$\xi = \alpha, \gamma$ as additional constraints for Σ decay amplitudes. In this case we prove that $\alpha_+ = -\alpha_-$ and $\gamma_+ = -\gamma_-$. This mirror symmetry is in a good agreement with the experimental data. Therefore, since the departures from the exact predictions (21a,b,c) or (22a,b,c) are in general higher than the values of the test quantities $T_{2,3}(\xi)$ and $T_{4,5}(\pm\xi)$ and are up to 50% from $T_1(i,0)$, $i=+,-$ we can conclude that the reliable tests and the equations for the $\Delta I = 1/2$ breaking parameters are those resulting from the test quantities (12a,b,d,e). We note of course that more accurate experiments with better statistics are needed.

We remark that the results of Sect. 2 and 3 are particularly important for testing the Lee-Sugawara relations [4,5] or different models such as vector-meson dominance model [6] which predicts the relation $(\Lambda \to n\gamma) - 2(\Xi \to \Lambda\gamma) = \sqrt{3}(\Sigma \to p\gamma)$ for both the parity-violating and the parity-conserving amplitudes.

Finally, we note that our results (Sect. 2) are of great interest for a systematic study of the breaking effects of the isospin invariance, $SU(3)$ -symmetry [7], quark models [8], etc., when complete experimental data for $(0\,1/2 \to 0'1'/2)$ reactions are available. If some experimental data are lacking then an investigation of the constraints on data and amplitude analysis, resulting from the triangular relations, can be improvized using the bounds (see ref. [9,10]) derived from the inequalities of form (4a,b,c) and their "integrated" analogous [10].

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