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ABSORPTIVE PART OF THE VVA TRIANGLE GRAPH: A COLLECTION OF FORMULAE

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1. INTRODUCTION

Let us consider the Fourier transform of a correlation function of three fermionic currents

$$iT_{a\mu\nu}(k,p) = \int d^4x\, d^4y\, e^{i(kx+py)} \langle 0|T(V_\mu(x)V_\nu(y)A_a(0))|0\rangle , \quad (1)$$

where V_μ and A_μ denote vector and axial currents resp.*:

$$V_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x), \quad A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x). \quad (2)$$

For convenience we suppress any internal-symmetry indices (which might eventually label the currents) since these are irrelevant for the subsequent discussion. In the one-loop approximation (i.e., using the free fields in (1), (2)) one gets formally

$$T_{a\mu\nu}(k,p) = \Gamma_{a\mu\nu}(k,p) + \Gamma_{a\nu\mu}(p,k), \quad (3)$$

where

$$\Gamma_{a\mu\nu}(k,p) = \int \frac{d^4r}{(2\pi)^4} \mathrm{Tr}\left(\frac{1}{\not r - \not k - m}\gamma_\mu \frac{1}{\not r - m}\gamma_\nu \frac{1}{\not r + \not p - m}\gamma_a\gamma_5\right). \quad (4)$$

The expressions (3), (4) represent the contribution of the familiar VVA triangle graph, which has been discussed in numerous papers in connection with the famous axial anomaly [1-3].

Let us first summarize several well-known facts concerning (3), (4) (see, e.g., ref. [4]): The definition of the integral in (4) requires special care; its contribution is finite (even without any explicit regularization) after the symmetric integrations, but ambiguous with respect to the shifts of the integration variables owing to the superficial linear divergence which persists in (4) even after performing the trace. In order to satisfy the usual vector Ward identities (gauge invariance)

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*Throughout the paper we employ the metric $g_{\mu\nu} = \mathrm{diag}(+---)$ and adopt the conventions $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\epsilon_{0123} = +1$.

$$k^{\mu}T_{\alpha\mu\nu}(k,p) = p^{\nu}T_{\alpha\mu\nu}(k,p) = 0, \qquad (5)$$

one has to make a shift $r \to r + (2+\beta)k + \beta p$ (with β being an arbitrary constant) in (4) and then integrate symmetrically around the origin. Another possibility would be to regularize the integral (4) from the very beginning in a gauge-invariant way (e.g., à la Pauli-Villars) and then let the cut-off go to infinity. If (5) holds, then the axial Ward identity picks up an anomalous term

$$q^{\alpha}T_{\alpha\mu\nu}(k,p) = 2mT_{\mu\nu}(k,p) + \frac{1}{2\pi^2}\epsilon_{\mu\nu\rho\sigma}k^{\rho}p^{\sigma}, \qquad (6)$$

where $q = k + p$ and the $T_{\mu\nu}(k,p)$(corresponding to the "normal term" which vanishes for $m \to 0$) is given by the formulae completely analogous to (3), (4) with γ_{α} replaced by the unit matrix. The integral representing $T_{\mu\nu}(k,p)$is convergent.

The amplitude (3) satisfying (5) has been first calculated by Rosenberg [5] who parametrized $T_{\alpha\mu\nu}(k,p)$ in terms of invariant amplitudes ("formfactors") and represented these in the form of the integrals over Feynman parameters (cf. also [2,3]). Dolgov and Zakharov [6], in an attempt to elucidate the origin of the axial anomaly, have calculated explicitly the imaginary ("absorptive") part of a formfactor for $k^2 = p^2 = 0$, $m \neq 0$ and shown that in the limit $m \to 0$ this is nonzero, proportional to $\delta(q^2)$ (see also ref. [7]). Nonvanishing of such an absorptive part in the massless limit is in turn related to the existence of the axial anomaly via a dispersion relation. The absorptive part of the VVA triangle diagram has been reconsidered later by Frishman et al. [8] in connection with a formal proof of the so-called 't Hooft anomaly condition [9] via dispersion relations (cf. also [10,11]). In ref. [8] the absorptive part of a particular formfactor has been given explicitly for $m = 0$, $k^2 = p^2 \le 0$.

