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ON THE PROBLEM OF AXIAL ANOMALY IN SUPERSYMMETRIC GAUGE THEORIES

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1. The problem of axial anomaly in supersymmetric gauge theories has recently attracted close attention /1-6/. The heart of the problem is that according to the Adler-Bardeen theorem the divergence of axial current contains only the one-loop contribution /7/

$$\partial_\mu j_\mu^5 = -\frac{\alpha N}{4\pi} \mathcal{F}_{\mu\nu} \widetilde{\mathcal{F}}_{\mu\nu} \tag{1}$$

(N belongs to the SU(N) gauge group), whereas the superconformal anomaly, including the axial one, is proportional to the β--function, i.e., receives contributions from all orders in α /8/

$$\partial_\mu J_\mu^5 = \frac{1}{3} \frac{B(\alpha)}{\alpha} \left[F_{\mu\nu} \widetilde{F}_{\mu\nu} + 2 \partial_\mu J_\mu^5 \right] . \tag{2}$$

Another source of discrepancy is the anomalous dimension of the current. For the AB current j_μ^5 it is not zero. At the same time the axial current J_μ^5 is the first component of the supermultiplet including also the energy-momentum tensor which, evidently has zero anomalous dimension. This appears to be inconsistent with a multiplet structure of the anomalies.

The solution of the paradox is concealed in the essence of renormalization procedure. The discrepancies are removed if one takes into account the fact that we have two different renormalization prescriptions. The AB theorem takes place only in a special scheme which is not supersymmetric. The quantum operators in the supersymmetric scheme and in the Adler-Bardeen one do not coincide, but are connected by finite multiplicative transformations. This is true not only for the currents, but also for the $F\widetilde{F}$ operators entering into the anomaly expression. (1),(2). The account of current renormalization only, as it will be shown, is not enough and leads to a number of contradictory statements in the literature /1-4/.

2. Consider the renormalized operators involved in the axial anomaly equations (1),(2). They obey the standard renormalization group equations written down in the corresponding renormalization scheme:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a} - \hat{\gamma}(a)\right]\begin{pmatrix}\partial_\mu j^5_\mu(a)\\ \mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}(a)\end{pmatrix} = 0, \tag{3}$$

$$\left[\mu\frac{\partial}{\partial\mu} + B(A)\frac{\partial}{\partial A} - \hat{\Gamma}(A)\right]\begin{pmatrix}\partial_\mu J^5_\mu(A)\\ F_{\mu\nu}\tilde{F}_{\mu\nu}(A)\end{pmatrix} = 0, \tag{4}$$

where $\beta(a)$ and $B(A)$ are the β-functions, $a \equiv \frac{\alpha_{AB}N}{2\pi}$ and $A \equiv \frac{\alpha_{SS}N}{2\pi}$ are the renormalized couplings, and the matrices of anomalous dimensions have the triangular form /6,9/:

$$\hat{\gamma} = \begin{pmatrix}\gamma_{11} & 0\\ \gamma_{21} & \gamma_{22}\end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix}\Gamma_{11} & 0\\ \Gamma_{21} & \Gamma_{22}\end{pmatrix}. \tag{5}$$

These matrices may be found by direct computation, however, some general properties follow already from the renormalization invariance of anomalies /6,9/:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a}\right]\left\{\partial_\mu j^5_\mu + \frac{a}{2}\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}\right\} = 0, \tag{6}$$

$$\left[\mu\frac{\partial}{\partial\mu} + B(A)\frac{\partial}{\partial A}\right]\left\{\partial_\mu J^5_\mu - \frac{B(A)}{3A}\left(F_{\mu\nu}\tilde{F}_{\mu\nu} + 2\partial_\mu J^5_\mu\right)\right\} = 0. \tag{7}$$

