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ON THE ROLE OF A RECOIL EFFECT IN THE BAG MODEL

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1. An original version of the bag model is known to be the quasi-independent quark model [1]. Its results are in a satisfactory agreement with experimental values of the proton magnetic moment, the ratio of the axial and vector coupling constants, and a number of other static values. Later on, this model has been further developed in ref. [2], where it has been formulated in relativistic-invariant form, and additional types of interaction between quarks have been taken into account [3].

An exact solution of equations of motion in the four-dimensional bag model is unknown. So, it is important to find approximate solutions. Usually one uses a solution in the static-cavity approach [3] for practical calculations. But in this approach an initial translational invariance is violated because there exists a classical object (the cavity), and as a consequence, momentum is not conserved. This causes a number of difficulties for the bag model in describing the structure functions of scattering [4] and in taking the recoil corrections to static properties of hadrons into account.

The present paper is devoted to the calculation of corrections to the proton electromagnetic radius and magnetic moment due to the recoil effect. For this purpose, we use a relativistic-covariant approximate solution of the Friedberg-Lee soliton model [5] obtained in ref. [6]. To derive such a solution, a general method of Bogolubov's canonical transformations [7,8] is applied. This method allows us to construct solutions of quantum models taking into account their initial symmetries.

2. The Lagrangian of the model with an interaction between a quark field $\Psi(\vec{x},t)$ and a scalar field $\sigma(\vec{x},t)$ has the form [5]

$$L = g^{-1}\int d^4x \left\{ \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}(\vec{\nabla}\sigma)^2 - V(\sigma) - \bar{\Psi}(-i\gamma^{\mu}\partial_{\mu} + h\sigma)\Psi \right\}, \qquad (1)$$

where g is a formal parameter, $V(\sigma) = a\sigma^4 + b\sigma^3 + c\sigma^2 + p,\ c > 0$.

The function $V(\sigma)$ has an absolute minimum at the point $\bar{\sigma}$, $V(\bar{\sigma}) = 0$, and a local one at zero: $V(0) = p$. Such a choice of $V(\sigma)$ corresponds to an assumption of the existence of two phases of vacuum. Values of $V(\sigma)$ at any point except $\bar{\sigma}$ and zero are of an order of $\mathcal{O}(m^4)$, where m is a mass of the scalar field. The field σ is responsible for the formation of the bag. The field Ψ and σ are determined in any space-time point and obey the standard commutation relations

$$[\sigma(\vec{x}, t), \sigma(\vec{y}, t)] = ig\delta(\vec{x} - \vec{y}), \tag{2}$$

$$\{\Psi(\vec{x}, t), \Psi^+(\vec{y}, t)\} = g\delta(\vec{x} - \vec{y}). \tag{3}$$

3. Bogolubov's transformation makes a substitution of the dynamical field variables in such a way as to emerge parameters of a symmetry group (collective coordinates) as new dynamical variables. The collective coordinates are introduced only for those symmetry transformations which are spontaneously broken. We find a solution of the model with the broken space translational invariance. Namely, the space of the collective variables is considered to be isotropic. As a consequence of this consideration, a solution of the zeroth order in the constant $\sqrt{g}$ is spherically symmetric. Consider a motion of a bag along an arbitrary axis (e.g., the z axis). An algebra of operators in the space of collective variables is defined by a set of $\{H, P_z, \vec{P}_{x,y}, K, L_z\}$ with the following commutation relations between its elements:

$$[H, P_z] = 0, \quad [H, K] = igH, \quad [H, L_z] = -igP_z,$$
$$[K, L_z] = 0, \quad [P_z, K] = igP_z, \quad [P_z, L_z] = -igH, \tag{4}$$
$$[\vec{P}_\perp, H] = [\vec{P}_\perp, P_z] = [\vec{P}_\perp, L_z] = [\vec{P}_\perp, K] = 0.$$

Moreover, introduce the following new variables:

$$\hat{\tau}(\vec{x}, t) = Ht - P_z z - K, \quad \hat{\xi}(\vec{x}, t) = Hz - P_z t - L_z, \quad \hat{\vec{\eta}}_\perp(\vec{x}) = \vec{x}_\perp - \vec{q}_\perp, \tag{5}$$

where coordinates $\vec{q}_\perp$ are canonically conjugated to $P_\perp$:

$$[q^i_\perp, P^j_\perp] = i\delta^{ij}.$$

It is necessary to note that the functions $\hat{\tau}$, $\hat{\xi}$, and $\hat{\vec{\eta}}_\perp$ commute with each other. Bogolubov's transformation with the help of these variables is written as /9/