In the present paper we extend and generalize the earlier results [6,8] by presenting the formulae for the absorptive part of the amplitude (3), (4) for $m \neq 0$ and $k^2 = p^2 \le 0$. There are essentially two independent formfactors describing the $T_{\alpha\mu\nu}(k,p)$ (if (5) is imposed as well) and we calculate the imaginary parts of both of them explicitly. The limit $m = 0$, $k^2 = p^2 = 0$ is also considered. The properties of the (pseudo) tensor basis employed for the decomposition of $T_{\alpha\mu\nu}(k,p)$ are discussed in a rather pedantic way, in view of an ambiguity in the definition of the formfactors encountered in the current literature (cf. [2,3,6-8,10,12]).

The paper is organized as follows: In Sec.2 the definition of the relevant invariant amplitudes is discussed. In Sec. 3

we present the general formulae for the absorptive parts of the invariant amplitudes for $m \neq 0$, $k^2 = p^2 \le 0$. In Sec.4 the limiting cases $k^2 = p^2 = 0$ and/or $m = 0$ are considered. We compare our results with those following from [3]; the comparison with [6,8] is straightforward. Section 5 contains a summary of the main results and some concluding remarks.

2. INVARIANT AMPLITUDES

We will summarize here the relevant formulae concerning the general tensor structure of the amplitude (1) (or (3) in particular). For the purpose of later references, our discussion will be somewhat more detailed than is usual in the standard literature (cf., e.g., [2-5,12]). We restrict ourselves to the external momenta k,p such that

$$k^2 = p^2; \qquad (7)$$

for definiteness, we shall therefore refer only to k^2 hereafter.

The amplitude (1) is a 3rd rank Lorentz pseudotensor, symmetric under the interchange (k,μ) and (p,ν).Further, one may impose the vector Ward identity (5). The $T_{\alpha\mu\nu}(k,p)$ may be then written as

$$T_{\alpha\mu\nu}(k,p) = \sum_{i=1}^{4} F_i(q^2,k^2)\,T^{(i)}_{\alpha\mu\nu}(k,p), \qquad (8)$$

where

$$T^{(1)}_{\alpha\mu\nu}(k,p) = \epsilon_{\mu\nu\rho\sigma}k^{\rho}p^{\sigma}q_{\alpha}, \quad T^{(2)}_{\alpha\mu\nu}(k,p) = (\epsilon_{\alpha\mu\rho\sigma}p_{\nu} - \epsilon_{\alpha\nu\rho\sigma}k_{\mu})k^{\rho}p^{\sigma},$$
$$T^{(3)}_{\alpha\mu\nu}(k,p) = (\epsilon_{\alpha\mu\rho\sigma}k_{\nu} - \epsilon_{\alpha\nu\rho\sigma}p_{\mu})k^{\rho}p^{\sigma}, \quad T^{(4)}_{\alpha\mu\nu}(k,p) = \epsilon_{\alpha\mu\nu\rho}(k^{\rho} - p^{\rho}), \qquad (9)$$

and the invariant amplitudes $F_i(q^2,k^2)$(occasionally called formfactors) satisfy

$$F_4 = k^2F_2 + (\tfrac{1}{2}q^2 - k^2)F_3, \qquad (10)$$

as a consequence of (5). However, the four (pseudo)tensors in (9) are linearly dependent [2,3,5,12] as the following identity holds

$$T^{(1)}_{\alpha\mu\nu}(k,p) + T^{(2)}_{\alpha\mu\nu}(k,p) + T^{(3)}_{\alpha\mu\nu}(k,p) + \frac{1}{2}q^2T^{(4)}_{\alpha\mu\nu}(k,p) = 0. \qquad (11)$$

To prove (11) one has to employ the identity (cf. [5,12])

$$g_{\lambda\alpha}\epsilon_{\mu\nu\rho\sigma} - g_{\mu\alpha}\epsilon_{\lambda\nu\rho\sigma} + g_{\nu\alpha}\epsilon_{\lambda\mu\rho\sigma} - g_{\rho\alpha}\epsilon_{\lambda\mu\nu\sigma} + g_{\sigma\alpha}\epsilon_{\lambda\mu\nu\rho} = 0. \qquad (12)$$