Combining eqs.(3) and (6) and eqs. (4) and (7) we obtain the matrices (5) in the form

$$\hat{\gamma}(a) = \begin{pmatrix}\gamma_{11} & 0\\ -\frac{2\gamma_{11}}{a} & -\frac{\beta(a)}{a}\end{pmatrix}, \quad \hat{\Gamma}(A) = \begin{pmatrix}0 & 0\\ -2A\left(\frac{B}{A}\right)' & -A\left(\frac{B}{A}\right)'\end{pmatrix}, \tag{8}$$

where we have taken into account the fact that the anomalous dimension of the SS current $\Gamma_{11}=0$ in SS scheme. On the contrary, in the AB scheme the anomalous dimension vanishes only in one-loop order being non-zero and equal to /9,2/ $\gamma_{11}(a) = -\frac{3}{2}a^2 + O(a^3)$.

3. Starting from the general statements of the renormalization theory we conclude that the renormalized operators of two different schemes are related by finite multiplicative renormalization of the form

$$\begin{pmatrix}\partial_\mu J^5_\mu(A)\\ F_{\mu\nu}\tilde{F}_{\mu\nu}(A)\end{pmatrix} = \begin{pmatrix}S_{11}(A) & 0\\ S_{21}(A) & S_{22}(A)\end{pmatrix}\begin{pmatrix}\partial_\mu j^5_\mu(a)\\ \mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}(a)\end{pmatrix}, \tag{9}$$

where $A = Z(A)a$.

With account of eq.(9) the consistency condition for eqs.(1) and (2) takes the form

$$B(A) = -\frac{3A^2}{2}\frac{S_{11}}{S_{22}Z}\left(1 - A\,\frac{S_{11}+\frac{1}{2}S_{21}}{S_{22}Z}\right)^{-1}. \tag{10}$$

Note that the first two coefficients of the β-function are scheme-invariant and are equal to $B(A) = -\frac{3}{2}A^2(1 + A + \cdots)$.

Combining now eqs. (3),(4) and (9) we get for the matrix $\hat{S}$ the equation

$$B(A)\frac{d\hat{S}(A)}{dA} = \hat{\Gamma}\hat{S} - \hat{S}\hat{\gamma} \tag{11}$$

with the initial condition $\hat{S}(0) = \mathbb{1}$.

With account of (8) eq. (11) has the following solution

$$\hat{S}(A) = \begin{pmatrix}e & 0\\ \frac{2B_0A}{B(A)}(1-e) - 2e & \frac{B_0A^2}{B(A)Z(A)}\end{pmatrix}, \tag{12}$$

where $B_0 = -3/2$ is the first β-function coefficient and $e \equiv \exp\left(-\int_0^A \gamma_{11}(Z^{-1}A)/B(A)\,dA\right)$.

Here we have taken into account the relation between the β-functions of two schemes:

$$\frac{\beta(a)}{a} = \frac{B(A)}{A}\left(1 - \frac{Z'(A)}{Z(A)}A\right). \tag{13}$$

Note that to satisfy the initial condition, it is essential that the expansion of γ_{11} starts from $-\frac{3}{2}a^2$.

Substituting eq.(12) into eq.(10) we convince ourselves that eq. (10) is satisfied identically for any γ_{11}, B and Z in all orders of PT.

4. Thus, the explicit transformation is found which converts the operators from the Adler-Bardeen scheme into the supersymmetric one. The axial anomalies (1) and (2) are consistent. The anomalous dimensions of the two currents are different. There arises no restriction on the form of the β-function. The constant $Z(A)$ is not specified and may be found, for example, from eq. (13). Hence, the transformation (9), (12) solves the problem of axial anomaly.

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К проблеме аксиальной аномалии в суперсимметричных калибровочных теориях

Найдена явная связь между оператором аксиального тока, удовлетворяющим теореме Адлера-Бардина, и суперсимметричным аксиальным током. При этом аксиальная и суперконформная аномалии взаимосогласованы во всех порядках теории возмущений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Problem of Axial Anomaly in Supersymmetric Gauge Theories

The explicit relation is found between the axial current obeying the Adler-Bardeen theorem and the supersymmetric one belonging to a supermultiplet. It is shown that the axial and superconformal anomalies are consistent in all orders of perturbation theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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