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INDUCED COLOUR DIPOLES IN HADRON-DEUTERON SCATTERING

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1. INTRODUCTION

Investigating the dynamics of the peripheral hadronic interaction one encounters the main unsolved problem of quantum chromodynamics, the problem of colour confinement. Nevertheless, the ideas of QCD became quite fruitful in this area, too, and led to a widely popular phenomenological picture of interaction. When colliding, hadrons exchange colour, and between the coloured objects flying away with a large relative momentum a colour flux tube [1-3] is stretched, which breaks into parts by quark pairs or gluons, produced by the colour field in the tube. The breaking continues until a number of small-mass white clusters are formed, filling a plateau in rapidity. The time of hadronisation (until all final hadrons are formed) is E/μ^2, where E is the energy of the incident hadron, and μ is some characteristic mass parameter. It is an old idea that high energy collisions off nuclei may cast a light on the problem of hadronisation, since the process is not finished inside the nucleus for energies $E > \mu^2 R_A$. In this case we may expect that colour forces show up in some observable effects.

The present paper is devoted to one of such possible effects: we suggest that colour fields excited in the nuclear matter should produce backward nucleons in high energy collisions with nuclei. This contribution is shown to possess salient features that could enable one to disentangle it on the background of other mechanisms.

In the second section of this paper we discuss the mechanism for backward proton production based on colour forces. The idea is explained in terms of classical mechanics. In section 3 cross sections for reactions $hd \to hp_B n$ and $hd \to p_B X$ are calculated. In section 4 a quantum-mechanical description is presented and it is shown that for backward proton momenta $p_B \gtrsim 550$ MeV/c the process goes through dibaryon resonances with separated colour. Parameters of these resonances are estimated and it is suggested that they should show up as structures in the momentum spectrum of backward protons.

In section 5 the contribution of this mechanism to the cross section of elastic backward proton - deuteron scattering is discussed. In the energy dependence of the cross section bumps are expected corresponding to dibaryon resonances.

Section 6 discusses polarisation effects in the cumulative process.

2. COLOUR DIPOLES IN hd SCATTERING. CLASSICAL CASE

In a previous letter [5] we have suggested a new mechanism for backward nucleon production off nuclei at high energies, based on diffractive excitation of colour dipole on deuteron.

The contribution of this mechanism is calculated below in detail for the case of the deuteron and some errors of ref. [5] are corrected.

In our calculations we shall use the colour flux tube model [1-3]. In this model the process is assumed to be adiabatic, i.e., hard gluon radiation is neglected and the colour field, being longitudinal, carries only energy but not momentum. The tension of the tube is found from the hadronic mass spectrum: $\kappa = (2\pi\alpha'_R)^{-1} \approx 1$ GeV/fm, where α'_R is the slope parameter of Regge trajectories. The effective tension of the colour tube in hadron-hadron collisions could appreciable differ from this value, since in these processes colour octets, not triplets fly away, and besides, the adiabatic approximation may turn to be quite crude. The second parameter of the model will also be used: the probability w of the quark-pair production in the colour field of the tube in unit time and per unit length of the tube. This value could be estimated by Schwinger's formula or from the width of heavy resonances [2,3], both giving $w \approx$ ≈ 2 fm^{-2}. One can also estimate these parameters from the momentum distribution of protons in the reaction $pp \to pX$. If the detected proton is in the target fragmentation region, its momentum is approximately equal to that acquired by the target proton under the influence of the tube tension for time interval τ from the collision until the first breaking of the tube. This momentum is $p \approx \kappa\tau$. The average time is determined by the condition $\tau^2 w/2 \approx 1$. On the other hand, p is connected with Feynman's x variable of the leading proton by $p = m(1 - x^2)/2x$ Since $\langle x \rangle \approx 0.5$, we have $\langle p \rangle \approx 1$ GeV/c and $w \approx 2/\tau^2 \approx 2\kappa^2/p^2 \approx 2$ fm^{-2}. Of course, these are only very approximate values, and they should be confronted with experiments in many possible ways.

Let's consider now the interaction of a high energy hadron with a pair of nucleons as described by the space-time picture of fig.1. The dashed lines represent trajectories of white particles; the solid lines, those of coloured particles. At point 1 the incident hadron exchanges colour with the first nucleon of the deuteron and a colour flux tube is stretched between them. The coloured nucleon of mass m starts to accelerate with acceleration κ/m. Note, that as the velocity of the coloured nucleon approaches the velocity of light, the length of the tube stops to increase, approaching the limit $m/\kappa \approx 1$ fm. At point 2 the coloured hadron exchanges its colour with the second nucleon and becomes colourless. The nucleon it-

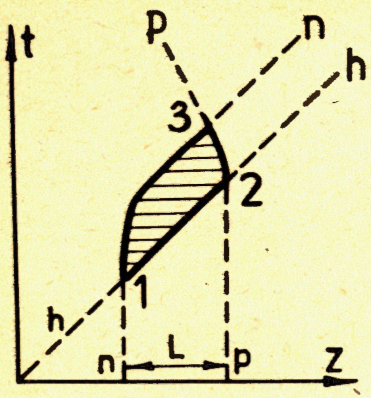

Fig.1. Space-time diagram for the process $hd \to p_B hn$; *solid lines represent trajectories of colour objects, dashed lines - those of colourless objects.*

self starts to accelerate with acceleration κ/m in the direction opposite to the momentum of the incident hadron. This nucleon reaches its maximal momentum p_B at point 3, where the third colour exchange occurs, after which the two nucleons fly away in colourless state.

Generally speaking, the tube could be broken by this time, and the probability of this is near to unity, if the distance between the nucleons in the deuteron is large. This possibility will be considered later; first we discuss the pionless process $hd \to p_B hn$.

The momentum of the backward proton is easily calculated from conditions $\Delta p_h \approx \Delta E_h = \kappa L$ for the incident hadron (assuming that its momentum is large $p_h \gg m$, κL). From these conditions we obtain

$$\frac{2(E-m)}{2m - p_L - E} = \frac{\kappa L}{m}, \qquad (1)$$

where $E = (m^2 + p_L^2 + p_T^2)^{1/2}$ is the energy, and p_L is the longitudinal momentum of the backward proton. As is seen from this expression, the backward momentum p_B grows with distance L between the nucleons and tends to the kinematic bound, which is $p_B^{max} = 3/4$ for $\theta = 180°$.

We note that this increase of significant distances with p_B does not contradict the principles of quantum-mechanics. Larger p_B's are due to production of colour dipoles of a larger mass, hence of a larger relative radius. Nevertheless, in section 4 it will be shown that quantum effects may change this relation between p_B and significant distances.

3. THE REACTION $hd \to p_B X$

As will be explained below, the contribution of the colour dipole production to the cross section for the reaction $hd \to p_B hn$

can be written in the following form:

$$\frac{d\sigma}{dp_L\, d^2p_T} = \frac{\beta\alpha_s^2 B}{8\pi} \exp(-Bp_T^2)\, (\sigma_{in}^{hN})^2\, |\Psi_d(L')|^2\, D(L)\, \frac{dL}{dp_B}. \qquad (2)$$

The cross section for the colour exchange on the first nucleon is σ_{in}^{hN}. The probability of the colour exchange on the second nucleon is given by $\sigma_{in}^{hN} |\Psi_d(L)|^2 dL$. (This factor corresponds to Glauber correction for screening). Note, that the length of the colour tube is less than the distance between the centers of the nucleons; hence we took $L' = L + R_0$ in the argument of the deuteron wave function in (2), where $R_0 = 0.5$ fm is the radius of the nucleon repulsive core.

The probability that no quark pair is created in the colour-electric field of the tube during this process is given in (2) by a factor

$$D(L) = \exp(-w \textstyle\int d\ell\, dt), \qquad (3)$$

where $\int d\ell dt$ is the shaded region in fig.1. Note, that for $L \gg m/\kappa$ we have $D(L) \approx \exp(-wLm/\kappa)$.