From (9) and (12) one obtains

$$T^{(2)}_{\alpha\mu\nu}(k,p)+T^{(3)}_{\alpha\mu\nu}(k,p)=(g_{\mu\lambda}\epsilon_{\nu\rho\sigma\alpha}-g_{\nu\lambda}\epsilon_{\mu\rho\sigma\alpha})(p^{\lambda}k^{\rho}p^{\sigma}+k^{\lambda}k^{\rho}p^{\sigma})=$$

$$=-(g_{\rho\lambda}\epsilon_{\mu\nu\sigma\alpha}-g_{\sigma\lambda}\epsilon_{\mu\nu\rho\alpha}+g_{\alpha\lambda}\epsilon_{\mu\nu\rho\sigma})q^{\lambda}k^{\rho}p^{\sigma}=$$

$$=-T^{(1)}_{\alpha\mu\nu}(k,p)-(k\cdot p)T^{(4)}_{\alpha\mu\nu}(k,p)-\epsilon_{\alpha\mu\nu\rho}(p^2k^{\rho}-k^2p^{\rho}),$$

and taking into account (7) we arrive at (11).

Thus, in view of (10) and (11), only two of the formfactors appearing on the r.h.s. of (8) are truly independent. In other words, the definition of the invariant amplitudes according to (8) through (10) is ambiguous owing to (11) and one must impose a subsidiary condition to fix them uniquely. Let us mention several conventions encountered in the current literature.

I. Following Bell and Jackiw [3] (see also [12]) one may require

$$F_2=-F_3\,. \tag{13}$$

It is easy to find the relation of any other set of the formfactors to those constrained by (13): Let the F_i, $i=1,\ldots,4$ in (8) be arbitrary; using (11), the decomposition (8) may be recast as

$$T_{\alpha\mu\nu}=(F_1-F_+)T^{(1)}_{\alpha\mu\nu}+F_-T^{(2)}_{\alpha\mu\nu}-F_-T^{(3)}_{\alpha\mu\nu}+(F_4-\frac{1}{2}q^2F_+)T^{(4)}_{\alpha\mu\nu}\,, \tag{14}$$

where

$$F_{\pm}=\frac{1}{2}(F_2\pm F_3)\,. \tag{15}$$

II. In (8) one may express $T^{(1)}$ in terms of $T^{(2)}$, $T^{(3)}$ and $T^{(4)}$ using (11), setting thus effectively (see, e.g., [2,5,8]),

$$F_1=0\,. \tag{16}$$

III. For $k^2=0$ a particularly convenient option would be to eliminate $T^{(3)}$ with the help of (11), setting thus

$$F_3=0; \tag{17}$$

according to (10) then also

$$F_4=0; \tag{18}$$

This is apparently the convention used in ref. [6]; note that in [6] the F_2 has been neglected as well, assuming tacitly that $T_{\alpha\mu\nu}$ is to be contracted with the polarization vectors of phy-

sical photons. Let us also remark that in ref. [7] (see p. 224 therein) the tensors $T^{(2)}$, $T^{(3)}$ and $T^{(4)}$ are discarded from the very beginning, since they do not in general satisfy (5) individually; however, in view of the preceding discussion such an approach is evidently implausible.

Let us also consider the 2nd rank pseudotensor $T_{\mu\nu}(k,p)$ appearing in the axial Ward identity (6). This is described by means of a single formfactor G, namely

$$T_{\mu\nu}(k,p)=G(q^2,k^2)\epsilon_{\mu\nu\rho\sigma}k^{\rho}p^{\sigma}; \tag{19}$$

(6) may be then recast as

$$q^2F_1-2F_4=2mG+\frac{1}{2\pi^2}\,. \tag{20}$$

Let us remind that $2mG\to 0$ for $m\to 0$.

If one considers the formfactors pertaining to the amplitude (3), (4) as functions of a complex variable q^2 at a fixed value of k^2, these possess a cut along the real axis, beginning at $q^2=4m^2$ (see [6,7]). The corresponding discontinuity of a formfactor F_j or the amplitude $T_{\alpha\mu\nu}$ resp., divided by 2i, will be called its absorptive (imaginary) part and denoted A_j or $A_{\alpha\mu\nu}$ resp. According to the preceding discussion we may write

$$A_{\alpha\mu\nu}(k,p)=\sum_{i=1}^{4}A_i(q^2;k^2,m^2)\,T^{(i)}_{\alpha\mu\nu}(k,p) \tag{21}$$