$$\sigma(\vec{x}, t) = \tilde{\sigma}(\hat{\xi}(\vec{x}, t), \hat{\vec{\eta}}_\perp(\vec{x}), \hat{\tau}(\vec{x}, t)) \equiv$$
$$\equiv \int df\, d\vec{f}_\perp\, d\omega \exp[i(\omega\hat{\tau} - \vec{f}_\perp\hat{\vec{\eta}}_\perp - f\hat{\xi})]\, \rho(f, \vec{f}_\perp, \omega), \tag{6}$$

$$\Psi(\vec{x}, t) = \tilde{\psi}(\hat{\xi}(\vec{x}, t), \hat{\vec{\eta}}_\perp(\vec{x}), \hat{\tau}(\vec{x}, t))D(\theta) \equiv$$
$$\equiv \int df\, d\vec{f}_\perp\, d\omega \exp[i(\omega\hat{\tau} - \vec{f}_\perp\hat{\vec{\eta}}_\perp - f\hat{\xi})]\, \Phi(f, \vec{f}_\perp, \omega)\, D(\theta). \tag{7}$$

The matrix $D(\theta)$ is an irreducible spinor part of the representation of the Lorentz group and has the form $D(\theta) = \exp\{i\theta(\vec{P})\sigma_{0z}\}$, where $\sigma_{0z} = 1/2[\gamma_0, \gamma_z]$ is a generator of the Lorentz transformation of the spinor field along the z axis.

It is not difficult to check up that the fields σ and Ψ expressed via the variables $\hat{\tau}$, $\hat{\xi}$, $\hat{\vec{\eta}}_\perp$ in (6,7) are properly transformed under the space-time translations and boost transformation along the z axis. A form of unknown functions ρ and Φ is found from the equations of motion for the fields σ and Ψ, validity of the commutation relations (2,3) and the operator constraints that are defined on state vectors $|\Psi\rangle$:

$$\{P_\mu - \hat{P}_\mu(\sigma, \Psi)\}|\Psi\rangle = 0, \quad \{L_z - \hat{L}_z(\sigma, \Psi)\}|\Psi\rangle = 0, \quad \{Q - \hat{Q}(\Psi)\}|\Psi\rangle = 0. \tag{8}$$

Here $\hat{P}$, $\hat{L}_z$, $\hat{Q}$ are the operators of four-momentum, boost and charge, respectively, constructed as functions of the fields σ and Ψ by Noether's theorem. The restrictions (8) ensure the number of dynamic variables unchanged after the transformations (6,7).

4. As a result, we get a covariant field function for the lowest-energy states ($\ell = 0$, $\mathcal{J}_z = -\frac{1}{2}$) to the first order in the constant $\sqrt{g}$:

$$\Psi_1(\vec{x}, t) = \int df\, d\vec{f}_\perp\, d\omega \exp\{i(\omega\hat{\tau}(\vec{x}, t) - \vec{f}_\perp\hat{\vec{\eta}}_\perp(\vec{x}) - f\hat{\xi}(\vec{x}, t))\}\Phi_1(\omega, \vec{f}_\perp, f)\times D(\theta(\vec{P})),$$

$$\Phi_1(\omega, \vec{f}) = \frac{1}{(2\pi)^4}\int d\beta\, d\vec{a}\, e^{-i(\beta\omega - \vec{f}\vec{a})}Q(\vec{a}, \beta), \tag{9}$$

$$Q(\vec{a}, \beta) = \sum_{m=-\infty}^{\infty} N_m b_m \begin{pmatrix} u_m \\ \ell_m \end{pmatrix} \chi e^{-ik_m\beta}, \quad \{b_m, b_n^+\} = \delta_{mn},$$

where $u_m = j_0(k_m r)$, $\ell_m = ij_1(k_m r)\vec{\sigma}\hat{r}$, $r = \sqrt{\vec{a}^2}$, k_m are energy eigenvalues for the static bag, χ is the Pauli spinor, and N_m are normalization constants. The bag state is a vector $|\Omega_p\rangle$, $P_\mu|\Omega_p\rangle = p_\mu|\Omega_p\rangle$, $p_\mu^2 = M^2$ and vacuum with respect to the secondary-quantized field Q: $b_n|\Omega_p\rangle = 0$.