The factor 1/8 stands for the relative probability to have all three particles in a colourless state after the three colour exchanges. Since the first two colour exchanges have chosen a deuteron configuration with the nucleons having the same impact parameters, the probability of the third colour exchange contains no additional small factors of the Glauber correction type; it gives only the factor $\beta\alpha_s^2$, where $\alpha_s = g_s^2/4\pi$ is the QCD coupling constant and the numerical coefficient β is estimated below.

The dependence of the cross section (2) on the transversal momentum p_T of the backward proton is written in a normalized Gaussian form with the slope parameter B.

In order to estimate the parameters β and B, let us consider the Feynman diagram shown in fig.2 corresponding to the given process, where the colour transfer is due to one gluon exchange. Naturally, this diagram does not reflect the effects of confinement forces. However, it seems reasonable to assume that these soft forces do not affect the total cross section of reaction, only change the momentum distribution of particles involved. Hence the contribution of the diagram in fig.2 to the cross section $hd \to hpn$ can be compared with the integral of (2) (with the factor D(L) omitted in it), giving the parameters β and B.

In the Appendix A we have calculated the contribution of the three-gluon diagram and found the values $B = 13\ (\mathrm{GeV}/c)^{-2}$, $\beta = 0.17$. Fig.3 shows the backward proton spectrum in the reaction $pd \to p_B pn$ at 180° calculated from (2). We have used the Hamada-Jonston wave function of ref. /6/.

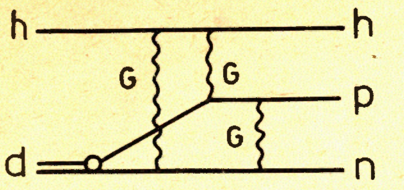

Fig. 2. Three-gluon exchange Feynman diagram for the process $hd \to p_B hn$.

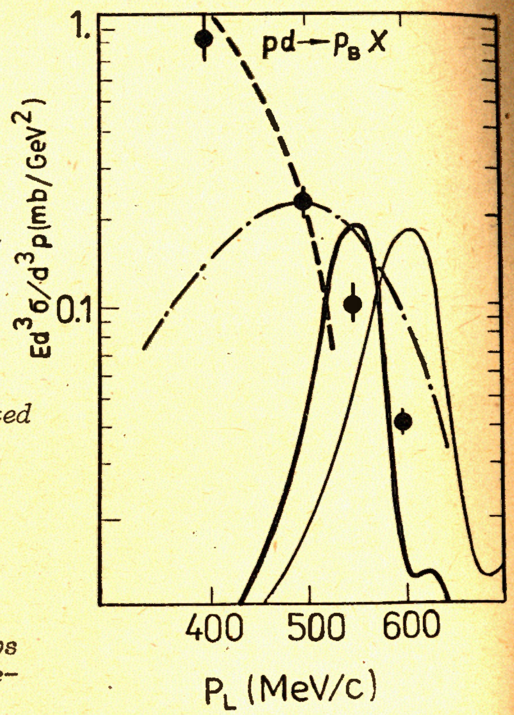

Fig. 3. Cross section for $pd \to p_B X$. *Data points are taken from ref.* /10/, p_{LAB} = *= 8.9 GeV/c; the dashed-dotted line denotes the result of (2), calculated for* κ = *= 1 GeV/fm and* $w = 2\ fm^{-2}$ *; the thick solid line shows the contribution of (35) at an angle* $\theta = 180°$*; the thin solid line the same for* θ = *= 140°; the dashed line shows the contribution of the spectator mechanism* /7/.

As is seen from fig.3, expression (2) gives a maximum at $p_B \approx 500$ MeV/c.

Note, that one should add the contribution of other mechanisms to this one: that of the spectator mechanism /7/ dominating for low backward momenta, of isobar mechanism /8/ dominating at low energies of the incident hadron, etc.

High energy data are available only for the inclusive process $pd \to p_B X$ /9/. Since the process under consideration is diffractive, the excitation of the incident hadron does not affect the spectrum of backward protons and can be taken into account by multiplying (2) by a factor $C_h = 1 + \sigma^{hN}_{diff}/\sigma^{hN}_{el}$. For incident protons $C_p \approx 1.4$, for pions $C_\pi \approx 1.6$.

The main decay mode of the colour dipole is due to $q\bar{q}$ production in the field of the tube, i.e., several dipoles of smaller mass are produced resulting in significantly smaller backward proton momenta. By our estimates these events do not strongly affect the spectrum for $p_B \gtrsim 500$ MeV/c.

One can see from fig.3 that in the region of small momenta $p_B \lesssim 400$ MeV/c the main contribution is given by the Fermi-motion of nucleons in the deuteron /7/. Experimental results for $pd \to p_B X$ obtained at the incident momentum 8.9 GeV/c are also shown here. The comparison with the data shows that the mechanism suggested gives the right order of magnitude for momenta $p_B \gtrsim 500$ MeV/c.

4. QUANTUM-MECHANICAL APPROACH

First we study the corresponding one-dimensional nonrelativistic problem. In section 4.3 the results will be generalized to a realistic case.

4.1. Two-Particle Scattering. Dibaryon Resonances

To begin with, let us consider a system of two particles both having two states: "white" and "colour" denoted by vectors $\binom{0}{1}$ and $\binom{1}{0}$, respectively. We write the Hamiltonian as

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \Pi_1 \Pi_2 V(x_1 - x_2) + \sigma_1 \sigma_2 v(x_1 - x_2). \qquad (4)$$

Here $\Pi_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_i$ is the projection operator to the colour state of the i-th particle; $V(x)$ is the confinement potential for colour particles with a relative distance x. Note that for the colour string $V(x) = \kappa |x|$. $\sigma_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_i$ is the colour changing operator. The last term in (4) corresponds to colour exchange between the particles. For simplicity we take

$$v(x_1 - x_2) = a\delta(x_1 - x_2). \qquad (5)$$

The sum of the first two terms (H_0) acts in two orthogonal subspaces: i) a system of two noninteracting white particles; ii) a system of two colour particles interacting via the potential $V(x_1 - x_2)$. The last term in (4) mixes these subspaces. We shall consider it perturbatively.

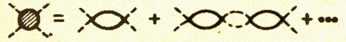

Fig. 4. The elastic scattering amplitude of two colourless objects.

The T-matrix for the scattering of two white particles, shown in Fig.4, is

$$T = vG_0 v + vG_0 vG_0 vG_0 v + .. \qquad (6)$$

Here $G_0 \equiv G_0(\mathcal{E} + i0)$, where $\mathcal{E}$ is the total energy of the two-particle system, $G_0(z) = (z - H_0)^{-1}$ is the resolvent of H_0. G_0 is the sum of two resolvents acting in two orthogonal subspaces:

$$G_0 = (1 - \Pi_1\Pi_2)\, G_f + \Pi_1\Pi_2 G_c . \tag{7}$$

Here G_f is the resolvent for the free motion, and

$$G_c(\mathcal{E}) = \sum_n \int \frac{dP}{2\pi} \frac{|P,n\rangle\langle P,n|}{\mathcal{E} - P^2/2M - E_n} \tag{8}$$

$|P,n\rangle$ is an eigenstate of $H_0^{conf} = p_1^2/2m + p_2^2/2m + V(x_1 - x_2)$ with total momentum P and c.m.s. energy E_n; the total mass of the system: $M = 2m$.

The matrix elements of $G_c(\mathcal{E})$ are reduced as

$$\langle x_1', x_2'|G(\mathcal{E})|x_1, x_2\rangle = \int \frac{dP}{2\pi} \exp[iP(X' - X)]\, g_c(E; x', x), \tag{9}$$

where $x = x_1 - x_2$; $X = (x_1 + x_2)/2$; $E = \mathcal{E} - P^2/2M$; and g_c is the resolvent for the relative motion:

$$g_c(E; x', x) = \sum_n \frac{\phi_n(x')\,\phi_n^*(x)}{E - E_n} . \tag{10}$$

The corresponding free resolvent is

$$g_f(E; x', x) = -\frac{i\mu}{\sqrt{2\mu E}} \exp(i\sqrt{2\mu E}\,|x' - x|), \tag{11}$$

where $\mu = m/2$ is the reduced mass.