with $T^{(i)}_{\alpha\mu\nu}$, $i=1,\ldots,4$ given by (9)* .The $A_{\alpha\mu\nu}$ may be calculated with the help of the well-known Cutkosky rules [13]. Using such a method, one deals only with truly convergent integrals; thus, the integration variables may be shifted with impunity and the vector Ward identity (cf. (10))

$$A_4=k^2A_2+(\frac{1}{2}q^2-k^2)A_3\,, \tag{22}$$

should be satisfied automatically. For the same reason, the "normal" axial Ward identity must hold for the absorptive parts, i.e. (cf. ref. [8])

$$q^2A_1-2A_4=2mB\,, \tag{23}$$

where B is the absorptive part of the formfactor G (see (19)). Needless to say, (23) may be trivially recovered from (20), as the anomalous term in (20) is real.

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*For convenience, the A_i's will be frequently called simply invariant amplitudes (corresponding to $A_{\alpha\mu\nu}$).

3. GENERAL FORMULAE FOR $m \neq 0$, $k^2 \leq 0$

The direct evaluation of the quantity $2iA_{\alpha\mu\nu}(k,p)$ by means of the Cutkosky rules [13] amounts to the following replacement in (3), (4):

$$(\not r - \not k - m)^{-1} \to -2\pi i(\not r - \not k + m)\,\delta((r-k)^2 - m^2)\,\theta(k_0 - r_0),$$
$$(\not r + \not p - m)^{-1} \to -2\pi i(\not r + \not p + m)\,\delta((r+p)^2 - m^2)\,\theta(p_0 + r_0). \quad (24)$$

Note that for calculational convenience it is then also helpful to shift the integration variables, e.g., so that $r \to r - p$. Further, as we have already mentioned in the preceding section, (24) implies that $A_{\alpha\mu\nu}(k,p)$ can be nonzero only if $q^2 = (k+p)^2 > 4m^2 \geq 0$. One may therefore choose to work in the rest frame of $\vec{q}$, which greatly facilitates the calculation (cf. [14]). Thus, the external momenta may be conveniently parametrized as

$$k^\mu = (a,0,0,b), \quad p^\mu = (a,0,0,-b). \quad (25)$$

Then in turn

$$a^2 = \frac{1}{4}q^2, \quad b^2 = \frac{1}{4}q^2 - k^2. \quad (26)$$

Substituting (25) into the general decomposition (21), and using (9), one finds that among the components of $A_{\alpha\mu\nu}$ there are essentially three independent combinations of the invariant amplitudes A_i (we do not impose (22) beforehand and rather verify it by an explicit calculation, see below), e.g., A_{012}, A_{102} and A_{123}:

$$A_{012} = -4a^2bA_1 + 2bA_4, \quad (27a)$$

$$A_{102} = 2a^2b(A_2 + A_3) - 2bA_4, \quad (27b)$$

$$A_{123} = -2ab^2(A_2 - A_3), \quad (27c)$$

A straightforward calculation based on the Cutkosky rules (24) yields the following result for the components on the l.h.s. of (27):

$$A_{012} = -\frac{1}{4\pi}\theta(a^2 - m^2)\frac{m^2}{a}\ln\frac{a^2+b^2-2abR}{a^2+b^2+2abR}, \quad (28a)$$

$$A_{102} = \frac{1}{8\pi}\theta(a^2 - m^2)\left[\frac{3a^2-b^2}{b}R + \frac{(3a^2+b^2)(a^2-b^2)+4m^2b^2}{4ab^2}\ln\frac{a^2+b^2-2abR}{a^2+b^2+2abR}\right], \quad (28b)$$

$$A_{123} = -\frac{a}{b}A_{102}, \quad (28c)$$

where we have denoted

$$R = \sqrt{1 - \frac{m^2}{a^2}}. \quad (29)$$

From (27), (28) it is easy to see that the vector Ward identity (22) is satisfied. Indeed, using (28c), one gets immediately from (27b), (27c) $A_4 = (a^2 - b^2)A_2 + (a^2 + b^2)A_3$ and this coincides with (22) owing to (26). It can be shown that (27a), (28a) constitute the axial Ward identity (23).