5. Consider a process of an electromagnetic scattering on a proton in the Breit system. With the aid of the solution (9), the matrix element for this process is presented in the general form by the expression

$$\langle\frac{\vec{q}}{2}|\Psi_1^+(0)\Gamma_\mu\Psi_1(0)| - \frac{\vec{q}}{2}\rangle =$$
$$= M\int\frac{d\alpha}{2\pi}\,\frac{1}{\cosh\frac{\alpha}{2}}\int dt\, dx\, d\vec{x}_\perp e^{-i(\alpha t - \beta x)M}D^+(\theta)Q^+(t, x, \vec{x}_\perp)\Gamma_\mu Q(t, x, \vec{x}_\perp)D(\theta). \tag{10}$$

Here $\beta = 2\,\text{atanh}\,\frac{q/2}{E}$, $E = \sqrt{M^2 + \frac{q^2}{4}}$ and $\Gamma_\mu = \lambda \gamma_0 \gamma_\mu$ (λ is a quark charge). Parameter M is an energy corresponding to the solution of the zeroth-order approximation in the constant $\sqrt{g}$. Taking into account relative motion of quarks inside the bag, it is chosen as [10]

$$M \simeq M_{st} - \frac{\langle P^2_{cm} \rangle}{2M_{st}} = (4 - \frac{9}{8})\, k_0,$$

where $M_{st} = 4k_0$ is the energy of the classical static solution to the equations of motion for the model (1). The electric, magnetic, and axial form factors as functions of q^2,

$$G_E(q^2) = e \int_0^\infty dx\, x^2 j_0(\beta M \cdot x)\,[u^2(x) + \ell^2(x)],$$

$$G_M(q^2) = 2e\{I_1 + \frac{1}{2} I_2\} \equiv 2e\{\int_0^\infty dx \cdot x^2 \frac{1}{(\frac{q/2}{E})} j_1(\beta M x)\, u(x)\,\ell(x) + \tag{11}$$

$$+ \frac{1}{2}\int_0^\infty dx \cdot x^2 [j_0(\beta M x)(u^2(x) - \ell^2(x)) + 2\frac{j_1(\beta M x)}{\beta M x}\ell^2(x)]\},$$

$$G_A(q^2) = I_1 + 2(\frac{q/2}{E})^2 I_2,$$

are presented in Figs.1-3. Expanding the form factors (11) at low speeds ($v = \frac{q/2}{E}$) we obtain the expressions for the proton electromagnetic radius and its magnetic moment bearing the recoil effect in mind

$$\langle r^2 \rangle = \langle r^2 \rangle^{(st)}, \tag{12}$$

$$\mu_p = \frac{M}{M_{st}} \mu_p^{(st)} + \Delta\mu_p, \tag{13}$$

where

$$\Delta\mu_p = \frac{e}{2M} \int_0^\infty dx\, x^2 [u^2(x) - \frac{1}{3}\ell^2(x)]. \tag{14}$$

If the model parameters are fixed by an experimental value of the proton radius $\langle r^2 \rangle^{1/2} = 0.83$ fm, the correction to the magnetic moment value calculated in the static cavity approximation is positive and equals $\Delta\mu_p = 0.65 \frac{e}{2M}$.

6. It should be noted that a number of papers [11,12] are devoted to the description of an electromagnetic scattering pro-

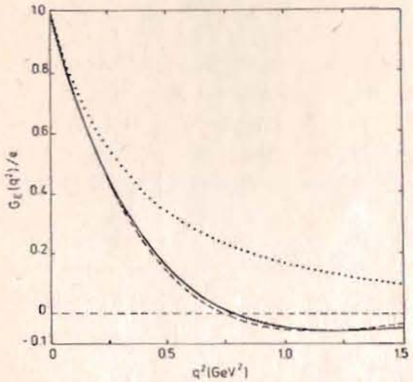

Fig.1. Proton electric form factors. The dotted line denotes the dipole-fit approximation. The static and recoil-corrected results are denoted by the dashed and solid lines, respectively.

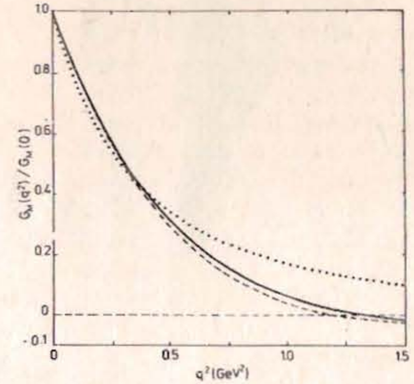

Fig.2. Proton magnetic form factors. Notation of lines is the same as in Fig.1.