The series

$$vg_c v + vg_c vg_f vg_c v + \ldots \tag{12}$$

can be summed up easily and it gives the following reflection amplitude

$$A = \frac{a^2 \mu g_c}{ik - a^2 \mu g_c}, \tag{13}$$

where k is the momentum of the particles in c.m.s. and

$$g_c = g_c\left(\frac{k^2}{2\mu} + i0; 0, 0\right) = \sum_n \frac{|\phi_n(0)|^2}{k^2/2\mu - E_n + i0} .$$

By inspecting the poles of (13) the two-particle system can be shown to have one bound state with negative energy and a spectrum of "dibaryon" resonances in the $N_c N_c$ system with energies $E = E_n - i\Gamma_n^{el}/2$, where

$$\Gamma_n^{el} \approx a^2 |\phi_n(0)|^2 m/k . \tag{14}$$

The width Γ_n^{el} is due to the possibility to exchange colour and decay in the NN channel.

In the case of the linear potential $V(x) = \kappa|x|$ the wave functions of the resonances are (see Appendix B):

$$\phi_n(x) = \sqrt{\frac{\epsilon}{2a_n'}}\, Ai(\epsilon|x| - a_n') / Ai(-a_n') , \tag{15}$$

where $\epsilon = (2\mu\kappa)^{1/3}$; $Ai(y)$ is the Airy function; $-a_n'$ are the zeros of $Ai'(y)$. The corresponding energy spectrum is given by

$$E_n = a_n' \left(\frac{\kappa^2}{m}\right)^{1/3} . \tag{16}$$

The possibility of $q\bar{q}$ production in the tube can be taken into account by introducing an imaginary part of the potential by replacement $\kappa \to \kappa - iw/2$, where w is the probability for $q\bar{q}$ production introduced in section 2. In this way we obtain for the total width of the multiparticle decays

$$\Gamma_n^{in} \approx \frac{2w}{3\kappa} E_n . \tag{17}$$

E_n in (10) should also be replaced by $E_n - i\Gamma_n/2$. However, the expression obtained for g_c is a good approximation only if $\Gamma_n \ll E_{n+1} - E_n$, which is not true for large n values. This condition means that the life-time of the resonance should be larger than the period of motion on the classical orbit. However, for high excitations the tube breaks with a large probability, and the notion of resonance loses sense.

Nevertheless, in the linear potential one can derive an exact expression for the propagator

$$g_c(E; x, 0) = \frac{\mu}{\epsilon} Ai\left(\epsilon|x| - \frac{\epsilon E}{\kappa}\right) / A_i'\left(-\frac{\epsilon E}{\kappa}\right) . \tag{18}$$

By using the asymptotic form of the Airy function /10/, one can show that at high energies this expression tends to the free propagator (11) with a damping factor for breaking of the string (cf. Appendix B). Of course, this result is more general and valid for a large class of potentials.

4.2. Duality of the Scattering Amplitude

The amplitude (12) is dual in the following sense. As it has been pointed out, at high energies the propagator g_c could be replaced by the free one. This corresponds to the Pomeron exchange in the simplified model discussed above. On the other hand, the amplitude at low energies has a resonance behaviour as is clear from expression (10) for g_c.

It is interesting to note, that duality holds in average as well. The contribution of dibaryon resonances to the imaginary part of the NN elastic scattering amplitude, averaged over an energy interval ΔE is:

$$\overline{\mathrm{Im}A} = \left(\frac{dE_n}{dn}\right)^{-1} \int dE \frac{\Gamma_n^{el}\,\Gamma_n^{t}}{(E - E_n)^2 + (\Gamma_n^t/2)^2} \cdot \frac{1}{2}\sqrt{\frac{2E}{\mu}} = \pi a^2 \left(\frac{dE_n}{dn}\right)^{-1} |\phi_n(0)|^2 . \qquad (19)$$

By using the asymptotic form of g_c, i.e., the free propagator, we obtain:

$$\mathrm{Im}A = a^2 \sqrt{\frac{\mu}{2E}} . \qquad (20)$$

Expressions (19) and (20) coincide if the following relation holds true:

$$\phi_n^2(0)\, \pi \left(\frac{dE_n}{dn}\right)^{-1} \sqrt{\frac{2E_n}{\mu}} \approx 1 . \qquad (21)$$

This relation is fulfilled approximately for different confinement potentials and has a clear physical interpretation. Using the quasi-classical relations

$$\frac{dE_n}{dn} = \frac{2\pi}{T_{cl}} ; \qquad \sqrt{\frac{2E_n}{\mu}} = V_{cl}(0) ,$$

where $V_{cl}(x)$ and T_{cl} are the velocity and the period of motion on the classical trajectory, one can rewrite (21) in the following form:

$$\phi_n^2(0)\, dx = \frac{dx/V_{cl}(0)}{T_{cl}/2} .$$

This relation means the equality of quantum-mechanical and classical probabilities to find the particle in the interval dx.

One can hope, that the duality between the Pomeron and dibaryon resonances remains valid for a more realistic case as well. Note, that in the meson-nucleon scattering the Pomeron corresponds to 5-quark resonances with separated colour.

4.3. Scattering of Three Particles

The Hamiltonian (4) is easily generalized to several particles:

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} \Pi_i \Pi_j V(x_i - x_j) + \sum_{i<j} \sigma_i \sigma_j v(x_i - x_j) . \qquad (22)$$

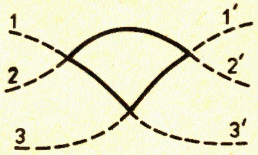

Fig.5. The elastic scattering amplitude of three colourless particles in the lowest order.

Consider the scattering amplitude for three white particles: $1 + 2 + 3 \to 1' + 2' + 3'$. The Hamiltonian H_0 consisting of the first two terms of (22) does not mix the different orthogonal subspaces: when all three particles are white, and when, e.g., particle 3 is white and free, 1 and 2 are in a colour state and interact via $V(x_1 - x_2)$. The last term in (22) will be considered again as perturbation.

In the lowest order the scattering amplitude for three particles is shown in fig.5. This corresponds to the diagram in fig.2, considered above.

We have to derive the three-particle resolvent acting in the subspace where particles 1 and 2 are coloured while particle 3 is white. The matrix elements of this resolvents are simply expressed by g_c defined in (10):

$$\langle x_1', x_2', x_3' | G_0^{(12)}(E + i0) | x_1, x_2, x_3 \rangle =$$

$$= \int \frac{dp_3}{2\pi} \frac{dP_{12}}{2\pi} \exp[ip_3(x_3' - x_3) + iP_{12}(X_{12}' - X_{12})] g_c(E_{12} + i0, x_{12}', x_{12}),$$

where $X_{12} = (x_1 + x_2)/2$; $x_{12} = x_1 - x_2$; $E_{12} = E - p_3^2/2m - P_{12}^2/2M$, $(M = 2m)$

is the energy of the relative motion in the system (1-2). (p_1, p_2, p_3 are the momenta of particles, and $P_{12} = p_1 + p_2$). The amplitude corresponding to fig.5 is written as

$$A \equiv \langle p_1', p_2', p_3' | T | p_1, p_2, p_3 \rangle =$$

$$= \langle p_1', p_2', p_3' | v^{(12)} G_0^{(12)}(E + i0)\, v^{(13)} G_0^{(23)}(E + i0)\, v^{(23)} | p_1, p_2, p_3 \rangle ,$$

where $v^{(12)} \equiv v(x_1 - x_2) = a\delta(x_1 - x_2)$; $2mE = \sum_{i=1}^{3} p_i^2 = \sum_{i=1}^{3} p_i'^2$.