As for the invariant amplitudes, we shall give them explicitly, e.g., for the convention II (i.e., $A_1 = 0$, cf. (16)); the passage to any other convention mentioned in Sec.2 may be easily accomplished with the help of the identity (11). For $A_1 = 0$ we obtain from (26), (27), (28) (we omit the ubiquitous factor $\theta(q^2 - 4m^2)$ hereafter)

$$A_2^{(II)}(q^2;k^2,m^2) = \frac{1}{2\pi}\frac{q^2-2k^2}{q^2}\left[\frac{q^2+2k^2}{(q^2-4k^2)^2}R + \frac{2k^2}{\sqrt{q^2}(q^2-4k^2)^{5/2}}\left(q^2-k^2+2m^2\frac{q^2-4k^2}{q^2-2k^2}\right)\ln S\right], \quad (30a)$$

$$A_3^{(II)}(q^2;k^2,m^2) = \frac{1}{2\pi}\frac{-2k^2}{q^2}\left[\frac{q^2+2k^2}{(q^2-4k^2)^2}R + \frac{2k^2(q^2-2k^2)}{\sqrt{q^2}(q^2-4k^2)^{5/2}}\left(\frac{q^2-k^2}{q^2-2k^2}+m^2\frac{q^2-4k^2}{2(k^2)^2}\right)\ln S\right], \quad (30b)$$

$$A_4^{(II)}(q^2;k^2,m^2) = -\frac{1}{2\pi}\frac{m^2}{\sqrt{q^2(q^2-4k^2)}}\ln S, \quad (30c)$$

where (cf. (26), (29))

$$R = \sqrt{1 - \frac{4m^2}{q^2}}, \quad S = \frac{q^2 - 2k^2 - R\sqrt{q^2(q^2-4k^2)}}{q^2 - 2k^2 + R\sqrt{q^2(q^2-4k^2)}}. \quad (31)$$

4. LIMITING CASES $k^2 = 0$ AND/OR $m = 0$

For $k^2 = 0$ and $m \neq 0$ the formulae (30) take the simple form

$$A_2^{(II)}(q^2;m^2) = \frac{1}{2\pi}\frac{R}{q^2}, \qquad (32a)$$

$$A_3^{(II)}(q^2;m^2) = -\frac{1}{\pi}\frac{m^2}{(q^2)^2}\ln\frac{1-R}{1+R}, \qquad (32b)$$

$$A_4^{(II)}(q^2;m^2) = \frac{1}{2}q^2 A_3^{(II)}(q^2;m^2). \qquad (32c)$$

For a comparison with the result of ref./6/ we must transform (32) with the help of the identity (11) to comply with the convention III (cf. (17), (18)/. We thus obtain

$$A_1^{(III)}(q^2;m^2) = \frac{1}{\pi}\frac{m^2}{(q^2)^2}\ln\frac{1-R}{1+R}, \qquad (33a)$$

$$A_2^{(III)}(q^2;m^2) = \frac{1}{2\pi}\left(\frac{R}{q^2} + \frac{2m^2}{(q^2)^2}\ln\frac{1-R}{1+R}\right), \qquad (33b)$$

$$A_3^{(III)}(q^2;m^2) = A_4^{(III)}(q^2;m^2) = 0. \qquad (33c)$$

Now (33a) coincides with the result presented in/6/ (up to an inessential overall factor and an obvious misprint occurring in the formula (14) of ref./6/).

As a pedagogical exercise we may also compare our results with those following (after some manipulations) from ref. /3/. To this end, we have to make a transformation to the convention I according to (14), (15):

$$A_1^{(I)}(q^2;m^2) = -\frac{1}{4\pi}\frac{1}{q^2}\left(R - \frac{2m^2}{q^2}\ln\frac{1-R}{1+R}\right), \qquad (34a)$$

$$A_2^{(I)}(q^2;m^2) = -A_3^{(I)}(q^2;m^2) = \frac{1}{4\pi}\frac{1}{q^2}\left(R + \frac{2m^2}{q^2}\ln\frac{1-R}{1+R}\right), \qquad (34b)$$

$$A_4^{(I)}(q^2;m^2) = \frac{1}{2}q^2 A_3^{(I)}(q^2;m^2). \qquad (34c)$$

In ref./3/ an integral representation of the formfactors $F_i(q^2;m^2)$ corresponding to (3), (4), (8) and (13) has been given, which for our purposes may be written as

$$F_1(q^2;m^2) = -\frac{1}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \frac{1}{m^2 - q^2 xy}[x+y-(x-y)^2], \qquad (35a)$$

$$F_2(q^2;m^2) = -F_3(q^2;m^2) = \frac{1}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \frac{1}{m^2 - q^2 xy}(1-x-y)(x+y), \qquad (35b)$$

$$F_4(q^2;m^2) = -\frac{1}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \frac{1}{m^2 - q^2 xy}[(\alpha-2)(m^2 - q^2 xy) + \frac{1}{2}q^2(1-x-y)(x+y)]. \qquad (35c)$$

