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A THEORETICAL INTERPRETATION OF THE NUCLEON FORM FACTOR DEPENDENCE ON THE MOMENTUM TRANSFER

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In papers [1] the advantages were studied of the description of the nucleon spatial distribution through the use of the relativistic configurational representation introduced in ref. [2]. Instead of the Fourier transform for transition to a new coordinate representation there is taken the expansion over the principal series of unitary irreducible representations of the Lorentz group [3]. Due to the spherical symmetry this transformation for a form factor F(t) has the form

$$F(t) = 4\pi \int_0^\infty \frac{\sin rMy}{rM\,\mathrm{sh}\,y} F(r) r^2 dr, \tag{1}$$

where M is the nucleon mass and the hyperbolic angle y is the rapidity corresponding to the momentum transfer $t = (p-k)^2$: $y = \mathrm{Ar\,cosh}\left(\frac{2M^2-t}{2M^2}\right)$. As has been shown [1], the nucleon invariant mean-square radius can be defined as an eigenvalue of the Casimir operator of the Lorentz group $\hat{C} = \vec{N}^2 - \vec{M}^2$. In terms of an invariant-function F(r) describing the spatial distribution in an arbitrary reference frame (and not only in the Breit one) this radius is defined as follows *)

$$\langle r_0^2 \rangle \equiv \frac{6 \frac{\partial F(t)}{\partial t}\big|_{t=0}}{F(0)} = \frac{\{\hat{C} F(t)\}\big|_{t=0}}{F(0)} = \frac{\hbar^2}{M^2 c^2} + \langle r^2 \rangle, \tag{2}$$

where

$$\langle r^2 \rangle = \frac{\int r^2 F(r)\, d\vec{r}}{\int F(r)\, d\vec{r}} \tag{3}$$

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*) Here the units $\hbar$ and c are restored for a more clear representation.

From eq. (2) it follows that the squared distance from the particle center is defined as the eigenvalue χ^2 of the Casimir operator $\hat{C} = \chi^2$ where for the principal series $\chi^2 = \frac{1}{M^2} + r^2$; $0 \le r < \infty$.

The square of the nucleon Compton wave length $\frac{\hbar^2}{M^2c^2}$ is small as compared with the experimentally measured value of $\langle r_0^2 \rangle_N$. This means that the quantity $\langle r^2 \rangle$, eq. (3), for nucleon should be positive *) . It can be considered that both the relativistic coordinate r and the function F(r) describe only the part of nucleon which is outside the sphere of radius of the nucleon Compton wave length. Really, as is clear from (2), to the sphere of $R = \frac{\hbar}{Mc}$ there corresponds $r=0$, i.e. $F(r) = \frac{\delta(r)}{4\pi r^2}$. Substitution of this F(r) into (1) gives the expression

$$F_{R=\frac{\hbar}{Mc}}(t) = \frac{\sin rMy}{rM\sinh y}\Big|_{r=0} = \left(\frac{y}{\sinh y}\right) = 2M^2\frac{\ln\left(1-\frac{t}{2M^2}+\frac{1}{2M^2}\sqrt{t(t-4M^2)}\right)}{\sqrt{t(t-4M^2)}}, \tag{4}$$

which is the contribution of the central part with $R = \frac{\hbar}{Mc}$. Accordingly, the form factor F(t) is represented as $F(t) = \left(\frac{y}{\sinh y}\right)\Phi(y)$ where the "outer" form factor $\Phi(y)$ corresponds to the nucleon distribution outside the sphere of $R = \frac{\hbar}{Mc}$. The factor $\left(\frac{y}{\sinh y}\right)$ and central sphere of $R = \frac{\hbar}{Mc}$ corresponding to it have no nonrelativistic analog as for $c \to \infty$ $\left(\frac{y}{\sinh y}\right) \xrightarrow[c\to\infty]{} 1$ and $R = \frac{\hbar}{Mc} \xrightarrow[c\to\infty]{} 0$. Note that such a separation of the central part contribution is not always possible. It is based on that for $\langle r^2 \rangle > 0$ the new coordinate is interpreted as the one describing the particle

*) To achieve this it is necessary that the function F(r) be of constant sign.