After simple calculations one obtains in the coordinate representation:

$$A = a^3 \int dx \exp[ix(p_3 - p_1')/2]\, g_c(E_{23}'; 0, x)\, g_c(E_{12}; x, 0) . \qquad (23)$$

From this we get the scattering amplitude for the case when particle 1 is scattered by a resting "deuteron" - a bound state of two white particles 2 and 3:

$$A_d \equiv \langle p_1', p_2', p_3' | T | p_1, d\rangle =$$

$$\doteq \alpha^3 \int \frac{dq}{2\pi} dx \tilde{\Psi}_d (q) \exp[ix(q - p_1')/2] g_c (E_{23}', 0, x) g_c (E_{12}; x, 0), \tag{24}$$

where $p_3 = q$, $p_2 = -q$ and $\tilde{\Psi}_d(q)$ is the wave function of the "deuteron" in the momentum representation. We make some approximations in (24). The incident particle is assumed to be very fast: $p_1 \gg q, p_2', p_3' \gg p_1 - p_1'$. Then one has $p_3' = -p_2' \equiv -p_B$. The last factor in (24) can be replaced by the asymptotic expression (B3).

Then one obtains:

$$A_d = -\frac{im\alpha^3}{p_1} \int_0^\infty dx \Psi_d (x) g_c(E_{23}'; 0, x) \exp(-\frac{w\mu}{4p_1} x^2), \tag{25}$$

where $E_{23}' = p_B/m$. This expression has a clear physical meaning: the potential accelerates the particles 2,3 from their original distance x till 0, transferring momenta $\pm p_B$ to them.

Inserting the sum over resonances (10) into (25) one gets overlap integrals between the wave functions of the resonances and the deuteron. It does not mean, of course, that colour separated states are assumed in the deuteron. The colour is transferred by the incident hadron, and the amplitude for creating a colour dipole of size x is proportional to the deuteron wave function $\Psi_d(x)$.

The main difference between expression (25) and the classical expression (2) is that in (25) there is no one-to-one relation between the backward momentum p_B (i.e., energy E_{23}') and the "prepared" distance x in the deuteron.

We shall investigate the dependence of relevant longitudinal distances x on p_B. If the momentum p_B is large enough, one can use the quasi-classical approximation:

$$g_c(E_{23}'; 0, x) = \frac{-im}{2\sqrt{q(0)\,q(x)}} \exp\{i \int_0^x q(y)\, dy - \frac{1}{2} \int_0^T W[y(t)]\, dt\}, \tag{26}$$

where

$$q(x) = \sqrt{p_B^2 - mV(x)}\,. \tag{27}$$

The second term in the exponent takes into account the possibility of breaking of the tube described by a complex potential

$V(x) - iW(x)/2$. The function $y(t)$ is the solution of classical equation of motion

$$\frac{dy}{dt} = \frac{1}{\mu} \sqrt{2\mu(E_{23}' - V(y))}$$

with the boundary condition $y(0) = x$. The time interval $T = T(x)$ is given by $y(T) = 0$. For the string with the potential $(\kappa - iw/2)|x|$

$$\frac{1}{2} \int_0^T W[y(t)]\, dt = \frac{w}{3} (\frac{m}{\kappa})^{1/2} x^{3/2}\,.$$

Let us estimate the integral (25) by the stationary-phase method

$$A_d \approx (\frac{-im}{p_1}) \frac{-im}{2\sqrt{q(0)\,q(x)}} (-\frac{1}{2\pi} \frac{\partial q(x)}{\partial x})^{-1/2} \Psi_d (x) D^{1/2} (x)|_{x=L(p_B)}, \tag{28}$$

where the value $x = L(p_B)$ given by the stationary phase condition $q(x, p_B)|_{x=L(p_B)} = 0$ is just the classical distance corresponding to the expression (1).

The function $D(x)$ in (28) includes exponential factors from (25) and (26) taking into account breaking of the tube.

$D(x)$ coincides with the classical expression (3) for the nonrelativistic case.

Further we have

$$q(x) \frac{dq}{dx}\Big|_{x=L} = q(x) \frac{dq}{dp_B}\Big|_{x=L} (\frac{dL}{dp_B})^{-1} = q(0) (\frac{dL}{dp_B})^{-1}\,.$$

Inserting this into (28) we finally get

$$A_d \approx -\frac{m^2}{p_1 p_B} \sqrt{\frac{\pi}{2}} \Psi_d (L) \sqrt{\frac{dL}{dp_B}} D^{1/2}(L)\,. \tag{29}$$

This approximation of (25) corresponds to expression (2).

In the quasiclassical approximation the significant distances grow with p_B. However it is obvious from (25) and (26) that this increase is limited because of rapid decrease of the wave function $\Psi_d(x)$ and the factor $D^{1/2}(x)$ at large x values. At realistic values of parameters the integration in (25) is cut off at $x \approx \kappa/(wm) \approx 1\div 2$ fm corresponding to $p_B \approx 0.5$ GeV/c. At larger values of p_B the quasiclassical approach does not work, and the physical meaning of expression (25) is as follows. The energy E_{23}' of the colour dipole comes from work done by colour forces $\sim \kappa^2/(mw)$ and the kinetic energy of the nucleon "prepared" in the deuteron. However, if E_{23}' becomes too large, the two-nucleon interpretation of the deuteron wave function loses sense and it is necessary to take into account the quark structure of the deuteron.

Note that in the spectator mechanism [7] the total backward momentum should be prepared in the deuteron. Hence the present mechanism remains valid at much larger values of E'_{23} than the spectator one.

4.4. Generalization to a Realistic Case

The problem above contains all the main features of the quantum-mechanical approach. However, it has been formulated in one-dimensions, nonrelativistic case, for the δ-type colour exchange potential $v(x)$ and for a very simplified colour structure. We generalize the problem to a more realistic case.

We assume that the colour state has 8 components (colour octet). The colour exchange potential in Hamiltonian (4) is replaced by $(1/\sqrt{8})\,\delta_{ab}\, v(r)$, where $a, b = 1,\dots,8$.

The wave functions (15) of dibaryon resonances in the S-state are modified in the following way:

$$\phi_n^{ab}(r) = \frac{1}{\sqrt{8}}\delta^{ab}\phi_n(r) = \frac{\delta^{ab}}{\sqrt{8}}\sqrt{\frac{\epsilon}{4\pi}}\,\frac{Ai(\epsilon r - a_n)}{Ai'(-a_n)\cdot r}\,, \qquad (30)$$

where $a_n = 2.3;\ 4.1,\ 5.5\dots$ are the zero's of the Airy function: $Ai(-a_n) = 0$.

Note that a more realistic potential can be used instead of the potential $V(r) = \kappa r$

$$V(r) = \begin{cases} \infty & \text{if} \quad r < R_0 \\ \kappa(r - R_0) & \text{if} \quad r > R_0\,. \end{cases}$$

Here $R_0 \approx 0.5$ fm is the radius of the repulsive core. The corresponding modification of the wave function is very simple: it is enough to replace r by $r - R_0$ in the argument of the Airy function and take $\phi_n = 0$ for $r < R_0$. The masses of the corresponding resonances are

$$M_n \approx 2m + a_n\left(\frac{\kappa^2}{m}\right)^{1/3}. \qquad (31)$$

It is seen that already the first dibaryon resonance with separated colour has a large mass of about 3 GeV/c^2. Their width Γ_n^{in} is still given by expression (17), and for the first resonance it is about 200 MeV/c. Note that values for masses and widths have to be considered as rough estimates only since beside other approximations the values of κ and w are not well determined.