In conformity with ref./4/, we have taken here into account the afore mentioned ambiguity with respect to a general shift $r \to r + (\alpha+\beta)k + \beta p$ in the integral (4). Notice that the vector Ward identity (10) is satisfied just for $\alpha = 2$. The formula (35c) demonstrates once again that the ambiguity (the dependence on α) resides solely in the real part of F_4. Performing now in (35) the integration over y, one gets for the imaginary parts (dropping the manifestly real terms):

$$\mathrm{Im}\,F_1(q^2;m^2) =$$

$$= -\frac{1}{4\pi^2}\frac{1}{q^2}\,\mathrm{Im}\int_0^1 dx\left[x-1+\frac{m^2}{(q^2)^2}\,\frac{m^2-q^2x(2x+1)}{x^3}\right]\ln\left[1-\frac{q^2}{m^2}x(1-x)\right], \qquad (36a)$$

$$\mathrm{Im}\,F_2(q^2;m^2) = -\mathrm{Im}\,F_3(q^2;m^2) =$$

$$= \frac{1}{4\pi^2}\frac{1}{q^2}\,\mathrm{Im}\int_0^1 dx\left[x-1+\frac{m^2}{(q^2)^2}\,\frac{m^2+q^2x(2x-1)}{x^3}\right]\ln\left[1-\frac{q^2}{m^2}x(1-x)\right], \qquad (36b)$$

$$\mathrm{Im}\,F_4(q^2;m^2) = \frac{1}{2}q^2 F_3(q^2;m^2). \qquad (36c)$$

Obviously, the logarithm in (36) may develop a nonzero imaginary part only if $q^2 > 4m^2$ and $x \in (x_-, x_+)$, where $x_\pm = (1 \pm R)/2$. Such an imaginary part may be then set equal to $-\pi$ according to the rule $m^2 \to m^2 - i0$. The integration over x in (36) is then elementary and one arrives at the desired result

$$\mathrm{Im}\,F_1(q^2;m^2) = A_1(q^2;m^2), \quad \mathrm{Im}\,F_2(q^2;m^2) = A_2(q^2;m^2),$$

where A_1, A_2 are given by (34a), (34b).

from the absorptive parts $A_i(q^2)$. Obviously, it is most convenient to consider only such sets (44), for which the integrals in the unsubtracted dispersion relations

$$F_i^{(un)}(q^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{A_i(t)}{t - q^2} dt, \quad i = 1, \ldots, 4 \tag{45}$$

converge (cf. [6,8]). This is the case, e.g., for the conventions II and III (see (30), (32)), while the convention I certainly does not meet such a requirement (see (34b), (34c) or (39) resp.). Note that similarly inconvenient would be the option $A_2 = 0$. On the basis of (10), (22), and (45) it is then easy to realize that in the technical sense one may encounter just two alternatives, when defining the formfactors $F_i(q^2)$ by means of the dispersion relations: First, $A_3 = 0$ (the convention III); then $F_i^{(un)}(q^2)$ given by (45) satisfy automatically both (10) and (20), and the anomalous term in (20) is due to (38) (cf. [6]). Second, $A_3 \neq 0$; then one has to modify the definition of the F_4 in order to satisfy (10), namely (see also [8])

$$F_4(q^2) = F_4^{(un)}(q^2) - \frac{1}{2\pi} \int_{4m^2}^{\infty} A_3(t)\, dt .$$

The anomalous term in (20) is in such a case due to (37b) or (43) resp. More precisely, the result

$$q^2 F_1(q^2; k^2, m^2) - 2F_4(q^2; k^2, m^2) \xrightarrow[m \to 0]{} \frac{1}{2\pi^2}$$

may be reproduced (at least for $k^2 \leq 0$ considered in this paper) with the help of the dispersion relations (45), (46) in the following way (adopting for definiteness the convention II): For an arbitrary $k^2 < 0$ one uses (40a) and (cf. (40c), (41), (43))

$$\int_0^{\infty} A_3(t; k^2, 0)\, dt = \frac{1}{2\pi} .$$

For $k^2 = 0$ one has to employ (37c) and the fact that (cf. (32b), (37b))

$$\int_{4m^2}^{\infty} A_3(t; 0, m^2)\, dt = \frac{1}{2\pi} ,$$

for any $m \neq 0$.