distribution at distances larger than the Compton wave length. As is known, the pion form factor is well described by the vector dominance (VD) model where $F_\pi(t) = \frac{m_\rho^2}{m_\rho^2 - t}$. The form of such a relativistic propagator in the r-space depends considerably on a relation between the mass m_ρ and the mass of the particle itself M_π [2]. Therefore from formulae (1),(3) for $\langle r^2 \rangle$ we obtain:

$$\langle r^2 \rangle = \frac{6M_\pi^2 - m_\rho^2}{M_\pi^2 m_\rho^2}. \tag{5}$$

It is easy to see that the propagator gives the negative value of $\langle r^2 \rangle$ for pion (and the positive one for nucleon when $M_\pi \to M_N$). Consequently, for pion $\langle r_0^2 \rangle = \frac{1}{M_\pi^2} - |\langle r^2 \rangle|$ and the above interpretation admitting to separate the contribution of the nucleon central part is not possible. Thus, for the nucleon of which the mean-square radius is larger than its Compton wave length it is possible to factorize the central part contribution (with $R = \frac{\hbar}{Mc}$). This difference of nucleon from pion suggests the idea that when using the VD model for description of the nucleon form factor it is necessary to allow for this additional contribution. As a results, for the nucleon form factor we obtain the following expression:

$$F_N(t) = \left(\frac{y}{\sinh y}\right)\sum_V \frac{m_V^2}{m_V^2 - t}. \tag{6}$$

For large $-t$ the factor $\left(\frac{y}{\sinh y}\right)$ decreases by the law $\left(\frac{y}{\sinh y}\right) \xrightarrow[|t| \gg M^2]{} \frac{\ln |t|/M^2}{|t|}$ according to (4), that gives the correct "almost dipole" asymptotic behaviour $F_N(t) \xrightarrow[|t| \gg M^2]{} \frac{\ln |t|/M^2}{t^2}$ for the form factor. It is interesting to note that at $|t|$ up to 1 (GeV/c) the factor $\left(\frac{y}{\sinh y}\right)$ is about unity. It is just the region where the pure VD model is consistent with experimental data.

The above considered case of nucleon is on a distinct status for its form factor is a square-integrable function, i.e., $\int_0^\infty |F_N(t)|^2 sh^2 y \, dy < \infty$ (as is seen from the dipole formula fitting the experimental data). By the theorem proven in [5] such functions are expanded over representations of the principal series only (see also [6]), i.e., formulae of transition to the coordinate space are of the form of (1).

The form factors which are not square-integrable functions, i.e., which decrease as $\frac{1}{t}$ and more slowly at large $-t$ are expanded in a direct sum of the principal and complementary series [4]. When allowing for the complementary series the eigenvalues of the Casimir operator $\hat{C}=\chi^2$, i.e., the squared distance from the particle center, are not bounded from below by $\frac{\hbar^2}{M^2c^2}$:

$$\hat{C} = \chi^2 = \begin{cases} \frac{1}{M^2} + r^2 & 0 \le r < \infty \quad \text{for the principal series} \\ \frac{1}{M^2} - \rho^2 & 0 \le \rho \le \frac{1}{M} \quad \text{for the compl. series} \end{cases} .$$

The parameter ρ can be treated as the coordinate describing the sphere of $R=\frac{\hbar}{Mc}$ measured from the sphere boundary to its center. To the particle localized at the center, i.e., $\chi=0$, there corresponds $\rho=\frac{\hbar}{Mc}$. In this case for elementary spherical functions (with $\ell=0$) of the complementary series $\frac{\sinh \rho M y}{\rho M \sinh y}$ [5] the equality $\frac{\sinh \rho M y}{\rho M \sinh y}\Big|_{\rho=\frac{\hbar}{Mc}} = 1$ holds which results in F(t)=1 (unlike (5) in the nucleon case).

Thus it can be expected that for the particles with the mean square radius larger than their Compton wave length the

form factor will decrease more rapidly than $\frac{\ln |t|/M^2}{|t|}$ and vice versa for the particles with $\langle r_0^2 \rangle < \frac{\hbar^2}{M^2c^2}$ it will decrease more slowly than $\frac{\ln |t|/M^2}{|t|}$.

The main result of this paper is formula (6) for the nucleon form factor which provides the correct asymptotic behaviour of the form factor in the VD model with ρ, ω and φ mesons without extra ρ' and ω' mesons [7]. Detailed calculations of the nucleon form factor in the VD model formula (6) will be presented in a subsequent paper.

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