Expression (14) giving the decay width to two nucleons is replaced by

$$\Gamma_n^{el} = \frac{mQ_n}{2\pi}\left|\int d^3\vec{r}\, v(\vec{r})\,\phi_n(\vec{r})\,\exp(i\vec{Q}_n\vec{r})\right|^2, \qquad (32)$$

where $Q_n = \sqrt{2M_n - 4m^2}/2$ is the momentum of outgoing nucleons in the c.m.s. Let us estimate Γ_n^{el}. We take $v(r)$ of the form

$$v(\vec{r}) = v(0)\exp(-\vec{r}^{\,2}/4B)\,. \qquad (33)$$

The parameters $v(0)$ and B can be found from the NN scattering. The parameter B turns to be equal to the slope parameter in the elastic NN scattering, $B \approx 10\ (\mathrm{GeV/c})^{-2}$, and

$$v(0) = \frac{\sqrt{2\sigma_{in}^{NN}}}{4\pi B} \approx 0.1\ \mathrm{GeV}. \qquad (34)$$

We have taken into account the fact that the colour exchange is vector like and not scalar; this yields a constant cross section. Then from (32) we obtain for the first resonance $\Gamma_1^{el} \approx$ ≈ 10 MeV. It's obvious that $\Gamma_1^{el} \ll \Gamma_1^{in}$. Note that Γ^{el} has been calculated much less reliably than Γ_n^{in} since the decay to two nucleons is due to the colour exchange within the resonance and its quark structure is important.

It has also to be noted that the parameters of $v(r)$ depend on energy. The estimate (34) corresponds to large energies. At lower energies besides gluon exchange quark exchanges are also important, they can increase $v^2(0)$ by a factor ≈ 2.

The cross section for $dd \to p_B hn$ acquires the form (cf.(25)):

$$E\frac{d^3\sigma}{d^3p_B} = \frac{2(\sigma_{in}^{hN})^2 B}{\pi Q}\left|\sum_n \frac{\sqrt{\Gamma_n^{el}}}{M - M_n + i\Gamma_n^t/2}\int dz\,\Psi_d(z)\,\phi_n(z)\,D^{1/2}(z)\right|^2. \qquad (35)$$

Here $Q = m(\alpha - 1)/\sqrt{\alpha(2-\alpha)}$ is the relative momentum of two outgoing nucleons in their c.m.s.; $M = 2m/\sqrt{\alpha(2-\alpha)}$ is their effective mass; $\alpha = (E + p_L)/m$ is the light cone variable (we have neglected the transverse momentum of the outgoing hadron h). A combinatorical factor 4 is included, it takes into account permutation of nucleons.

The results of calculations using formula (35) multiplied by the factor $C_N \approx 1.4$ (see sec.3) are shown in fig.3 for angles 180° and 140°. We have used here the value $\Gamma_1^{el} = 30$ MeV. Expression (35) has the following scaling property: at a fixed value of α the cross section does not depend on the backward angle. It is easy to verify that the spectator mechanism has the same feature, hence their relative contribution does not depend on the production angle.

One has to note that the real location of structures in backward spectrum may be different from that in fig.4 because the masses and widths of resonances are calculated only approximately.

The normalization of the cross section is also uncertain because the parameters of the model are not well known. Nevertheless fig.3 shows that the results of calculations agree in order of magnitude with experimental data for $p_B \geq 550$ MeV/c.

As it has already been mentioned, the contribution of the present mechanism to the cross section of backward proton production at large energies does not depend on the incident energy. However at intermediate energies of about several GeV there is a specific energy dependence. Really, the amplitude (23) contains $g_c(E_{12})$, the propagator function of the system consisting of the incident hadron and the target nucleon after the first colour exchange. At high energies we replaced it by free propagator (26). However, as is shown by (10), at intermediate energies $g_c(E_{12})$ has resonant dependence on $E_{12} = \sqrt{2mT_{kin} + 4m^2}$, where T_{kin} is the kinetic energy of the incident hadron. With the first resonance having 3 GeV/c^2 mass, for the fixed value of p_B one expects a maximum in the cross section at $T_{kin} \approx$ ≈ 2.6 GeV. The present mechanism starts to give considerable contribution only from these incident energies. In the case of the incident pion the resonances of $g_c(E_{12})$ are 5-quark pion-nucleon resonances with a separated colour. In our approximation the excitation spectrum of these resonances is close to that of dibaryon resonances.

5. ELASTIC pd BACKWARD SCATTERING

In the pionless process $pd \to p_B pn$ at intermediate energies the proton and the neutron flying forward can have comparable momenta and form a deuteron. Hence the mechanism of colour forces contributes to the pd backward scattering, as is shown by the diagram in fig.6. The corresponding amplitude is:

$$A^{pd\to dp} = \int \bar{\Psi}_d(\vec{q}\,')\, v(\vec{x}\,')\, g_c(E'_{23}\,;\vec{x}\,',\vec{z})\, v(\vec{x}-\vec{y})\, g_c(E_{12}\,;\vec{y},\vec{x}) \times$$
$$\times\, v(\vec{x})\, \bar{\Psi}_d(\vec{q}) \exp[i\vec{x}\,'\vec{q}\,'_{32} - i\vec{z}(\tfrac{1}{2}\vec{P}'_{32} - \vec{p}\,) + i\vec{y}(\tfrac{1}{2}\vec{P}_{12} - \vec{p}\,'_1) + i\vec{x}\vec{q}_{12}]\,do\,, \qquad (36)$$

where

$$\vec{q}_{12} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_2);\quad \vec{q}\,'_{32} = \tfrac{1}{2}(\vec{p}\,'_3 - \vec{p}\,'_2);\quad \vec{P}_{12} = \vec{p}_1 + \vec{p}_2;\quad \vec{P}'_{32} = \vec{p}\,'_3 + \vec{p}\,'_2;$$

$$\vec{p}_3 = \tfrac{1}{2}\vec{p}_d + \vec{q};\quad \vec{p}_2 = \tfrac{1}{2}\vec{p}_d - \vec{q};\quad \vec{p}\,'_1 = \tfrac{1}{2}\vec{p}\,'_d + \vec{q};\quad \vec{p}\,'_2 = \tfrac{1}{2}\vec{p}_d - \vec{q}\,;$$

$$do = d^3\vec{x}\,d^3\vec{x}\,'d^3\vec{y}\,d^3\vec{z}\,d^3\vec{q}\,d^3\vec{q}\,'.$$

Taking g_c as a sum over the resonances, and neglecting the momenta of the nucleons inside the deuteron, one gets for the cross section in c.m.s.:

$$\frac{d\sigma}{d\Omega_{cms}} = \frac{25|\Psi_d(0)|^4}{18Q^2}\left|\sum_{n,n'} \frac{\sqrt{\Gamma_n^{el}\Gamma_{n'}^{el}}\; F_{nn'}(\vec{p},\vec{p}\,')}{(M - M_n + i\Gamma_n^t/2)(M - M_{n'} + i\Gamma_{n'}^t/2)}\right|^2, \qquad (37)$$

where

$$F_{nn'}(\vec{p},\vec{p}\,') = \int d^3\vec{r}\,d^3\vec{r}\,'\phi_n(\vec{r})\, v(\vec{r}-\vec{r}\,')\,\phi_{n'}(\vec{r}\,')\exp[i(\tfrac{1}{2}\vec{p}+\vec{p}\,')\tfrac{\vec{r}}{2} - i(\tfrac{1}{2}\vec{p}\,'+\vec{p})\tfrac{\vec{r}\,'}{2}]. \qquad (38)$$

Here $M^2 = 4m^2 + 2mT_{kin}$; $Q^2 = mT_{kin}/2$, where T_{kin} is the kinetic energy of the incident proton in the laboratory system; p and p′ are the initial and final momenta of the proton in the c.m.s. The combinatorical factor 25 comes from different permutations of the nucleons.