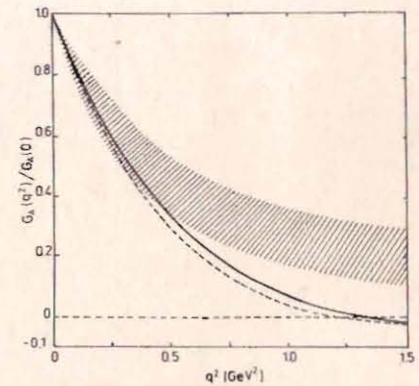

Fig.3. Proton axial-vector form factors. The experimental results are given with the dipole-fit uncertainty.

cess in the bag model. Studying the results of these papers, one may conclude that either the corrections are great and caused by the recoil effect [11] or the relativistic retarding effect compensates them considerably [12]. To remove an ambiguity in the interpretation of the results, in the present paper we use the solution of the model (1) obtained on the basis of the relativistic-covariant perturbation theory. In this approach, the corrections to magnetic moments are seen from formulae (11,12) to emerge large.

It is worth reminding that the Bogolubov canonical transformations method is the most consistent operator method to solve models with symmetry degeneracies. A solution of the two-dimensional bag model obtained by this method has been earlier established [6] to coincide with the known solution of the translationally invariant L_0-approximation [13,14]. In the four-dimensional bag model, no exact solution has been found, and so, it is impossible to make use of the L_0-approximation. Drawing a parallel with the case of the two-dimensional model, one may probably think that the field functions (9) found by the method of canonical transformations serve as solutions to be obtained in the L_0-approximation. Moreover, the use of the covariant solutions (9) enables us to solve problems that arise in calculating the structure functions of the deep inelastic lepton-nucleon scattering we mentioned in the introduction. These results are generally in agreement with the discussions from ref. [14].

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REFERENCES

1. Bogolubov P.N. Ann.Inst. Henri Poincarè, 1968, 8, p.163; Bogolubov P.N. Fiz.Elem.Chast. i Atom.Yadra, 1972, 3, p.144.
2. Chodos A. et al. Phys.Rev., 1974, D9, p.3471.
3. De Grand T. et al. Phys.Rev., 1975, D12, p.2060.
4. Jaffe R.L. Phys.Rev., 1975, D11, p.1953.
5. Friedberg R., Lee T.D. Phys.Rev., 1977, D15, p.1694; ibid, 1977, D16, p.1096.
6. Dorokhov A.E. Teor.Mat.Fiz., 1984, 61, p.64.
7. Bogolubov N.N. Ukr.Mat.Zh., 1950, 2, p.3.
9. Solodovnikova E.P., Tavkhelidze A.N., Khrustalev O.A. Teor.Mat.Fiz., 1972, 11, p.317; Khrustalev O.A., Razumov A.V., Taranov A.Yu. Nucl.Phys., 1980, B172, p.44.
10. Sveshnikov K.A. Teor.Mat.Fiz., 1983, 55, p.361.
11. Donoghue J.F., Johnson K. Phys.Rev., 1980, D21, p.1975; Barnhill M.V. III, Phys.Rev., 1984, D30, p.1126.
11. Picek I., Tadic D. Phys.Rev., 1983, D27, p.665; Hwang W.-Y.P. Z.Phys., 1983, C16, p.327; Wang X.-M., Yin D.C. Phys.Lett., 1984, B140, p.249; Wang. X.-M. Phys.Lett., 1984, B140, p.413.
12. Guichon P.A.M. Phys.Lett., 1983, B129, p.108; Betz M., Goldflam. Phys.Rev., 1983, D28, p.2848; Tegen R., Brockman R., Weise W. Z.Phys., 1983, A307, p.339.
13. Krapchev V. Phys.Rev., 1976, D13, p.329.
14. Jaffe R.L. Ann.of Phys., 1981, 132, p.32.

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Чижов А.В., Дорохов А.Е. E2-84-835

О роли эффекта отдачи в модели мешков

На основе решения солитонной модели мешка Фридберга-Ли, полученного в рамках ковариантной теории возмущений, вычислены формфакторы электромагнитного рассеяния на протоне и связанные с ними статические характеристики протона. Выделены вклады, обусловленные эффектом отдачи. В частности, возникает большая положительная поправка к значению величины магнитного момента, вычисленного в приближении полости.

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On the Role of a Recoil Effect in the Bag Model

The electromagnetic form factors of a proton and its static characteristics are calculated on the basis of the Friedberg-Lee soliton bag model solution obtained in the covariant perturbation theory. The contributions due to the recoil effect are separated. In particular, the correction to the magnetic moment calculated in the cavity approximation turns out to be large and positive.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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