The peculiar behaviour of the relevant absorptive parts $A_i(q^2; k^2, m^2)$ for $k^2 = m^2 = 0$ (see (37b), (38), (43)) has been

established so far by means of particular limiting procedures, namely $k^2 \to 0$ followed by $m \to 0$ and vice versa. It would be interesting to use the formulae (30b), (31) for an investigation of more general limiting procedures, e.g., $m \to 0$, $k^2 \to 0$ simultaneously. This point will be discussed elsewhere.

REFERENCES

1. Schwinger J. Phys.Rev., 1951, 82, p. 664.
2. Adler S.L. Phys.Rev., 1969, 177, p. 2426.
3. Bell J.S., Jackiw R. Nuovo Cimento, 1969, 60A, p. 47.
4. Jackiw R. In: Lectures on Current Algebra and Its Applications, eds. D.Gross, R.Jackiw, S.Treiman (Princeton University Press, Princeton 1972).
5. Rosenberg L. Phys.Rev., 1963, 129, p. 2786.
6. Dolgov A.D., Zakharov V.I. Nucl.Phys., 1971, B27, p. 525.
7. Huang K. Quarks, Leptons and Gauge Fields. Chapter XI (World Scientific, Singapore 1982).
8. Frishman Y. et al. Nucl.Phys., 1981, B177, p. 157.
9. 't Hooft G. In: Recent Developments in Gauge Theories, Cargèse 1979, p. 135, eds. G.'t Hooft et al. (Plenum Press, New York 1980).
10. Peskin M.E. In: Recent Developments in Quantum Field Theory and Statistical Mechanics, Les Houches 1982, ed. R.Stora (North-Holland, Amsterdam 1983).
11. Coleman S., Grossman B. Nucl.Phys., 1982, B203, p. 205.
12. Llewellyn Smith C.H. In: Quantum Flavordynamics, Quantum Chromodynamics and Unified Theories, Boulder,1979, eds. K.T.Mahanthappa, J.Randa (Plenum Press, New York 1980).
13. Cutkosky R.E. J.Math.Phys., 1960, 1, p. 429; Eden R. et al. The Analytic S-Matrix (Cambridge University Press, Cambridge 1966).
14. Berestetskii V.B., Lifshitz E.M., Pitaevskii L.P. Quantum Electrodynamics, p. 567 (Nauka, Moscow 1980) (in Russian).

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Абсорбтивная часть треугольной диаграммы типа VVA: сводка формул

Представляется сводка некоторых формул для абсорбтивной части известной треугольной диаграммы с двумя векторными и одной аксиальной вершиной. Детально обсуждаются свойства псевдотензорного базиса, используемого для определения соответствующих инвариантных амплитуд,и неоднозначность такого определения. Существуют две независимые инвариантные амплитуды, и мы представляем обе в явном виде. Таким образом, мы обобщаем результаты прежних работ других авторов по этой теме. Обсуждаются также предельные случаи нулевой массы фермиона или светоподобных внешних импульсов. Рассмотрен также вопрос о восстановлении аксиальной аномалии при помощи дисперсионных соотношений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Absorptive Part of the VVA Triangle Graph: A Collection of Formulae

A set of formulae is presented for the absorptive part of the familiar VVA triangle graph. Properties of the pseudotensor basis used for the definition of the corresponding invariant amplitudes and the ambiguity of such a definition are discussed in detail. There are essentially two independent invariant amplitudes and we give both of them explicitly. We thereby extend and generalize results of the previous treatments dealing with the subject. The limiting cases of vanishing fermion mass and/or some of the external momenta being lightlike are briefy discussed, as well as the recovering of the axial anomaly via dispersion relations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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