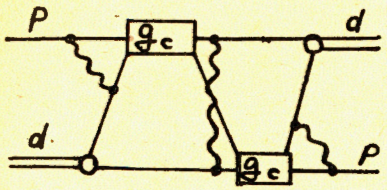

Fig.6. The diagram for pd *backward elastic scattering.*

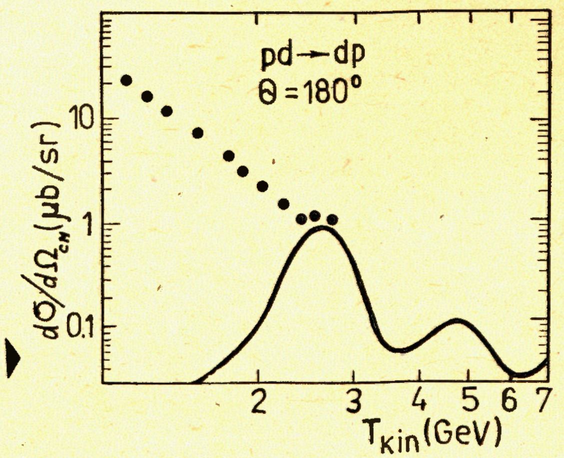

Fig.7. The cross section for the reaction pd → dp *at θ = 180°; the data points are from ref.* /14/. *the solid line shows the contribution of (37).*

The expression (37) has a clear interpretation (cf.fig.6). The incident proton hitting the target nucleon forms a dibaryon resonance with probability proportional to Γ_n^{el}. Then this resonance scatters backward on the second target nucleon by a colour-nucleon-exchange. The amplitude of the last process is described by expression (38). It is interesting to note that this scattering process takes place at c.m.s. momentum $\sim p/2$.

Let us estimate (38) for scattering angles near to 180 at $n = n' = 1$, using in (30) the following approximation for the Airy function:

$$Ai(x - a_1) \approx 0.70 \cdot x \cdot \exp(-0.29x^2), \quad x \geq 0,$$

where $a_1 \approx 2.34$ is the first zero of $Ai(-x)$. From (38) we find

$$F_{11}(p, p') \approx v(0)\left(\frac{4\pi}{\lambda}\right)^{3/2} \exp\left[-\frac{p^2}{16(\lambda - 1/B)} - \frac{9p_T'^2}{64\lambda}\right], \quad (39)$$

where $\lambda = 2\epsilon \cdot 0.29 + 1/B$ (cf. (15) and (33)).

To estimate the cross section for elastic pd scattering at 180° we neglect the nondiagonal terms $n \neq n'$ in (37) and the n dependence of $F_{nn}(p, p')$. The results of calculations with the parameters fixed above are compared with the experimental data/11/ in fig.7. It should be mentioned that this cross section contains an additional factor $\Gamma^{el} v^2(0)$ with respect to the cross section of $pd \to p_B X$. At the values of parameters fixed in the previous section the $pd \to dp$ cross section comes out too small almost by an order of magnitude. However, these factors are very uncertain and within their limits one can change the normalization, what has been done in fig.7.

As is seen in fig.7, the observed change in the energy dependence of the cross section at $T_{kin} \approx 2.5$ GeV may be connected with a dibaryon resonance of mass around 3 GeV/c^2. Figure 7 shows that the contribution of this mechanism at smaller energies is negligible. It would be important to obtain experimental data at higher energies.

6. POLARIZATION EFFECTS

Effects connected with the polarization of the incident particle in the reaction $hd \to p_B X$ are small at high energies and decrease as an inverse power of energy. However, the polarization of the backward nucleon could be large in principle. Nevertheless, in the domain where the first dibaryon resonances dominate ($p_B \approx$ 500-600 MeV/c), the polarization of the backward protons is zero if one can neglect the interference of different resonances and of the background. If p_B is near to the kinematical boundary, the polarization of the backward protons is determined by the interference of gluon- and quark-exchanges in the last colour exchange, which takes place at a finite energy. Hence the polarization would depend on p_B but not on the incident energy. It is interesting to note that a polarization near

to zero can be expected in this case. Actually, in the elastic NN scattering the polarization is due to the interference of the imaginary non-spin-flip Pomeron amplitude $Im f^P_{++}$ with the real part of the leading Reggeon's contribution to the spin-flip amplitude, $Re f^R_{+-}$. The exchange degeneracy of $f - \omega$ and $\rho - A_2$ leads to their compensation in $Im f^R$, while their contributions add in $Re f^R$. In the process $N_c N_c \to NN$ considered here the polarization is due to the interference of the real gluon-exchange non-spin-flip amplitude $Re f^G_{++}$ with the imaginary part of the spin-flip Reggeon amplitude, $Im f^{R_c}_{+-}$. The later is zero if the colour Reggeons R_c are exchange degenerate. If the exchange degeneracy for colour Reggeons is strongly violated, the backward protons can have polarization of several per cent/5/. Note, that the size of this polarization is equal to the azimuthal asymmetry of backward protons in the case of polarized deuteron.

7. DISCUSSION

A high energy hadron, interacting with the deuteron, can transfer colour from one of the nucleons to another, turning the deuteron into a colour dipole. The decaying dipole can emit a nucleon into the backward hemisphere. This mechanism is based on the popular model of colour strings, which reflects, we hope, properly the space-time development of hadron-hadron interactions. The study of hadron-nucleus interactions provides a unique possibility to verify these ideas.

We note, that the kinematics of production and decay of the colour dipole resembles the mechanism of intermediate production of resonance/8/ when this resonance interacts with the second nucleon in the reaction $N^*N \to NN$, and due to the excess of mass it produces a nucleon in the backward hemisphere. At small momenta $p_{lab} \approx 1.5$ GeV/c of the incident proton the production of Δ_{33} isobar/8,12/ gives a large contribution, which decreases with energy as an inverse power. The contribution of diffractive excitations does not depend on the incident energy. However, the cross section of diffractive dissociation, summed over the final states is suppressed compared to σ_{in} by an order of magnitude. This smallness enters quadratically into the cross section $pd \to p_B pn$, therefore at high energies the contribution of white intermediate states is negligible with respect to the coloured ones.

Another mechanism with nonvanishing contribution at high energies - the spactator mechanism/7/ - has been mentioned earlier. This dominates in the $pd \to p_B X$ cross section in the soft part of the backward spectrum $p_B \leq 500$ MeV/c. At large momenta the calculations in this model lose sense because the usage of the two-nucleon wave function of the deuteron is unjustified.

For the same reason the calculations presented above also have limited domain of validity, extending, however, until higher momenta, than for the spectator mechanism. A colour dipole with a large mass can be formed not only by increasing the prepared momenta of the nucleons within the deuteron but also by the energy of the colour flux tube.

The comparison with the available experimental data allows us to conclude only that the contribution of the mechanism proposed here agrees in order of magnitude with the cross section of reaction $pd \to p_B X$ at $p_B \gtrsim 500$ MeV/c and with the cross section of backward elastic pd scattering at $T_{kin} \approx 2.5$ GeV.

A great part of the available experimental data for backward proton production has been obtained on nuclei with A > 2. Cascading effects and the Fermi-motion of a nucleon pair as a whole makes the theoretical study of these process difficult.

For that reason is more favourable to study different processes on deuterons. Several examples are listed below.

1. A change in the character of the reaction $hd \to p_B X$ with the increasing p_B. At $p_B \lesssim 500$ MeV/c the spectator mechanism dominates. It is characterized by large multiplicity $\langle n\rangle_{hd} \approx \langle n\rangle_{hN}$ and large momentum loss of the leading particle $\langle x_F\rangle \approx 0.5$, where x_F is the Feynman variable.

If the contribution of intermediate colour dipole dominates at $p_B \gtrsim 500$ MeV/c then - since this process is diffractive - the leading hadron should have quantum numbers of the incident particle and x_F in the diffractive region. Accordingly, the mean multiplicity in this process is small.

By selecting the diffractive part in the reaction $hd \to p_B X$, e.g., $hd \to p_B nh(h^*)$, one can suppress the background of the spectator mechanism.

2. Observation of bumps in the momentum spectrum of the backward protons would be a serious argument for the existence of heavy dibaryon resonances with separated colour. These resonances are analogous to giant nuclear resonances, which are collective excitations of nuclei.

We would like to stress that the calculation of the mass spectrum of dibaryon resonances presented above bears an illustrative character only, since no quark structure has been taken into account, a linear confinement potential has been used, etc. Values of parameters κ and w strongly affect the masses and widths of the resonances, e.g., at κ = 3 GeV/fm the first resonance gives a peak in the backward spectrum at $p_B \approx 0.65$ GeV/c.

It should be noted that search for such dibaryon resonances in the NN scattering is difficult due to their small production cross section. Indeed, the contribution of dibaryon resonance into the cross section at $E = E_n$ is $(4\pi/k^2)\Gamma_n^{el}/\Gamma_n^t$, which is only 1% of σ_{tot}^{NN}.

Therefore search for heavy dibaryon resonances in reactions with energetic backward nucleon production $p_B \gtrsim 500$ MeV/c seems to be favourable with respect to the signal to noise ratio. Really, at high incident energies the known mechanisms do not contribute (the spectator contribution can be suppressed by selecting the diffractive events).

Light nuclei can be used as targets if the effective mass distribution of two protons is studied, one of which flies backward.

3. The importance of obtaining data on elastic backward scattering at $T_{kin} \gtrsim 3$ GeV is obvious from fig.6.

The mechanism considered above should be noted to give contribution also to the πd backward elastic scattering. In this case five-quark intermediate resonances of $\pi_c N_c$ type are excited. The cross section (37) for πd backward scattering has an addtional factor of 4/25 taking into account fewer permutations of nucleons and less number of quarks in the pion.

4. It is important to note that this mechanism does not contribute to the backward proton production on the deuteron if the incident particle is a lepton. On the contrary, the type of the incident particle is irrelevant for the spectator mechanism. Therefore a characteristic dependence on p_B of the ratio $R = \sigma(\ell d \to p_B X)/\sigma(hd \to p_B X)$ can be expected. At $p_B \lesssim 500$ MeV/c, where the spectator mechanism dominates, R should be constant ($\sim\sigma_{tot}(\ell N)/\sigma_{tot}(hN)$), while it should fall down at increasing p_B, because the colour excitation mechanism becomes relatively more important.

This statement does not apply to heavier nuclei since the quark kicked out by the lepton can convert a pair of nucleons into a colour dipole.

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APPENDIX A

For convenience of notations let us consider the problem where all three interacting particles A, B and C are mesons, consisting of quarks and antiquarks denoted by $q_1\bar{q}_2$, $q_3\bar{q}_4$, $q_5\bar{q}_6$. Particles B and C form a bound state on which particle A is scattered. We shall calculate the scattering amplitude in the eikonal approximation, valid for large relative energies.

In this approximation we have

$$A(\vec{k}_A, \vec{k}_B, \vec{k}_C) = 2s \int d^2\vec{x}_1 \ldots d^2\vec{x}_6 \Psi^A_{i_1 i_2}(\vec{x}_1 - \vec{x}_2) [\Psi^A_{j_2 j_1}(\vec{x}_1 - \vec{x}_2)]^* \exp(-i\vec{k}_A \vec{X}_{12}) \times$$

$$\times \Psi^B_{i_3 i_4}(\vec{x}_3 - \vec{x}_4) [\Psi^B_{j_4 j_3}(\vec{x}_3 - \vec{x}_4)]^* \exp(-i\vec{k}_B \vec{X}_{34}) \Psi^C_{i_5 i_6}(\vec{x}_5 - \vec{x}_6) [\Psi^C_{i_5 i_6}(\vec{x}_5 - \vec{x}_6)]^* \times$$

$$\times \exp(-i\vec{k}_C \vec{X}_{56}) \Psi_d(\vec{X}_{34} - \vec{X}_{56}) \exp[-\frac{ig^2}{4} \sum_{a=1}^{8} \sum_{\alpha, \beta=1}^{6}{}' t^a(\alpha) t^a(\beta) V(\vec{x}_\alpha - \vec{x}_\beta)]. \tag{A1}$$

Here $\vec{x}_\alpha$ is the transversal coordinate of α' th quark (antiquark); $\vec{k}_A$, $\vec{k}_B$ and $\vec{k}_C$ are the transverse momenta of corresponding particles after the interaction; $\Psi^A_{i_1 i_2}(\vec{x}_1 - \vec{x}_2)$ is the wave function of hadron A with quark colour indices i_1 and i_2 ; $X_{\alpha\beta} = (\vec{x}_\alpha + \vec{x}_\beta)/2$; ; $\Psi_d(\vec{X}_{34} - \vec{X}_{56})$ is the "deuteron" wave function; the sum Σ' in the exponent is done over quark pairs belonging to different hadrons; matrices t are defined as

$$t^a(\alpha) = \begin{cases} \lambda^a(\alpha) & \text{for quarks } \alpha = 1,3,5 \\ -[\lambda^a(\alpha)]^T & \text{for antiquarks } \alpha = 2,4,6, \end{cases}$$

$\lambda^a(\alpha)$ are the Gell-Mann matrices acting on quark α ; g is the QCD coupling constant;

$$V(x) = \int \frac{d^2\vec{q}}{(2\pi)^2} e^{i\vec{q}\vec{x}} \frac{1}{\vec{q}^{\,2}}.$$

An implicit dependence on longitudinal quark momenta in the infinite-momentum frame is also ment.

In the eikonal approximation the longitudinal momenta of quarks does not change in the course of interaction.

For colourless hadrons we have:

$$\Psi^A_{i_1 i_2}(\vec{x}_1 - \vec{x}_2) = \frac{1}{\sqrt{3}} \delta_{i_1 i_2} \Psi^A(\vec{x}_1 - \vec{x}_2).$$

The amplitude shown in fig.2 is given by the $(g^2)^3$ terms in (A1). After summing up over the colour indices and some integrations the amplitude becomes:

$$A(\vec{k}_A, \vec{k}_B, \vec{k}_C) = 16 s g^2 \frac{1}{27} \int \frac{d^2\vec{p}}{(2\pi)^2} \frac{d^2\vec{q}}{(2\pi)^2} \Phi(\vec{q}_1, \vec{q}_2) \Phi(-\vec{q}_1, \vec{q}_3) \times$$

$$\times \Phi(-\vec{q}_2, \vec{q}_3) \vec{q}_1^{\,-2} \vec{q}_2^{\,-2} \vec{q}_3^{\,-2} \Psi_d(\vec{p}). \tag{A2}$$

Here we use the notations $\vec{q}_1 = \vec{q}$; $\vec{q}_2 = \vec{k}_A - \vec{q}$; $\vec{q}_3 = \vec{k}_B - \vec{q} - \vec{p}$; $\Phi(\vec{k}, \vec{q}) = f(\vec{k} + \vec{q}) - f(\vec{k} - \vec{q})$, where

$$f(k) = \int d^2\vec{x} \int_0^1 \frac{d\alpha}{4\pi\alpha(1-\alpha)} |\Psi(\vec{x}, \alpha)|^2 \exp(i\vec{k}\vec{x}) \tag{A3}$$

is the one-quark form factor of the hadron. The coefficient $[4\pi\alpha(1-\alpha)]^{-1}$ appears in the infinite momentum frame; α is the momentum fraction carried by the quark.

In (A2) we have $\Psi_d(\vec{p}) \equiv \Psi_d(\vec{p}, \alpha_N) = \int d^2\vec{x} \Psi_d(\vec{x}, \alpha_N) \exp(-i\vec{p}\vec{x})$.

In the case of baryons the r.h.s. of (A2) should be multiplied by $(3/2)^3$.

The p dependence of the integrand in (A2) is determined mainly by $\Psi_d(\vec{p})$, which decreases rapidly with increasing p. Therefore all other factors can be evaluated at $p = 0$. Then after integration over the transverse components of p we obtain for the differential cross section:

$$\frac{d\sigma}{d^2\vec{k}_A d^2\vec{k}_B} = 256\pi^2 \alpha_s^6 |A_0(\vec{k}_A, \vec{k}_B)|^2 \int \frac{d\alpha_N}{4\pi\alpha_N(1-\alpha_N)} |\Psi_d(x = 0, \alpha_N)|^2 . \tag{A4}$$

Here

$$A_0(\vec{k}_A, \vec{k}_B) = \int \frac{d^2q}{(2\pi)^2} \Phi(\vec{q}_1, \vec{q}_2) \Phi(-\vec{q}_1, \vec{q}_3) \Phi(-\vec{q}_2, -\vec{q}_3) \vec{q}_1^{\,-2} \vec{q}_2^{\,-2} \vec{q}_3^{\,-2}. \tag{A5}$$

The integration over α_N in (A4) gives

$$\int \frac{d\alpha_N}{4\pi\alpha_N(1-\alpha_N)} |\Psi_d(x = 0, \alpha_N)|^2 = \int dL |\Psi_d(x = 0, L)|^2 . \tag{A6}$$

For $k_A = 0$ (A4) can be written in the form

$$\frac{d\sigma}{d^2\vec{k}_A d^2\vec{k}_B}\Big|_{k_A = 0} = \frac{2^{10}\alpha_s^6}{\mu^8} I(x) \int dL |\Psi_d(L)|^2 , \tag{A7}$$

where μ is a mass parameter in the nucleon form factor: $f(k) = \mu^2/(k^2 + \mu^2)$; $x = k_T^2/\mu^2$ and

$$I = \frac{2}{(1+x^2)} \int_0^\infty dy \frac{(x-y)^2}{(1+y)^2(1+2x+2y)} [\frac{1+x+y}{|x^2 - y^2|} - \frac{1}{\sqrt{(1+x+y)^2 - 4xy}}].$$

Note that $I(0) = 1$, hence

$$\frac{d\sigma}{d^2\vec{k}_A d^2\vec{k}_B}\Big|_{k_A = k_B = 0} = \frac{2^{10}\alpha_s^6}{\mu^8} \int dL |\Psi_d(L)|^2 . \tag{A8}$$

Numerical integration over $\vec{k}_B$ in (A7) gives

$$\frac{d\sigma}{d^2\vec{k}_A}\Big|_{k_A=0} = 0.3\,\frac{2^{10}\alpha_s^6}{\mu^6}\int dL\,|\Psi_d(L)|^2 . \qquad (A9)$$

One can define the average value of slope parameter B as

$$B = \int d^2\vec{k}_A\,\frac{d\sigma}{d^2\vec{k}_A}\left(\frac{d\sigma}{d^2\vec{k}_A}\Big|_{k_A=0}\right)^{-1} . \qquad (A10)$$

The outgoing transverse momenta $\vec{k}_A$, $\vec{k}_B$ and $-(\vec{k}_A+\vec{k}_B)$ enter into (A7) in a symmetric way, hence using the relation

$$\int d^2\vec{k}_2 d^2\vec{k}_3 \exp[-B_0(\vec{k}_1^2+\vec{k}_2^2+\vec{k}_3^2)]\,\delta^2(\vec{k}_1+\vec{k}_2+\vec{k}_3) = \frac{\pi}{2B_0}\exp(-\frac{3}{2}B_0\vec{k}_1^2) \qquad (A11)$$

and the definition (A7) we have

$$B = \frac{3}{2}B_0 . \qquad (A12)$$

On the other side, B_0 can be found from the ratio of (A8) and (A9) using the relation (A11):

$$\frac{2B_0}{\pi} = \left(\frac{d\sigma}{d^2\vec{k}_A d^2\vec{k}_B}\Big|_{k_A=k_B=0}\right)\left(\frac{d\sigma}{d^2\vec{k}_A}\Big|_{k_A=0}\right)^{-1} = \frac{3.3}{\mu^2} . \qquad (A13)$$

We determine the values of α_s and μ from elastic and total cross sections of pp scattering with the same parametrization of the form factor /14/

$$\sigma_{tot}^{NN} = \frac{32\pi}{\mu}\alpha_s^2 , \qquad \frac{d\sigma_{el}^{NN}}{dt} = \frac{64\pi\alpha_s^4}{\mu^4}\,J(-\frac{t}{\mu^2}),$$

where

$$J(x) = (1-x)^{-2}\,[1 + 2x(1-x)^{-2}\ln\frac{4x}{(1+x)^2}] .$$

Substituting the values $\sigma_{tot}^{NN} = 40$ mb, $\sigma_{el}^{NN} = 7$ mb we obtain $\alpha_s = 0.78$ and $\mu^2 = 0.62\ GeV^2$. From (A10) we also get the slope parameter $B = 12.8\ GeV^{-2}$. For the coefficient β in (2) the comparison gives the value $\beta \approx 0.17$.

APPENDIX B

We present here some formulae for the Green functions in the linear potential with complex string tension κ.

For the Hamiltonian

$$\hat{H} = -\frac{1}{2\mu}\frac{\partial}{\partial x^2} + \kappa|x|$$

in one dimensions we obtain the Grren function (18). In three dimensions the Green function is (for $\vec{r}' = 0$):

$$g(E; r, 0) = -\frac{\mu}{2\pi r}\,\frac{Ai(\epsilon r - \epsilon E/\kappa)}{Ai(-\epsilon E/\kappa)} , \qquad (B1)$$

where $\epsilon = (2\mu\kappa)^{1/3}$.

For real values of κ we obtain from (18) and (B1) the normalized wave functions (15) and (30).

One can verify the orthogonality and normalization of these wave functions using the following relations:

$$\int_x^\infty dy\,Ai(y)\,Ai(y+c) = \frac{1}{c}Ai'(x)\,Ai(x+c) - \frac{1}{c}Ai(x)\,Ai'(x+c),$$
$$\int_x^\infty dy[Ai(y)]^2 = -x\,[Ai(x)]^2 + [Ai'(x)]^2 \qquad (B2)$$

which can be proved directly from the differential equation for the Airy function $Ai''(x) = xAi(x)$.

For complex values of κ the corresponding wave functions are not orthogonal to each other, and the expression (16) is no more valid. However (18) and (B1) remain true, and the propagator has poles at complex energy values $E_n = E_n^0 - i\Gamma_n/2 = a_n'(\kappa^2/2\mu)^{1/3}$. At large energies, where $\Delta E_n \ll \Gamma_n$ one can use the asymptotic expression /10/

$$Ai(-z) = \pi^{-1/2}z^{-1/4}\sin(\frac{2}{3}z^{3/2} + \frac{1}{4}\pi)$$

and obtain

$$g(E; x, 0) \sim \frac{\mu}{ip}\exp(ip|x| - \frac{1}{2}x^2\frac{\mu}{p}\mathrm{Im}\,\kappa), \qquad (B3)$$

where $p = \sqrt{2\mu E}$.

The expression (B3) shows that g is given by the free propagator with the damping factor $D^{1/2}(x)$ introduced in section 3, which forbids the breaking of the string during propagation: $D^{1/2}(x) = \exp(-\mathrm{Im}\,\kappa\int dxdt)$.

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Копелиович Б.З., Нидермайер Ф. E2-84-786
Индуцированные цветовые диполи
в адрон-дейтронном рассеянии

При двукратной цветовой перезарядке налетающего адрона на разных нуклонах ядра может дифракционно возбудиться цветовой диполь, распад которого приводит к вылету одного из нуклонов мишени в заднюю полусферу. Вычислен вклад этого механизма в сечение процесса $pd \to p_B X$, который оказался велик в жесткой части импульсного спектра $p_B \geq$ 500 МэВ/с. В квантово-механическом подходе проанализирована зависимость существенных продольных расстояний от кумулятивного импульса. Рассмотрены предсказания для образования дибарионных резонансов с разделенным цветом, упругого pd-рассеяния назад, поляризационных эффектов и т.п.

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Kopeliovich B.Z., Niedermayer F. E2-84-786
Induced Colour Dipoles
in Hadron-Deuteron Scattering

An incident hadron can exchange colour on two different nucleons in a nucleus diffractively exciting a colour dipole - a dibaryon resonance with separated colour. The decay of this can lead to backward emission of one of the target nucleons. The contribution of this mechanism to the reaction cross section is calculated, and it turned out to be large in the hard part of the backward spectrum, $p_B \geq$ 500 MeV/c. Predictions for production of such dibaryon resonances, pd elastic backward scattering, and polarization effects are considered.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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